Lösungen

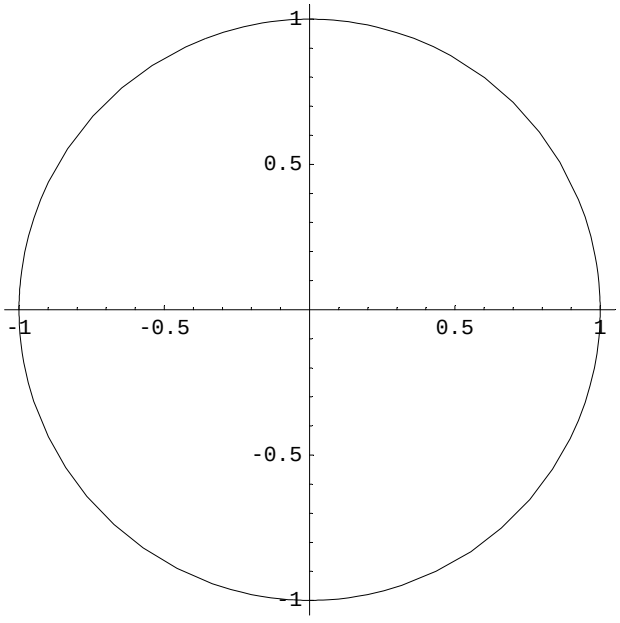
1

a

a1

```
r = 1;
xVec0[t_] := r {Cos[t], Sin[t]};

ParametricPlot[xVec0[t], {t, 0, 2 Pi}, AspectRatio → Automatic];
```

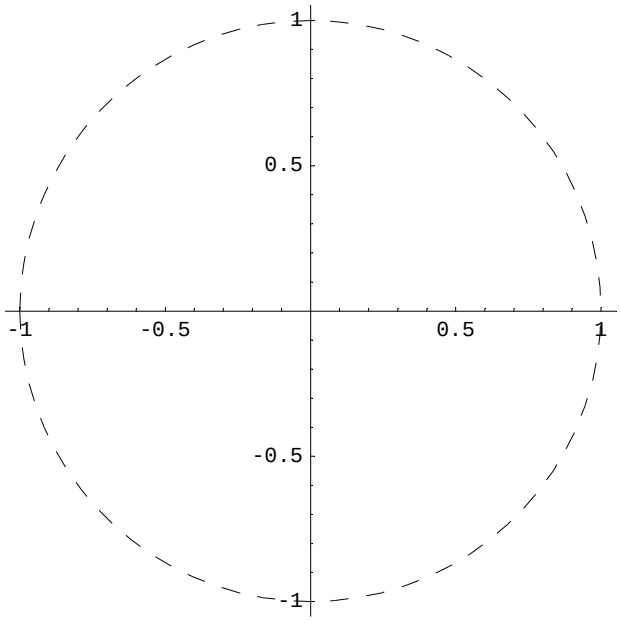


a2

```
<< Graphics`ImplicitPlot`

fig0 = ({x, y}.{x, y} - r^2 == 0);
```

```
ImplicitPlot[fig0, {x, -1, 1}, PlotStyle → {GrayLevel[0], Dashing[{.03}]}];
```



b

```
D1 = {{1, 0}, {0, 2}}; D1 // MatrixForm
```

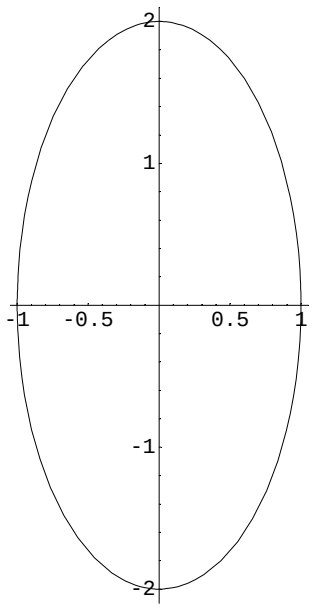
$$\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

b1

```
xVec1a[t_] := D1.{Cos[t], Sin[t]}; xVec1a[t]
```

```
{Cos[t], 2 Sin[t]}
```

```
ParametricPlot[xVec1a[t], {t, 0, 2 Pi}, AspectRatio → Automatic];
```

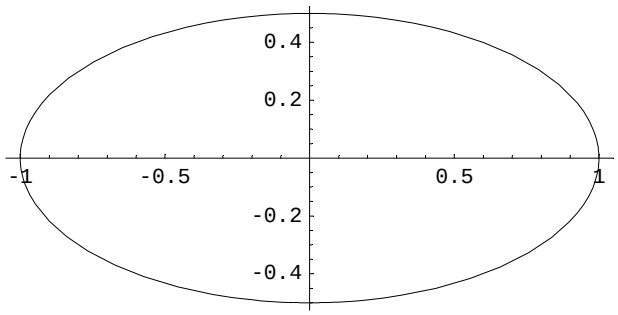


b2

```
xVec1b[t_] := Inverse[D1].{Cos[t], Sin[t]}; xVec1b[t]
```

$$\left\{\text{Cos}[t],\ \frac{\text{Sin}[t]}{2}\right\}$$

```
ParametricPlot[xVec1b[t], {t, 0, 2 Pi}, AspectRatio → Automatic];
```



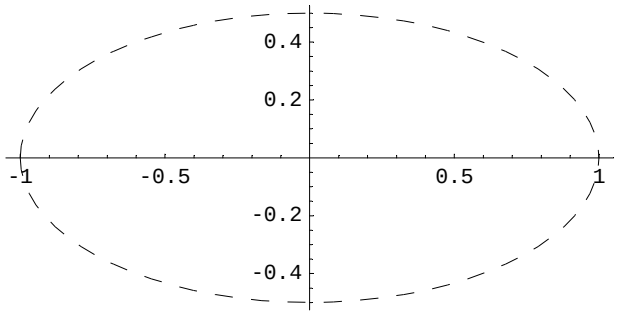
b3

```
(D1.{x, y}).(D1.{x, y}) - r^2 == 0
```

$$-1 + x^2 + 4\,y^2 == 0$$

```
fig1 = ((D1.{x, y}).(D1.{x, y}) - r^2 == 0);

ImplicitPlot[fig1, {x, -1, 1}, PlotStyle → {GrayLevel[0], Dashing[{.03}]}];
```



c

```
φ = -Pi / 6;
M = {{Cos[φ], -Sin[φ]}, {Sin[φ], Cos[φ]}};
M // MatrixForm
```

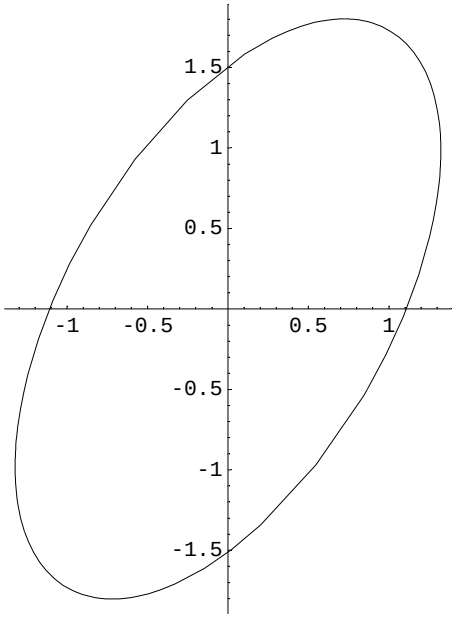
$$\begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

c1

```
xVec2a[t_] := M.xVec1a[t]; xVec2a[t]
```

$$\left\{\frac{1}{2}\sqrt{3}\,\text{Cos}[t] + \text{Sin}[t],\ -\frac{\text{Cos}[t]}{2} + \sqrt{3}\,\text{Sin}[t]\right\}$$

```
ParametricPlot[xVec2a[t], {t, 0, 2 Pi}, AspectRatio → Automatic];
```




```
Eigensystem[M.D1]
```

$$\left\{\left\{\frac{1}{4}\left(3\sqrt{3}+i\sqrt{5}\right),\ \frac{1}{4}\left(3\sqrt{3}-i\sqrt{5}\right)\right\},\ \left\{\left\{\frac{1}{2}\left(\sqrt{3}-i\sqrt{5}\right),\ 1\right\},\ \left\{\frac{1}{2}\left(\sqrt{3}+i\sqrt{5}\right),\ 1\right\}\right\}\right\}$$

```
Eigensystem[M.D1] // N
```

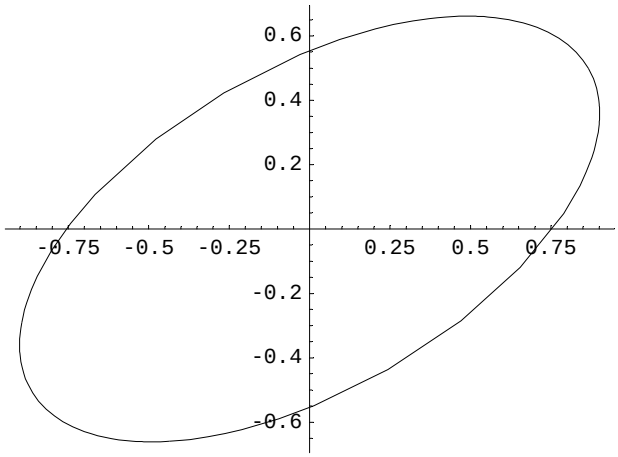
```
{{1.29904 + 0.559017 i, 1.29904 - 0.559017 i},
 {{0.866025 - 1.11803 i, 1.}, {0.866025 + 1.11803 i, 1.}}}
```

c2

```
xVec2b[t_] := Inverse[M].xVec1b[t]; xVec2b[t]
```

$$\left\{\frac{1}{2}\sqrt{3}\,\text{Cos}[t]-\frac{\text{Sin}[t]}{4},\ \frac{\text{Cos}[t]}{2}+\frac{1}{4}\sqrt{3}\,\text{Sin}[t]\right\}$$

```
ParametricPlot[xVec2b[t], {t, 0, 2 Pi}, AspectRatio → Automatic];
```




```
Eigensystem[Inverse[M].Inverse[D1]]
```

$$\left\{\left\{\frac{1}{8}\left(3\sqrt{3}+i\sqrt{5}\right),\ \frac{1}{8}\left(3\sqrt{3}-i\sqrt{5}\right)\right\},\ \left\{\left\{\frac{1}{4}\left(\sqrt{3}+i\sqrt{5}\right),\ 1\right\},\ \left\{\frac{1}{4}\left(\sqrt{3}-i\sqrt{5}\right),\ 1\right\}\right\}\right\}$$

```
Eigensystem[Inverse[M].Inverse[D1]] // N
```

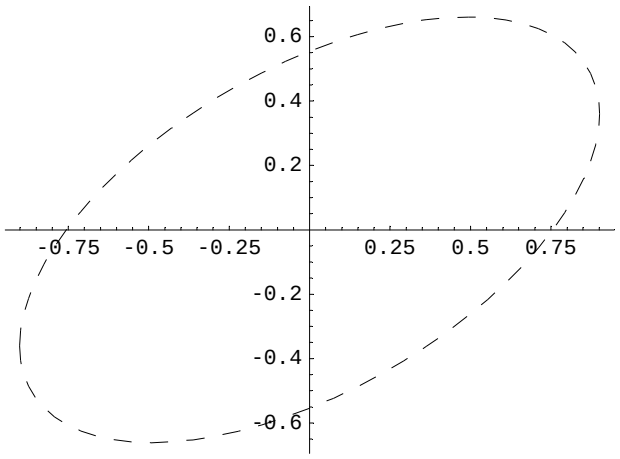
```
{{0.649519 + 0.279508 i, 0.649519 - 0.279508 i},
 {{0.433013 + 0.559017 i, 1.}, {0.433013 - 0.559017 i, 1.}}}
```

c3

```
fig2 = ((D1.M.{x, y}).(D1.M.{x, y}) - r^2 == 0)
```

$$-1 + \left(\frac{\sqrt{3}\ x}{2} + \frac{y}{2}\right)^2 + \left(-x + \sqrt{3}\ y\right)^2 == 0$$

```
ImplicitPlot[fig2, {x, -1, 1}, PlotStyle → {GrayLevel[0], Dashing[{.03}]}];
```




```
Eigensystem[D1.M]
```

$$\left\{\left\{\frac{1}{4}\left(3\sqrt{3} + i\sqrt{5}\right), \frac{1}{4}\left(3\sqrt{3} - i\sqrt{5}\right)\right\}, \left\{\left\{\frac{1}{4}\left(\sqrt{3} - i\sqrt{5}\right), 1\right\}, \left\{\frac{1}{4}\left(\sqrt{3} + i\sqrt{5}\right), 1\right\}\right\}\right\}$$

```
Eigensystem[D1.M] // N
```

```
{{1.29904 + 0.559017 i, 1.29904 - 0.559017 i},
 {{0.433013 - 0.559017 i, 1.}, {0.433013 + 0.559017 i, 1.}}}
```