

# Lösungen (Matrixkomposition)

---

1

a

```
 $\lambda_1 = 1; \lambda_2 = -1; \lambda_3 = 3;$   
 $v_1 = \{1, 0, 0\}; v_2 = \{1, 1, 0\}; v_3 = \{0, 1, -1\};$ 
```

a1

```
X = Transpose[{v1, v2, v3}]; X // MatrixForm
```

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix}$$

a2

```
Det[X]
```

```
-1
```

Vektoren l.u.

a3

```
D $\lambda$  = {{ $\lambda_1$ , 0, 0}, {0,  $\lambda_2$ , 0}, {0, 0,  $\lambda_3$ }}; D $\lambda$  // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

```
Xinv = Inverse[X]; Xinv // MatrixForm
```

$$\begin{pmatrix} 1 & -1 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix}$$

```
A = X.D $\lambda$ .Inverse[X]; A // MatrixForm
```

$$\begin{pmatrix} 1 & -2 & -2 \\ 0 & -1 & -4 \\ 0 & 0 & 3 \end{pmatrix}$$

**a4****Eigenvalues[A]** $\{3, -1, 1\}$ **Eigenvectors[A]** $\{\{0, -1, 1\}, \{1, 1, 0\}, \{1, 0, 0\}\}$ **Eigensystem[A]** $\{\{3, -1, 1\}, \{\{0, -1, 1\}, \{1, 1, 0\}, \{1, 0, 0\}\}\}$ **a5****Det[A] == Det[Dλ]**

True

**b****A1 = Inverse[A]; A1 // MatrixForm**

$$\begin{pmatrix} 1 & -2 & -2 \\ 0 & -1 & -\frac{4}{3} \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$$

**Eigenvalues[A1]** $\{-1, 1, \frac{1}{3}\}$ **Eigenvectors[A1]** $\{\{1, 1, 0\}, \{1, 0, 0\}, \{0, -1, 1\}\}$ **Eigensystem[A1]** $\{\{-1, 1, \frac{1}{3}\}, \{\{1, 1, 0\}, \{1, 0, 0\}, \{0, -1, 1\}\}\}$ **Det[A1] == Det[Inverse[Dλ]]**

True

**b****A2 = Transpose[A]; A2 // MatrixForm**

$$\begin{pmatrix} 1 & 0 & 0 \\ -2 & -1 & 0 \\ -2 & -4 & 3 \end{pmatrix}$$

```

Eigenvalues[A2]
{3, -1, 1}

Eigenvectors[A2]
{{0, 0, 1}, {0, 1, 1}, {-1, 1, 1}}

Eigensystem[A2]
{{3, -1, 1}, {{0, 0, 1}, {0, 1, 1}, {-1, 1, 1}}}

Det[A2] == Det[Dλ]
True

```

---

## 2

### a

```

λ1 = 1; λ2 = -1; λ3 = 3;
v1 = {1/2, 0, 1/2}; v2 = {1, 1, 0}; v3 = {0, 1, 1};

```

### a1

```
X = Transpose[{v1, v2, v3}]; X // MatrixForm
```

$$\begin{pmatrix} \frac{1}{2} & 1 & 0 \\ 0 & 1 & 1 \\ \frac{1}{2} & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{2} & 1 & 0 \\ 0 & 1 & 1 \\ \frac{1}{2} & 0 & 1 \end{pmatrix}$$

### a2

```
Det[X]
```

```
1
```

Vektoren l.u.

### a3

```
Dλ = {{λ1, 0, 0}, {0, λ2, 0}, {0, 0, λ3}}; Dλ // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

```
Xinv = Inverse[X]; Xinv // MatrixForm
```

$$\begin{pmatrix} 1 & -1 & 1 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}$$

```
A = X.Dλ.Inverse[X]; A // MatrixForm
```

$$\begin{pmatrix} 0 & -1 & 1 \\ -2 & 1 & 2 \\ -1 & 1 & 2 \end{pmatrix}$$

**a4**

```
Eigenvalues[A]
```

```
{3, -1, 1}
```

```
Eigenvectors[A]
```

```
{{0, 1, 1}, {1, 1, 0}, {1, 0, 1}}
```

```
Eigensystem[A]
```

```
{{3, -1, 1}, {{0, 1, 1}, {1, 1, 0}, {1, 0, 1}}}
```

**a5**

```
Det[A] == Det[Dλ]
```

```
True
```

**b**

```
A1 = Inverse[A]; A1 // MatrixForm
```

$$\begin{pmatrix} 0 & -1 & 1 \\ -\frac{2}{3} & -\frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix}$$

```
Eigenvalues[A1]
```

```
{-1, 1,  $\frac{1}{3}$ }
```

```
Eigenvectors[A1]
```

```
{{1, 1, 0}, {1, 0, 1}, {0, 1, 1}}
```

```
Eigensystem[A1]
```

```
{{-1, 1,  $\frac{1}{3}$ }, {{1, 1, 0}, {1, 0, 1}, {0, 1, 1}}}
```

```
Det[A1] == Det[Inverse[Dλ]]
```

```
True
```

**b**

```
A2 = Transpose[A]; A2 // MatrixForm
```

$$\begin{pmatrix} 0 & -2 & -1 \\ -1 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

```
Eigenvalues[A2]
```

```
{3, -1, 1}
```

```
Eigenvectors[A2]
```

```
{{-1, 1, 1}, {-1, -1, 1}, {1, -1, 1}}
```

```
Eigensystem[A2]
```

```
{{3, -1, 1}, {{-1, 1, 1}, {-1, -1, 1}, {1, -1, 1}}}
```

```
Det[A2] == Det[Dλ]
```

```
True
```