

Lösungen

Teil I: Wiederholung Matrizen und Eigenwertprobleme

I/1 Matrizenrechnung

```
Remove["Global`*"]
```

a

```
A = {{1,2,3},{3,5,6},{4,5,6}}; A//MatrixForm
```

$$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 5 & 6 \\ 4 & 5 & 6 \end{pmatrix}$$

```
A1=Inverse[A]; A1 //MatrixForm
```

$$\begin{pmatrix} 0 & -1 & 1 \\ -2 & 2 & -1 \\ \frac{5}{3} & -1 & \frac{1}{3} \end{pmatrix}$$

```
Det[A]
```

-3

```
Det[A1]
```

$-\frac{1}{3}$

b

```
e1={1,0,0}; e2={0,1,0}; e3={0,0,1};
```

```
A.e1
```

{1, 3, 4}

```
A.e2
```

{2, 5, 5}

```
A.e3
```

{3, 6, 6}

Spalten von A

c**A1.e1**

$$\left\{0, -2, \frac{5}{3}\right\}$$

A1.e2

$$\{-1, 2, -1\}$$

A1.e3

$$\left\{1, -1, \frac{1}{3}\right\}$$

Spalten der Inversen von A

I/2 Eigenwerte, Eigenvektoren

```
Remove["Global`*"]
```

a

```
A = {{1, 2}, {2, 3}}; A // MatrixForm
```

$$\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$$

```
Det[A]
```

```
-1
```

b1

```
A1=Inverse[A]; A1//MatrixForm
```

$$\begin{pmatrix} -3 & 2 \\ 2 & -1 \end{pmatrix}$$

```
EM=IdentityMatrix[2]; EM//MatrixForm
```

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

```
Det[A-λ EM]
```

```
-1 - 4 λ + λ2
```

$A \cdot x\text{Vec} = \lambda x\text{Vec} = EM \cdot x\text{Vec}$ bedeutet $(A - EM) \cdot x\text{Vec} = 0\text{Vec}$ und dass hier nicht nur die Null-Lösung existiert. Das heißt $\text{Det}[A - EM] = 0$. Diese Gleichung ist lösbar (charakteristische Gleichung)!

```
solv1=Solve[Det[A-λ EM]==0,{λ}]/Flatten
```

```
{λ → 2 - √5, λ → 2 + √5}
```

```
N[%]
```

```
{λ → -0.236068, λ → 4.23607}
```

```
λ1 = λ/.solv1[[1]]
```

```
2 - √5
```

```
λ2 = λ/.solv1[[2]]
```

```
2 + √5
```

b2

```
solv2=Solve[A.{x,y}==λ1{x,y},{x,y}]/Flatten
```

```
{x → - $\frac{1}{2}$  (1 + √5) y}
```

```
x1Vec = ({x,y}/.solv2)/.y->1
```

```
{ $\frac{1}{2}$  (-1 - √5), 1}
```

```
Simplify[A.x1Vec] == Simplify[λ1 x1Vec]
```

```
True
```

b3

```
N[λ1]
```

```
-0.236068
```

```
solv2a=Solve[A.{x,y}== -0.236068 {x,y},{x,y}]/Flatten
```

```
{x → 0., y → 0.}
```

b4

```
solv3=Solve[A.{x,y}==λ2{x,y},{x,y}]/Flatten
```

```
{x → - $\frac{1}{2}$  (1 - √5) y}
```

```
x2Vec = ({x,y}/.solv3)/.y->1
```

```
{ $\frac{1}{2}$  (-1 + √5), 1}
```

```
Simplify[A.x2Vec] == Simplify[λ2 x2Vec]
```

```
True
```

b5

```

N[λ2]

4.23607

Solve[A.{x,y]==4.23607 {x,y},{x,y}]/Flatten

{x → 0., y → 0.}

```

Teil II: Eigenwertprobleme und Matrixzerlegung

Definitionen

```

U[n_] := Inverse[S[n]];
M[n_] := S[n].Ei[n].U[n];
pr[n_] := Module[{}, Print["det(S) = ", Det[U[n]]];
  Print["Gegebene Matrix S = ", S[n] // MatrixForm];
  Print["Inverse Matrix U = ", U[n] // MatrixForm];
  Print["Diagonalmatrix D = ", Ei[n] // MatrixForm];
  Print["Resultat M = ", M[n] // MatrixForm];
  Print["Eigensystem von M = ", Eigensystem[M[n]]]]

```

II/1

```

Ei[1] = {{1, 0, 0}, {0, 2, 0}, {0, 0, 3}};
S[1] = {{1, 1, 1}, {1, -1, 1}, {1, 1, -1}};
pr[1]

```

$$\det(S) = \frac{1}{4}$$

$$\text{Gegebene Matrix } S = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$

$$\text{Inverse Matrix } U = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix}$$

$$\text{Diagonalmatrix } D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$\text{Resultat } M = \begin{pmatrix} \frac{5}{2} & -\frac{1}{2} & -1 \\ \frac{1}{2} & \frac{3}{2} & -1 \\ -\frac{1}{2} & -\frac{1}{2} & 2 \end{pmatrix}$$

$$\text{Eigensystem von } M = \{\{3, 2, 1\}, \{-1, -1, 1\}, \{1, -1, 1\}, \{1, 1, 1\}\}$$

```
M[1] // TeXForm
```

II/2

```

Ei[2] = {{1, 0, 0}, {0, 1, 0}, {0, 0, 1}};
S[2] = {{1, 1, 1}, {1, -1, 1}, {1, 1, -1}};
pr[2]

```

$$\det(S) = \frac{1}{4}$$

$$\text{Gegebene Matrix } S = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$

$$\text{Inverse Matrix } U = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix}$$

$$\text{Diagonalmatrix } D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\text{Resultat } M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Eigensystem von M = {{1, 1, 1}, {{0, 0, 1}, {0, 1, 0}, {1, 0, 0}}}

II/3

```

Ei[4] = {{1, 0, 0}, {0, 1, 0}, {0, 0, 2}};
S[4] = {{1, 1, 1}, {1, -1, 1}, {1, 1, -1}};
pr[4]

```

$$\det(S) = \frac{1}{4}$$

$$\text{Gegebene Matrix } S = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$

$$\text{Inverse Matrix } U = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix}$$

$$\text{Diagonalmatrix } D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$\text{Resultat } M = \begin{pmatrix} \frac{3}{2} & 0 & -\frac{1}{2} \\ \frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & 0 & \frac{3}{2} \end{pmatrix}$$

Eigensystem von M = {{2, 1, 1}, {{-1, -1, 1}, {1, 0, 1}, {0, 1, 0}}}

Eigensystem[M[4]]

{{2, 1, 1}, {{-1, -1, 1}, {1, 0, 1}, {0, 1, 0}}}

II/4

```

Ei[3] = {{1, 0, 0}, {0, 1, 0}, {0, 0, 0}};
S[3] = {{1, 1, 1}, {1, -1, 1}, {1, 1, -1}};
pr[3]

```

$$\det(S) = \frac{1}{4}$$

$$\text{Gegebene Matrix } S = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$

$$\text{Inverse Matrix } U = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix}$$

$$\text{Diagonalmatrix } D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\text{Resultat } M = \begin{pmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ -\frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}$$

Eigensystem von M = {{1, 1, 0}, {{1, 0, 1}, {0, 1, 0}, {-1, -1, 1}}}

Eigensystem[M[3]]

{{1, 1, 0}, {{1, 0, 1}, {0, 1, 0}, {-1, -1, 1}}}

II/5 Abbildung eines Kreises

```

Ei[5] = {{1, 0}, {0, 2}};
S[5] = {{1, 1}, {1, -1}};
pr[5]

```

$$\det(S) = -\frac{1}{2}$$

$$\text{Gegebene Matrix } S = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\text{Inverse Matrix } U = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$$

$$\text{Diagonalmatrix } D = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

$$\text{Resultat } M = \begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{pmatrix}$$

Eigensystem von M = {{2, 1}, {{-1, 1}, {1, 1}}}

M[5] . {**x**, **y**}

$$\left\{ \frac{3x}{2} - \frac{y}{2}, -\frac{x}{2} + \frac{3y}{2} \right\}$$

```
u[x_, y_] := (M[5].{x, y})[[1]];
```

```
u[x, y]
```

$$\frac{3x}{2} - \frac{y}{2}$$

```
v[x_, y_] := (M[5].{x, y})[[2]];
```

```
v[x, y]
```

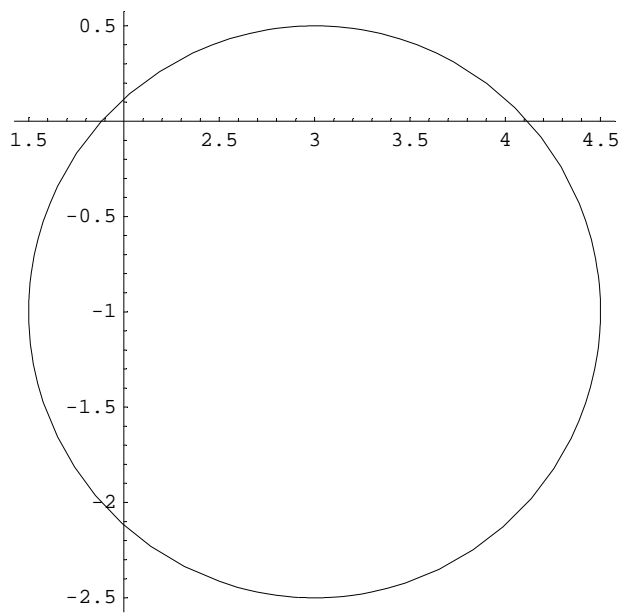
$$-\frac{x}{2} + \frac{3y}{2}$$

```
x[r_, φ_] := r Cos[φ] + 3;
```

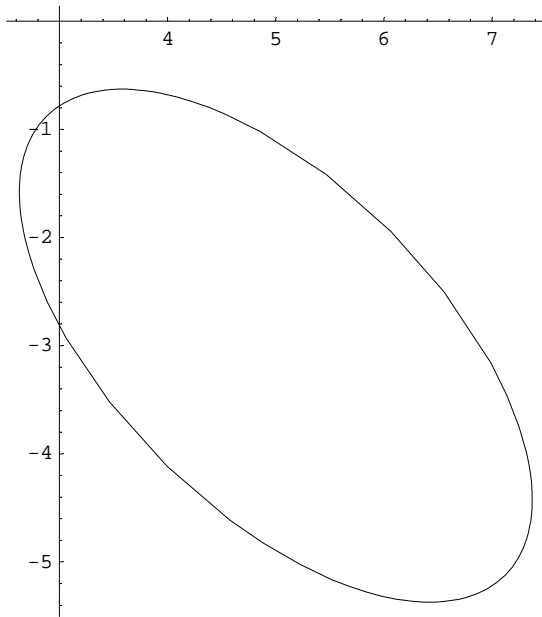
```
y[r_, φ_] := r Sin[φ] - 1;
```

```
r = 1.5;
```

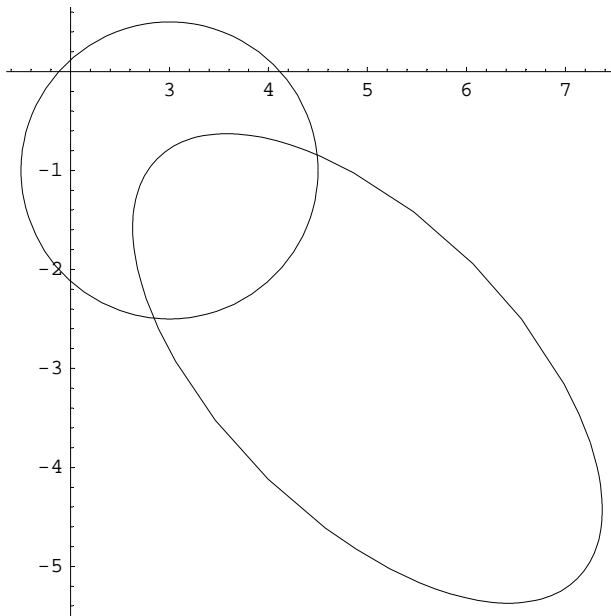
```
p1 = ParametricPlot[{x[r, φ], y[r, φ]}, {φ, 0, 2 Pi}, AspectRatio → Automatic];
```



```
p2 = ParametricPlot[{u[x[r,  $\phi$ ], y[r,  $\phi$ ]}, v[x[r,  $\phi$ ], y[r,  $\phi$ ]},  
  { $\phi$ , 0, 2 Pi}, AspectRatio  $\rightarrow$  Automatic];
```



```
Show[p1, p2];
```



Ellipse!