

Lösungen

1. Schwingungen

Materialbereitstellung

```

Remove["Global`*"];
T = 5;
A[1] = 2;
A[2] = 3;
α[1] = π/8;
α[2] = 3 π/6;
ω = 2 π/T;
φ[t_,k_]:= ω * t + α[k];
f[t_,k_]:= A[k] Cos[φ[t,k]];
φ[A1_,φ1_,A2_,φ2_]:= ArcTan[(A1 Sin[φ1]+A2 Sin[φ2])/(A1 Cos[φ1]+A2 Cos[φ2])];
A[A1_,A2_,φ1_,φ2_]:= Sqrt[A1^2+A2^2+2 A1 A2 Cos[φ2-φ1]];
f[A_,φ_]:= A Cos[φ]

```

a

```
A[A[1],A[1],φ[t,1],φ[t,2]] //Simplify
```

$$\sqrt{8 + 8 \cos\left[\frac{3\pi}{8}\right]}$$

```
N[%]
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3.32588

```
φ[A[1],A[1],φ[t,1],φ[t,2]] //Simplify
```

$$-\text{ArcTan}\left[\frac{\pi (5 + 16 t) \cos\left[\frac{2\pi t}{5}\right] + 80 \sin[2]}{-80 \cos[2] + \pi (5 + 16 t) \sin\left[\frac{2\pi t}{5}\right]}\right]$$

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N[%]
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$$-1. \text{ArcTan}\left[\frac{72.7438 + 3.14159 (5. + 16. t) \cos[1.25664 t]}{33.2917 + 3.14159 (5. + 16. t) \sin[1.25664 t]}\right]$$

```
f[A[A[1],A[1],φ[t,1],φ[t,2]], φ[A[1],A[1],φ[t,1],φ[t,2]]] //Simplify
```

$$\frac{2 \sqrt{2 (1 + \cos\left[\frac{3\pi}{8}\right])}}{\sqrt{1 + \frac{(\pi (5 + 16 t) \cos\left[\frac{2\pi t}{5}\right] + 80 \sin[2])^2}{(-80 \cos[2] + \pi (5 + 16 t) \sin\left[\frac{2\pi t}{5}\right])^2}}}$$

f[A[A[1],A[1],φ[t,1],φ[t,2]], φ[A[1],A[1],φ[t,1],φ[t,2]]]/.t->0 //Simplify

$$-32 \cos[2] \sqrt{\frac{2 (1 + \cos[\frac{3\pi}{8}])}{256 + \pi^2 + 32 \pi \sin[2]}}$$

N[%]

1.17157

f[A[A[1],A[1],φ[t,1],φ[t,2]], φ[A[1],A[1],φ[t,1],φ[t,2]]]/.t->Pi/6 //Simplify

$$\sqrt{\frac{2}{1 + \frac{\left(\frac{1}{120} \pi (15+8 \pi) \cos\left[\frac{\pi^2}{15}\right] + 2 \sin[2]\right)^2}{\left(2 \cos[2] - \frac{1}{120} \pi (15+8 \pi) \sin\left[\frac{\pi^2}{15}\right]\right)^2}}}{2 (1 + \cos\left[\frac{3\pi}{8}\right])}}$$

N[%]

1.61738

f[A[A[1],A[1],φ[t,1],φ[t,2]], φ[A[1],A[1],φ[t,1],φ[t,2]]]/.t->Pi/4 //Simplify

$$\sqrt{\frac{2}{1 + \frac{\left(\frac{1}{40} \pi (5+4 \pi) \cos\left[\frac{\pi^2}{10}\right] + 2 \sin[2]\right)^2}{\left(2 \cos[2] - \frac{1}{40} \pi (5+4 \pi) \sin\left[\frac{\pi^2}{10}\right]\right)^2}}}{2 (1 + \cos\left[\frac{3\pi}{8}\right])}}$$

N[%]

2.0275

f[A[A[1],A[1],φ[t,1],φ[t,2]], φ[A[1],A[1],φ[t,1],φ[t,2]]]/.t->Pi/3 //Simplify

$$\sqrt{\frac{2}{1 + \frac{\left(\frac{1}{120} \pi (15+16 \pi) \cos\left[\frac{2\pi^2}{15}\right] + 2 \sin[2]\right)^2}{\left(2 \cos[2] - \frac{1}{120} \pi (15+16 \pi) \sin\left[\frac{2\pi^2}{15}\right]\right)^2}}}{2 (1 + \cos\left[\frac{3\pi}{8}\right])}}$$

N[%]

2.4661

b

A[A[1],A[2],φ[t,1],φ[t,2]] //Simplify

$$\sqrt{13 + 12 \cos\left[\frac{3\pi}{8}\right]}$$

N[%]

4.19431

φ[A[1],A[2],φ[t,1],φ[t,2]] //Simplify

$$-\text{ArcTan}\left[\frac{\pi (5 + 16 t) \cos\left[\frac{2\pi t}{5}\right] + 80 \sin[3]}{-80 \cos[3] + \pi (5 + 16 t) \sin\left[\frac{2\pi t}{5}\right]}\right]$$

N[%]

$$-1. \operatorname{ArcTan}\left[\frac{11.2896 + 3.14159 (5. + 16. t) \operatorname{Cos}[1.25664 t]}{79.1994 + 3.14159 (5. + 16. t) \operatorname{Sin}[1.25664 t]}\right]$$

f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]] //Simplify

$$\frac{\sqrt{13 + 12 \operatorname{Cos}\left[\frac{3\pi}{8}\right]}}{\sqrt{1 + \frac{(\pi (5+16 t) \operatorname{Cos}\left[\frac{2\pi t}{5}\right] + 80 \operatorname{Sin}[3])^2}{(-80 \operatorname{Cos}[3] + \pi (5+16 t) \operatorname{Sin}\left[\frac{2\pi t}{5}\right])^2}}}}$$

f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]]/.t->0 //Simplify

$$-16 \operatorname{Cos}[3] \sqrt{\frac{13 + 12 \operatorname{Cos}\left[\frac{3\pi}{8}\right]}{256 + \pi^2 + 32 \pi \operatorname{Sin}[3]}}$$

N[%]

3.96999

f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]]/.t->Pi/6 //Simplify

$$\frac{1}{\sqrt{1 + \frac{\left(\frac{1}{120} \pi (15+8 \pi) \operatorname{Cos}\left[\frac{\pi^2}{15}\right] + 2 \operatorname{Sin}[3]\right)^2}{\left(2 \operatorname{Cos}[3] - \frac{1}{120} \pi (15+8 \pi) \operatorname{Sin}\left[\frac{\pi^2}{15}\right]\right)^2}}}}$$

N[%]

3.86067

f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]]/.t->Pi/4 //Simplify

$$\frac{1}{\sqrt{1 + \frac{\left(\frac{1}{40} \pi (5+4 \pi) \operatorname{Cos}\left[\frac{\pi^2}{10}\right] + 2 \operatorname{Sin}[3]\right)^2}{\left(2 \operatorname{Cos}[3] - \frac{1}{40} \pi (5+4 \pi) \operatorname{Sin}\left[\frac{\pi^2}{10}\right]\right)^2}}}}$$

N[%]

3.97943

f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]]/.t->Pi/3 //Simplify

$$\frac{1}{\sqrt{1 + \frac{\left(\frac{1}{120} \pi (15+16 \pi) \operatorname{Cos}\left[\frac{2\pi^2}{15}\right] + 2 \operatorname{Sin}[3]\right)^2}{\left(2 \operatorname{Cos}[3] - \frac{1}{120} \pi (15+16 \pi) \operatorname{Sin}\left[\frac{2\pi^2}{15}\right]\right)^2}}}}$$

N[%]

4.11581

c Sin[] = Cos[- /2]

$$\alpha[1] = \pi/8 - \text{Pi}/2;$$

A[A[1],A[2],φ[t,1],φ[t,2]] //Simplify

$$\sqrt{13 + 12 \operatorname{Cos}\left[\frac{7\pi}{8}\right]}$$

N[%]

1.38327

φ[A[1],A[2],φ[t,1],φ[t,2]] //Simplify

$$-\operatorname{ArcTan}\left[\frac{\pi(-15 + 16t) \operatorname{Cos}\left[\frac{2\pi t}{5}\right] + 80 \operatorname{Sin}[3]}{-80 \operatorname{Cos}[3] + \pi(-15 + 16t) \operatorname{Sin}\left[\frac{2\pi t}{5}\right]}\right]$$

N[%]

$$-1. \operatorname{ArcTan}\left[\frac{11.2896 + 3.14159(-15. + 16. t) \operatorname{Cos}[1.25664 t]}{79.1994 + 3.14159(-15. + 16. t) \operatorname{Sin}[1.25664 t]}\right]$$

f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]] //Simplify

$$\frac{\sqrt{13 + 12 \operatorname{Cos}\left[\frac{7\pi}{8}\right]}}{\sqrt{1 + \frac{(\pi(-15+16t) \operatorname{Cos}\left[\frac{2\pi t}{5}\right] + 80 \operatorname{Sin}[3])^2}{(-80 \operatorname{Cos}[3] + \pi(-15+16t) \operatorname{Sin}\left[\frac{2\pi t}{5}\right])^2}}}}$$

f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]]/.t->0 //Simplify

$$\sqrt{\frac{13 + 12 \operatorname{Cos}\left[\frac{7\pi}{8}\right]}{1 + \frac{1}{256} (3\pi \operatorname{Sec}[3] - 16 \operatorname{Tan}[3])^2}}$$

N[%]

1.26028

f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]]/.t->Pi/6 //Simplify

$$\frac{1}{\sqrt{\frac{1 + \frac{\left(\frac{1}{120}\pi(-45+8\pi) \operatorname{Cos}\left[\frac{\pi^2}{15}\right] + 2 \operatorname{Sin}[3]\right)^2}{\left(2 \operatorname{Cos}[3] - \frac{1}{120}\pi(-45+8\pi) \operatorname{Sin}\left[\frac{\pi^2}{15}\right]\right)^2}}{13 + 12 \operatorname{Cos}\left[\frac{7\pi}{8}\right]}}}}$$

N[%]

1.37911

f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]]/.t->Pi/4 //Simplify

$$\frac{1}{\sqrt{\frac{1 + \frac{\left(\frac{1}{40}\pi(-15+4\pi) \operatorname{Cos}\left[\frac{\pi^2}{10}\right] + 2 \operatorname{Sin}[3]\right)^2}{\left(2 \operatorname{Cos}[3] - \frac{1}{40}\pi(-15+4\pi) \operatorname{Sin}\left[\frac{\pi^2}{10}\right]\right)^2}}{13 + 12 \operatorname{Cos}\left[\frac{7\pi}{8}\right]}}}}$$

N[%]

1.37679

f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]]/.t->Pi/3 //Simplify

$$\frac{1}{\sqrt{1 + \frac{\left(\frac{1}{120} \pi (-45+16 \pi) \cos\left[\frac{2 \pi^2}{15}\right] + 2 \sin[3]\right)^2}{\left(2 \cos[3] - \frac{1}{120} \pi (-45+16 \pi) \sin\left[\frac{2 \pi^2}{15}\right]\right)^2}}}$$

N[%]

1.36797

d 1 neu definiert

α[1] = -π/8;

A[A[1],A[2],φ[t,1],φ[t,2]] //Simplify

$$\sqrt{13 + 12 \cos\left[\frac{5 \pi}{8}\right]}$$

N[%]

2.89962

φ[A[1],A[2],φ[t,1],φ[t,2]] //Simplify

$$-\text{ArcTan}\left[\frac{\pi (-5 + 16 t) \cos\left[\frac{2 \pi t}{5}\right] + 80 \sin[3]}{-80 \cos[3] + \pi (-5 + 16 t) \sin\left[\frac{2 \pi t}{5}\right]}\right]$$

N[%]

$$-1. \text{ArcTan}\left[\frac{11.2896 + 3.14159 (-5. + 16. t) \cos[1.25664 t]}{79.1994 + 3.14159 (-5. + 16. t) \sin[1.25664 t]}\right]$$

f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]] //Simplify

$$\frac{\sqrt{13 + 12 \cos\left[\frac{5 \pi}{8}\right]}}{\sqrt{1 + \frac{\left(\pi (-5+16 t) \cos\left[\frac{2 \pi t}{5}\right] + 80 \sin[3]\right)^2}{\left(-80 \cos[3] + \pi (-5+16 t) \sin\left[\frac{2 \pi t}{5}\right]\right)^2}}}$$

f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]]/.t->0 //Simplify

$$-16 \cos[3] \sqrt{\frac{13 + 12 \cos\left[\frac{5 \pi}{8}\right]}{256 + \pi^2 - 32 \pi \sin[3]}}$$

N[%]

2.89512

f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]]/.t->Pi/6 //Simplify

$$\frac{1}{\sqrt{1 + \frac{\left(\frac{1}{120} \pi (-15+8 \pi) \cos\left[\frac{\pi^2}{15}\right] + 2 \sin[3]\right)^2}{\left(2 \cos[3] - \frac{1}{120} \pi (-15+8 \pi) \sin\left[\frac{\pi^2}{15}\right]\right)^2}}}$$

N[%]

2.82601

f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]]/.t->Pi/4 //Simplify

$$\frac{1}{\sqrt{\frac{1 + \frac{\left(\frac{1}{40} \pi (-5+4\pi) \cos\left[\frac{\pi^2}{10}\right] + 2 \sin[3]\right)^2}{\left(2 \cos[3] - \frac{1}{40} \pi (-5+4\pi) \sin\left[\frac{\pi^2}{10}\right]\right)^2}}{13+12 \cos\left[\frac{5\pi}{8}\right]}}$$

N[%]

2.81547

f[A[A[1],A[2],φ[t,1],φ[t,2]], φ[A[1],A[2],φ[t,1],φ[t,2]]]/.t->Pi/3 //Simplify

$$\frac{1}{\sqrt{\frac{1 + \frac{\left(\frac{1}{120} \pi (-15+16\pi) \cos\left[\frac{2\pi^2}{15}\right] + 2 \sin[3]\right)^2}{\left(2 \cos[3] - \frac{1}{120} \pi (-15+16\pi) \sin\left[\frac{2\pi^2}{15}\right]\right)^2}}{13+12 \cos\left[\frac{5\pi}{8}\right]}}$$

N[%]

2.85414

2. Komplexe Zahlen

Materialbereitstellung

```
Remove["Global`*"];
z1 = 2+I;
z2 = 3-2I;
z3 = -4+3I;
z4 = -6-5I;
```

a

```
z = z1 + z2
```

```
5 - i
```

b

```
z = z2 - z3
```

```
7 - 5 i
```

c

$$z = z_1 + 2 z_2 - 3 z_3 + 4 z_4$$
$$-4 - 32 i$$

d

$$z = z_1 - 2 z_2 + 3 z_3 - 4 z_4$$
$$8 + 34 i$$

e

```
Remove[z];  
Solve[3 z1 - 2 z2 + 4 z3 + 6 z == 5 z2 - 3 z, {z}]
```

$$\left\{ \left\{ z \rightarrow \frac{31}{9} - \frac{29 i}{9} \right\} \right\}$$

```
N[%]
```

$$\left\{ \left\{ z \rightarrow 3.44444 - 3.22222 i \right\} \right\}$$

f

```
Solve[2 (z1 - 3 z2) + 4 (z3 - z ) - 5 (z4 + 2 z - z2) == 8 z1 - 8 z, {z}]
```

$$\left\{ \left\{ z \rightarrow -\frac{1}{6} + \frac{11 i}{2} \right\} \right\}$$

```
N[%]
```

$$\left\{ \left\{ z \rightarrow -0.166667 + 5.5 i \right\} \right\}$$