

Populationsdynamik

1 Erste Vorgehensweise

```
Remove["Global`*"]
```

Bemerkung zur Populationsmatrix B

```
B = {{3.5, -25}, {0.05, 0.5}};  
B // MatrixForm
```

$$\begin{pmatrix} 3.5 & -25 \\ 0.05 & 0.5 \end{pmatrix}$$

```
B = {{7/2, -25}, {5/100, 5/10}};  
B // MatrixForm
```

$$\begin{pmatrix} \frac{7}{2} & -25 \\ \frac{1}{20} & \frac{1}{2} \end{pmatrix}$$

```
Eigenvalues[B]
```

```
{3, 1}
```

```
Eigenvectors[B]
```

```
{{50, 1}, {10, 1}}
```

a) Negative Resultate

```

x[1]={1,1}; B={{3.5,-25},{0.05,0.5}};
x[n_]:= B.x[n-1];
z[k_]:=Prepend[Append[x[k],x[k][[1]]/x[k][[2]]],k];
Prepend[Table[z[k],{k,1,20}],{"Nr.", "x", "y", "Quot.
x/y"}]//N//MatrixForm

```

| Nr. | x | y | Quot. x/y |
|-----|---------------------------|------------------------|-----------|
| 1. | 1. | 1. | 1. |
| 2. | -21.5 | 0.55 | -39.0909 |
| 3. | -89. | -0.8 | 111.25 |
| 4. | -291.5 | -4.85 | 60.1031 |
| 5. | -899. | -17. | 52.8824 |
| 6. | -2721.5 | -53.45 | 50.9167 |
| 7. | -8189. | -162.8 | 50.301 |
| 8. | -24591.5 | -490.85 | 50.0998 |
| 9. | -73799. | -1475. | 50.0332 |
| 10. | -221422. | -4427.45 | 50.0111 |
| 11. | -664289. | -13284.8 | 50.0037 |
| 12. | -1.99289×10^6 | -39856.9 | 50.0012 |
| 13. | -5.9787×10^6 | -119573. | 50.0004 |
| 14. | -1.79361×10^7 | -358721. | 50.0001 |
| 15. | -5.38084×10^7 | -1.07617×10^6 | 50. |
| 16. | -1.61425×10^8 | -3.2285×10^6 | 50. |
| 17. | -4.84276×10^8 | -9.68551×10^6 | 50. |
| 18. | -1.45283×10^9 | -2.90565×10^7 | 50. |
| 19. | -4.35848×10^9 | -8.71696×10^7 | 50. |
| 20. | -1.30754×10^{10} | -2.61509×10^8 | 50. |

```

x[1]={5,1}; B={{3.5,-25},{0.05,0.5}};
x[n_]:=B.x[n-1];
z[k_]:=Prepend[Append[x[k],x[k][[1]]/x[k][[2]]],k];
Prepend[Table[z[k],{k,1,20}],{"Nr.", "x", "y", "Quot.
x/y"}]//N//MatrixForm

```

| Nr. | x | y | Quot. x/y |
|-----|------------------------|----------------------------|--------------------------|
| 1. | 5. | 1. | 5. |
| 2. | -7.5 | 0.75 | -10. |
| 3. | -45. | -2.08167×10^{-17} | 2.16173×10^{18} |
| 4. | -157.5 | -2.25 | 70. |
| 5. | -495. | -9. | 55. |
| 6. | -1507.5 | -29.25 | 51.5385 |
| 7. | -4545. | -90. | 50.5 |
| 8. | -13657.5 | -272.25 | 50.1653 |
| 9. | -40995. | -819. | 50.0549 |
| 10. | -123008. | -2459.25 | 50.0183 |
| 11. | -369045. | -7380. | 50.0061 |
| 12. | -1.10716×10^6 | -22142.3 | 50.002 |
| 13. | -3.3215×10^6 | -66429. | 50.0007 |
| 14. | -9.96451×10^6 | -199289. | 50.0002 |
| 15. | -2.98935×10^7 | -597870. | 50.0001 |
| 16. | -8.96807×10^7 | -1.79361×10^6 | 50. |
| 17. | -2.69042×10^8 | -5.38084×10^6 | 50. |
| 18. | -8.07126×10^8 | -1.61425×10^7 | 50. |
| 19. | -2.42138×10^9 | -4.84276×10^7 | 50. |
| 20. | -7.26413×10^9 | -1.45283×10^8 | 50. |

b) Stabilität

```
x[1]={10,1}; B={{3.5,-25},{0.05,0.5}};  
x[n_]:=B.x[n-1];  
z[k_]:=Prepend[Append[x[k],x[k][[1]]/x[k][[2]]],k];  
Prepend[Table[z[k],{k,1,20}],{"Nr.", "x", "y", "Quot.  
x/y"}]//N//MatrixForm
```

```
(Nr.  x   y   Quot.  x/y )  
1.  10.  1.  10.  
2.  10.  1.  10.  
3.  10.  1.  10.  
4.  10.  1.  10.  
5.  10.  1.  10.  
6.  10.  1.  10.  
7.  10.  1.  10.  
8.  10.  1.  10.  
9.  10.  1.  10.  
10. 10.  1.  10.  
11. 10.  1.  10.  
12. 10.  1.  10.  
13. 10.  1.  10.  
14. 10.  1.  10.  
15. 10.  1.  10.  
16. 10.  1.  10.  
17. 10.  1.  10.  
18. 10.  1.  10.  
19. 10.  1.  10.  
20. 10.  1.  10.)
```

c) Zunahme, positive Resultate, Quotient konvergiert von unten her

```
x[1]={25,1}; B={{3.5,-25},{0.05,0.5}};  
x[n_]:=B.x[n-1];  
z[k_]:=Prepend[Append[x[k],x[k][[1]]/x[k][[2]]],k];  
Prepend[Table[z[k],{k,1,20}],{"Nr.", "x", "y", "Quot.  
x/y"}]//N//MatrixForm
```

| Nr. | x | y | Quot. x/y |
|-----|--------------------------|-----------------------|-----------|
| 1. | 25. | 1. | 25. |
| 2. | 62.5 | 1.75 | 35.7143 |
| 3. | 175. | 4. | 43.75 |
| 4. | 512.5 | 10.75 | 47.6744 |
| 5. | 1525. | 31. | 49.1935 |
| 6. | 4562.5 | 91.75 | 49.7275 |
| 7. | 13675. | 274. | 49.9088 |
| 8. | 41012.5 | 820.75 | 49.9695 |
| 9. | 123025. | 2461. | 49.9898 |
| 10. | 369063. | 7381.75 | 49.9966 |
| 11. | 1.10718×10^6 | 22144. | 49.9989 |
| 12. | 3.32151×10^6 | 66430.8 | 49.9996 |
| 13. | 9.96453×10^6 | 199291. | 49.9999 |
| 14. | 2.98936×10^7 | 597872. | 50. |
| 15. | 8.96807×10^7 | 1.79361×10^6 | 50. |
| 16. | 2.69042×10^8 | 5.38084×10^6 | 50. |
| 17. | 8.07126×10^8 | 1.61425×10^7 | 50. |
| 18. | 2.42138×10^9 | 4.84276×10^7 | 50. |
| 19. | 7.26413×10^9 | 1.45283×10^8 | 50. |
| 20. | 2.17924×10^{10} | 4.35848×10^8 | 50. |

d) Proportionale Zunahme

```

x[1]={50,1}; B={{3.5,-25},{0.05,0.5}};
x[n_]:=B.x[n-1];
z[k_]:=Prepend[Append[x[k],x[k][[1]]/x[k][[2]]],k];
Prepend[Table[z[k],{k,1,20}],{"Nr.", "x", "y", "Quot.
x/y"}]//N//MatrixForm

```

| Nr. | x | y | Quot. x/y |
|-----|--------------------------|-----------------------|-----------|
| 1. | 50. | 1. | 50. |
| 2. | 150. | 3. | 50. |
| 3. | 450. | 9. | 50. |
| 4. | 1350. | 27. | 50. |
| 5. | 4050. | 81. | 50. |
| 6. | 12150. | 243. | 50. |
| 7. | 36450. | 729. | 50. |
| 8. | 109350. | 2187. | 50. |
| 9. | 328050. | 6561. | 50. |
| 10. | 984150. | 19683. | 50. |
| 11. | 2.95245×10^6 | 59049. | 50. |
| 12. | 8.85735×10^6 | 177147. | 50. |
| 13. | 2.65721×10^7 | 531441. | 50. |
| 14. | 7.97162×10^7 | 1.59432×10^6 | 50. |
| 15. | 2.39148×10^8 | 4.78297×10^6 | 50. |
| 16. | 7.17445×10^8 | 1.43489×10^7 | 50. |
| 17. | 2.15234×10^9 | 4.30467×10^7 | 50. |
| 18. | 6.45701×10^9 | 1.2914×10^8 | 50. |
| 19. | 1.9371×10^{10} | 3.8742×10^8 | 50. |
| 20. | 5.81131×10^{10} | 1.16226×10^9 | 50. |

e) Zunahme, positive Resultate, Quotient konvergiert von oben her

```

x[1]={70,1}; B={{3.5,-25},{0.05,0.5}};
x[n_]:=B.x[n-1];
z[k_]:=Prepend[Append[x[k],x[k][[1]]/x[k][[2]]],k];
Prepend[Table[z[k],{k,1,20}],{"Nr.", "x", "y", "Quot.
x/y"}]//N//MatrixForm

```

| Nr. | x | y | Quot. x/y |
|-----|--------------------------|-----------------------|-----------|
| 1. | 70. | 1. | 70. |
| 2. | 220. | 4. | 55. |
| 3. | 670. | 13. | 51.5385 |
| 4. | 2020. | 40. | 50.5 |
| 5. | 6070. | 121. | 50.1653 |
| 6. | 18220. | 364. | 50.0549 |
| 7. | 54670. | 1093. | 50.0183 |
| 8. | 164020. | 3280. | 50.0061 |
| 9. | 492070. | 9841. | 50.002 |
| 10. | 1.47622×10^6 | 29524. | 50.0007 |
| 11. | 4.42867×10^6 | 88573. | 50.0002 |
| 12. | 1.3286×10^7 | 265720. | 50.0001 |
| 13. | 3.98581×10^7 | 797161. | 50. |
| 14. | 1.19574×10^8 | 2.39148×10^6 | 50. |
| 15. | 3.58723×10^8 | 7.17445×10^6 | 50. |
| 16. | 1.07617×10^9 | 2.15234×10^7 | 50. |
| 17. | 3.2285×10^9 | 6.45701×10^7 | 50. |
| 18. | 9.68551×10^9 | 1.9371×10^8 | 50. |
| 19. | 2.90565×10^{10} | 5.81131×10^8 | 50. |
| 20. | 8.71696×10^{10} | 1.74339×10^9 | 50. |

```
x[1]={100,1}; B={{3.5,-25},{0.05,0.5}};
x[n_]:=B.x[n-1];
z[k_]:=Prepend[Append[x[k],x[k][[1]]/x[k][[2]]],k];
Prepend[Table[z[k],{k,1,20}],{"Nr.", "x", "y", "Quot.
x/y"}]//N//MatrixForm
```

| Nr. | x | y | Quot. x/y |
|-----|--------------------------|-----------------------|-----------|
| 1. | 100. | 1. | 100. |
| 2. | 325. | 5.5 | 59.0909 |
| 3. | 1000. | 19. | 52.6316 |
| 4. | 3025. | 59.5 | 50.8403 |
| 5. | 9100. | 181. | 50.2762 |
| 6. | 27325. | 545.5 | 50.0917 |
| 7. | 82000. | 1639. | 50.0305 |
| 8. | 246025. | 4919.5 | 50.0102 |
| 9. | 738100. | 14761. | 50.0034 |
| 10. | 2.21433×10^6 | 44285.5 | 50.0011 |
| 11. | 6.643×10^6 | 132859. | 50.0004 |
| 12. | 1.9929×10^7 | 398580. | 50.0001 |
| 13. | 5.97871×10^7 | 1.19574×10^6 | 50. |
| 14. | 1.79361×10^8 | 3.58723×10^6 | 50. |
| 15. | 5.38084×10^8 | 1.07617×10^7 | 50. |
| 16. | 1.61425×10^9 | 3.2285×10^7 | 50. |
| 17. | 4.84276×10^9 | 9.68551×10^7 | 50. |
| 18. | 1.45283×10^{10} | 2.90565×10^8 | 50. |
| 19. | 4.35848×10^{10} | 8.71696×10^8 | 50. |
| 20. | 1.30754×10^{11} | 2.61509×10^9 | 50. |

2 Andere Vorgehensweise

```
Remove[x]
```

```
x[1] = 25; y[1] = 1;
x[n_] := (B.{x[n-1], y[n-1]})[[1]];
y[n_] := (B.{x[n-1], y[n-1]})[[2]];
```

```
{x[1], y[1]}
```

```
{25, 1}
```

```
{x[2], y[2]} // N
```

```
{62.5, 1.75}
```

```
Table[{x[k], y[k]}, {k, 1, 15}] // N // MatrixForm (* Langsamer *)
```

```
( 25.          1.  
 62.5         1.75  
 175.         4.  
 512.5        10.75  
 1525.        31.  
 4562.5       91.75  
 13675.       274.  
 41012.5      820.75  
 123025.      2461.  
 369063.      7381.75  
 1.10718×106 22144.  
 3.32151×106 66430.8  
 9.96453×106 199291.  
 2.98936×107 597872.  
 8.96807×107 1.79361×106 )
```