

Lösungen

1

```
Remove["Global`*"]
```

a

```
X = Transpose[{{2, 1}, {-1, 2}}]; X // MatrixForm
```

$$\begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$$

```
Dλ = {{2, 0}, {0, 1}}; Dλ // MatrixForm
```

$$\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

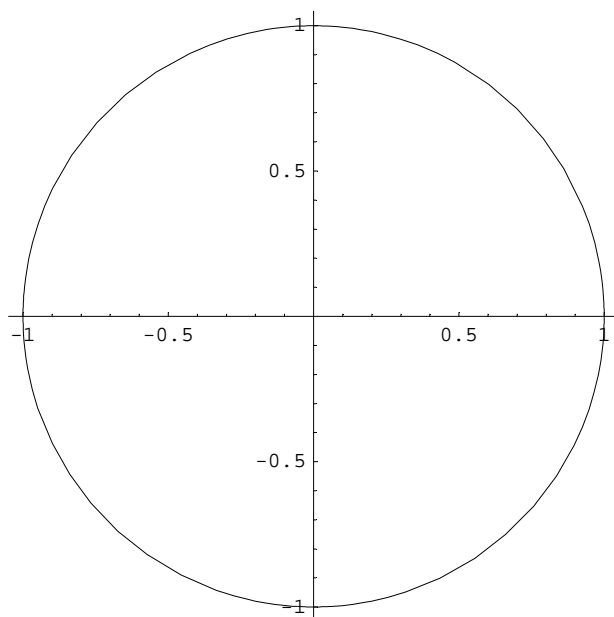
```
A = X.Dλ.Inverse[X]; A // MatrixForm
```

$$\begin{pmatrix} \frac{9}{5} & \frac{2}{5} \\ \frac{2}{5} & \frac{6}{5} \end{pmatrix}$$

b

```
v1[t_] := {Cos[t], Sin[t]};
```

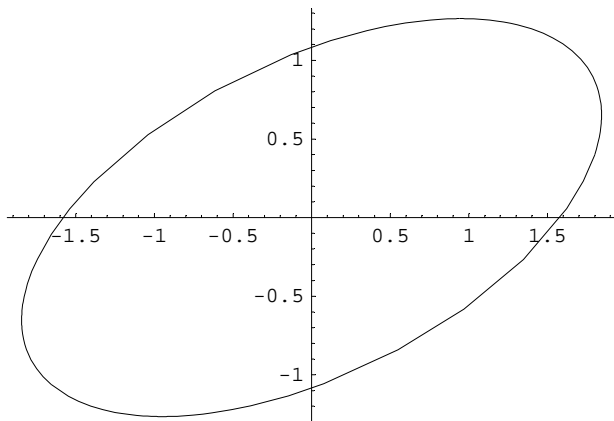
```
pl1 = ParametricPlot[v1[t], {t, 0, 2 Pi}, AspectRatio -> Automatic];
```



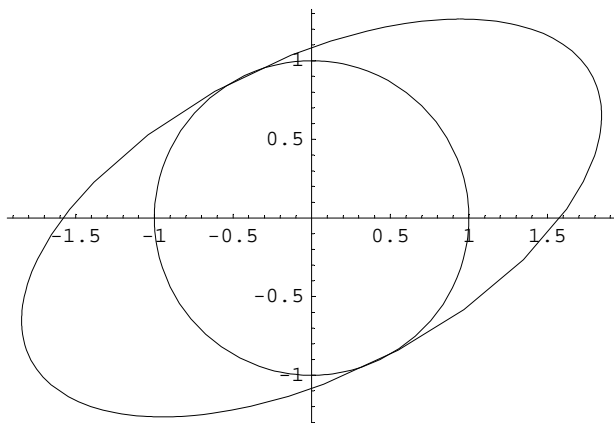
```
v2[t_] := A.v1[t]; v2[t]
```

$$\left\{ \frac{9 \cos[t]}{5} + \frac{2 \sin[t]}{5}, \frac{2 \cos[t]}{5} + \frac{6 \sin[t]}{5} \right\}$$

```
pl2 = ParametricPlot[v2[t], {t, 0, 2 Pi}, AspectRatio -> Automatic];
```



```
Show[pl1, pl2];
```

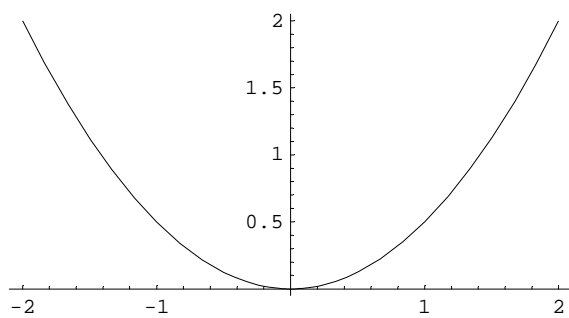


C

```
v3[t_] := {t, 1/2 t^2}; v3[t]
```

$$\left\{ t, \frac{t^2}{2} \right\}$$

```
pl3 = ParametricPlot[v3[t], {t, -2, 2}, AspectRatio -> Automatic];
```



```
Dreh[φ_] := {{Cos[φ], -Sin[φ]}, {Sin[φ], Cos[φ]}}; Dreh[φ] // MatrixForm
```

$$\begin{pmatrix} \cos[\varphi] & -\sin[\varphi] \\ \sin[\varphi] & \cos[\varphi] \end{pmatrix}$$

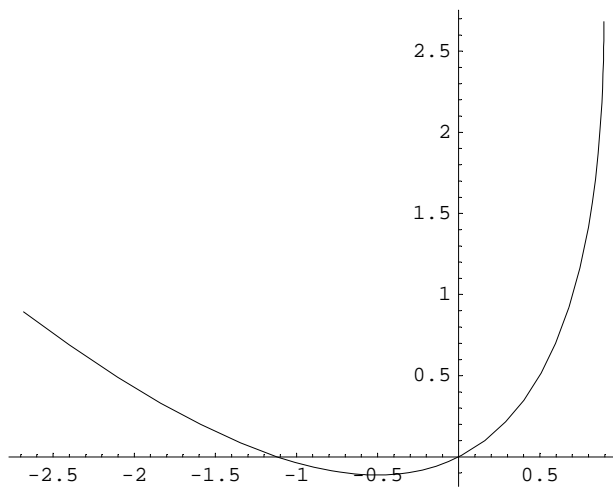
```
φ = ArcTan[1 / 2]
```

$$\text{ArcTan}\left[\frac{1}{2}\right]$$

```
v4[t_] := Dreh[φ].v3[t]; v4[t]
```

$$\left\{ \frac{2t}{\sqrt{5}} - \frac{t^2}{2\sqrt{5}}, \frac{t}{\sqrt{5}} + \frac{t^2}{\sqrt{5}} \right\}$$

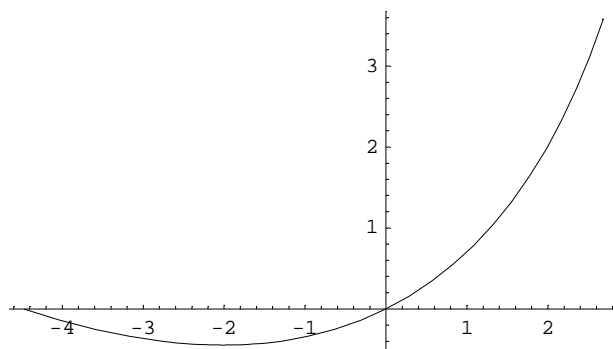
```
pl4 = ParametricPlot[v4[t], {t, -2, 2}, AspectRatio -> Automatic];
```



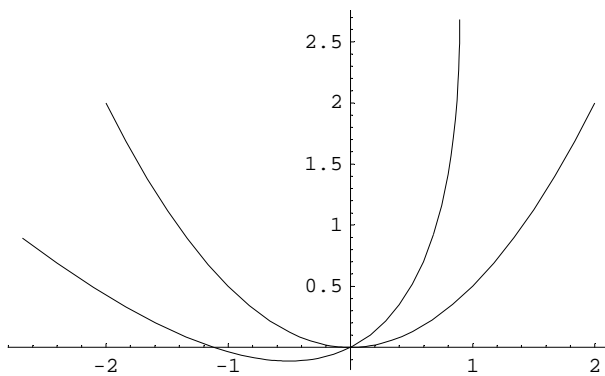
```
v5[t_] := A.v4[t]; v5[t]
```

$$\left\{ \frac{9}{5} \left(\frac{2t}{\sqrt{5}} - \frac{t^2}{2\sqrt{5}} \right) + \frac{2}{5} \left(\frac{t}{\sqrt{5}} + \frac{t^2}{\sqrt{5}} \right), \frac{2}{5} \left(\frac{2t}{\sqrt{5}} - \frac{t^2}{2\sqrt{5}} \right) + \frac{6}{5} \left(\frac{t}{\sqrt{5}} + \frac{t^2}{\sqrt{5}} \right) \right\}$$

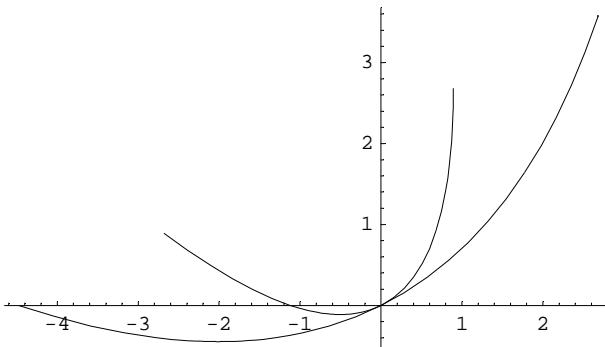
```
pl5 = ParametricPlot[v5[t], {t, -2, 2}, AspectRatio -> Automatic];
```



```
Show[p13, p14];
```



```
Show[p14, p15];
```



2

```
Remove["Global`*"]
```

a

```
X[φ_] := Transpose[{{Cos[φ], Sin[φ]}, {-Sin[φ], Cos[φ]}}]; X[φ] // MatrixForm
```

$$\begin{pmatrix} \cos[\varphi] & -\sin[\varphi] \\ \sin[\varphi] & \cos[\varphi] \end{pmatrix}$$

```
λ2 = 1.5;
```

```
Dλ[λ_] := {{λ, 0}, {0, λ2}}; Dλ[λ] // MatrixForm
```

$$\begin{pmatrix} \lambda & 0 \\ 0 & 1.5 \end{pmatrix}$$

```
A[φ_, λ_] := X[φ].Dλ[λ].Inverse[X[φ]]; A[φ, λ] // MatrixForm
```

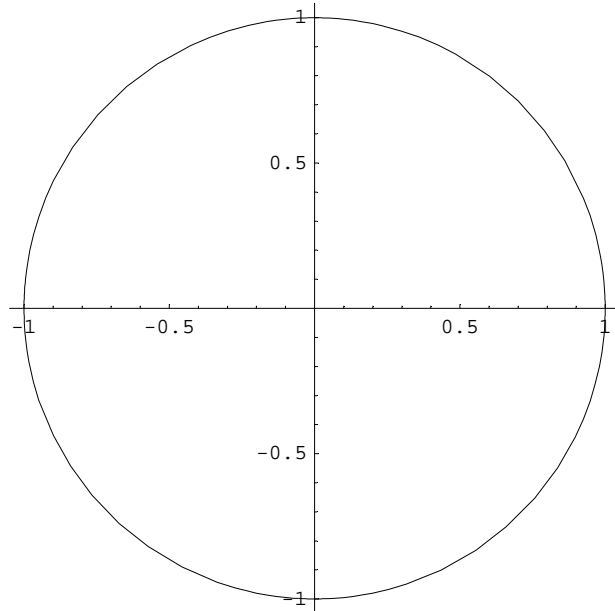
$$\begin{pmatrix} \frac{\lambda \cos[\varphi]^2}{\cos[\varphi]^2 + \sin[\varphi]^2} + \frac{1.5 \sin[\varphi]^2}{\cos[\varphi]^2 + \sin[\varphi]^2} & -\frac{1.5 \cos[\varphi] \sin[\varphi]}{\cos[\varphi]^2 + \sin[\varphi]^2} + \frac{\lambda \cos[\varphi] \sin[\varphi]}{\cos[\varphi]^2 + \sin[\varphi]^2} \\ -\frac{1.5 \cos[\varphi] \sin[\varphi]}{\cos[\varphi]^2 + \sin[\varphi]^2} + \frac{\lambda \cos[\varphi] \sin[\varphi]}{\cos[\varphi]^2 + \sin[\varphi]^2} & \frac{1.5 \cos[\varphi]^2}{\cos[\varphi]^2 + \sin[\varphi]^2} + \frac{\lambda \sin[\varphi]^2}{\cos[\varphi]^2 + \sin[\varphi]^2} \end{pmatrix}$$

b

```

v1[t_] := {Cos[t], Sin[t]};
pl1 = ParametricPlot[v1[t], {t, 0, 2 Pi}, AspectRatio -> Automatic];

```



```

v2[t_, φ_, λ_] := A[φ, λ].v1[t]; v2[t, φ, λ]

```

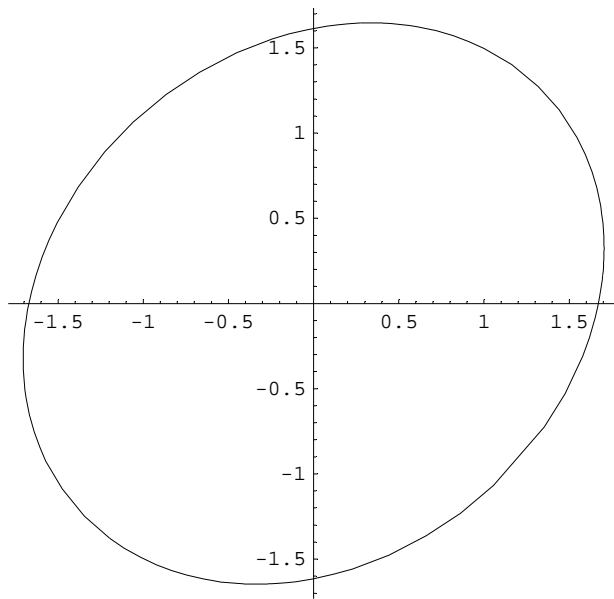
$$\begin{aligned}
& \left\{ \begin{aligned} & \text{Sin}[t] \left(-\frac{1.5 \text{Cos}[\varphi] \text{Sin}[\varphi]}{\text{Cos}[\varphi]^2 + \text{Sin}[\varphi]^2} + \frac{\lambda \text{Cos}[\varphi] \text{Sin}[\varphi]}{\text{Cos}[\varphi]^2 + \text{Sin}[\varphi]^2} \right) + \\ & \text{Cos}[t] \left(\frac{\lambda \text{Cos}[\varphi]^2}{\text{Cos}[\varphi]^2 + \text{Sin}[\varphi]^2} + \frac{1.5 \text{Sin}[\varphi]^2}{\text{Cos}[\varphi]^2 + \text{Sin}[\varphi]^2} \right), \\ & \text{Cos}[t] \left(-\frac{1.5 \text{Cos}[\varphi] \text{Sin}[\varphi]}{\text{Cos}[\varphi]^2 + \text{Sin}[\varphi]^2} + \frac{\lambda \text{Cos}[\varphi] \text{Sin}[\varphi]}{\text{Cos}[\varphi]^2 + \text{Sin}[\varphi]^2} \right) + \\ & \text{Sin}[t] \left(\frac{1.5 \text{Cos}[\varphi]^2}{\text{Cos}[\varphi]^2 + \text{Sin}[\varphi]^2} + \frac{\lambda \text{Sin}[\varphi]^2}{\text{Cos}[\varphi]^2 + \text{Sin}[\varphi]^2} \right) \end{aligned} \right\}
\end{aligned}$$

```

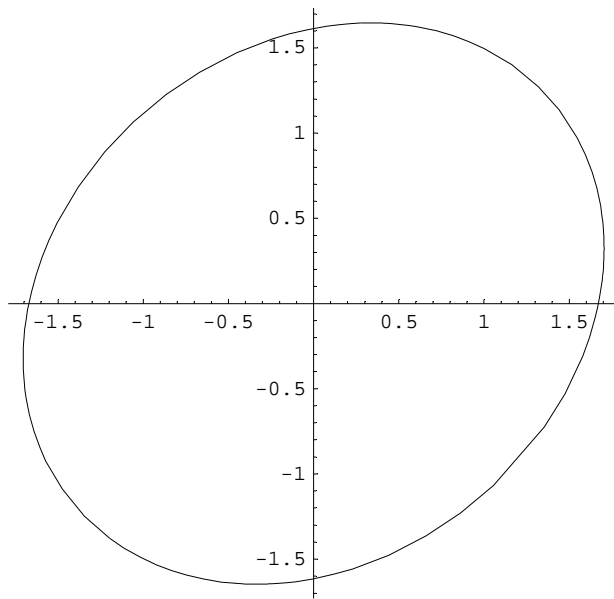
pl2[φ_, λ_] :=
  ParametricPlot[Evaluate[v2[t, φ, λ]], {t, 0, 2 Pi}, AspectRatio -> Automatic];

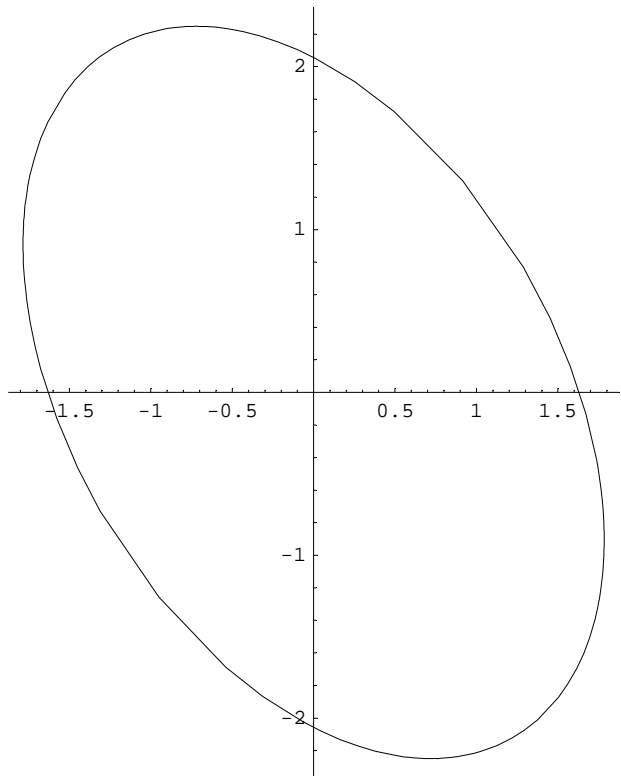
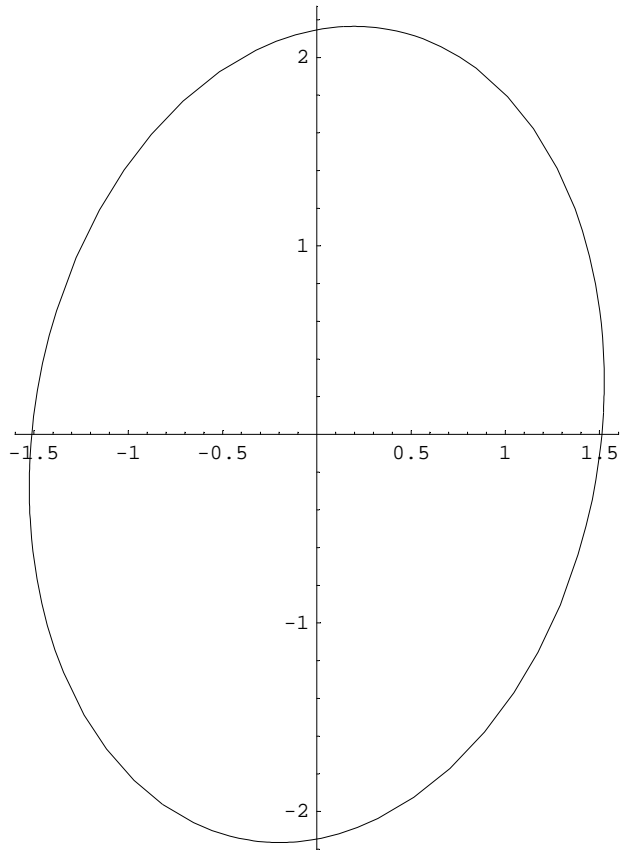
```

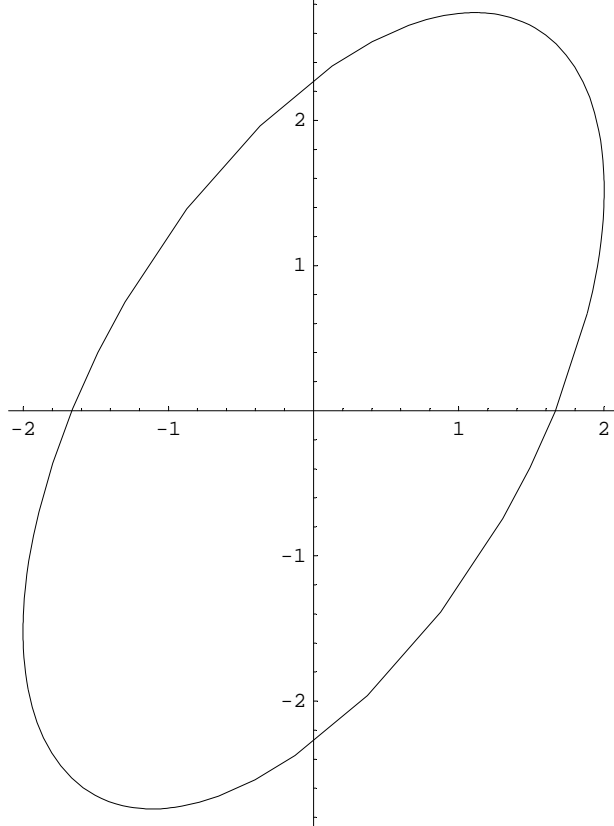
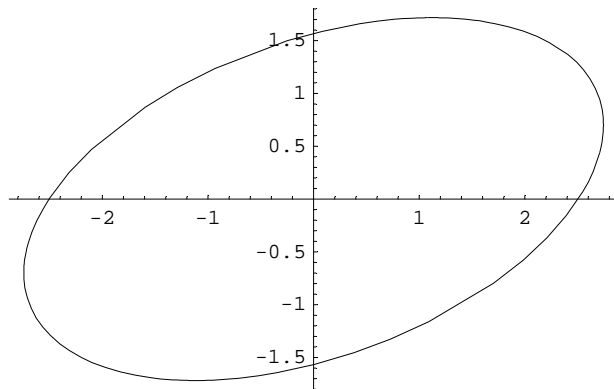
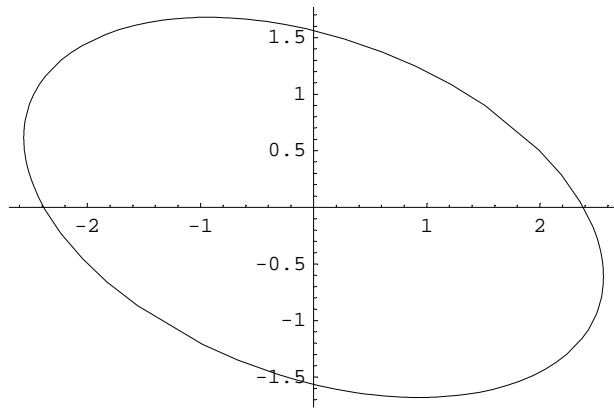
```
p12[ 2 Pi / 9, 1 + Sqrt[ 2 Pi / 9]];
```

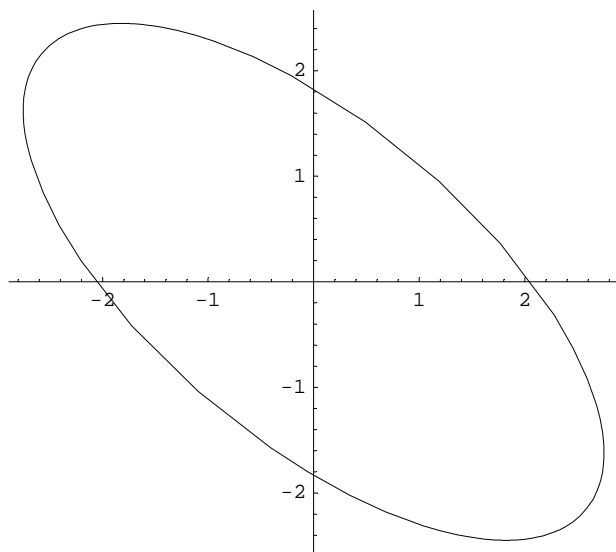
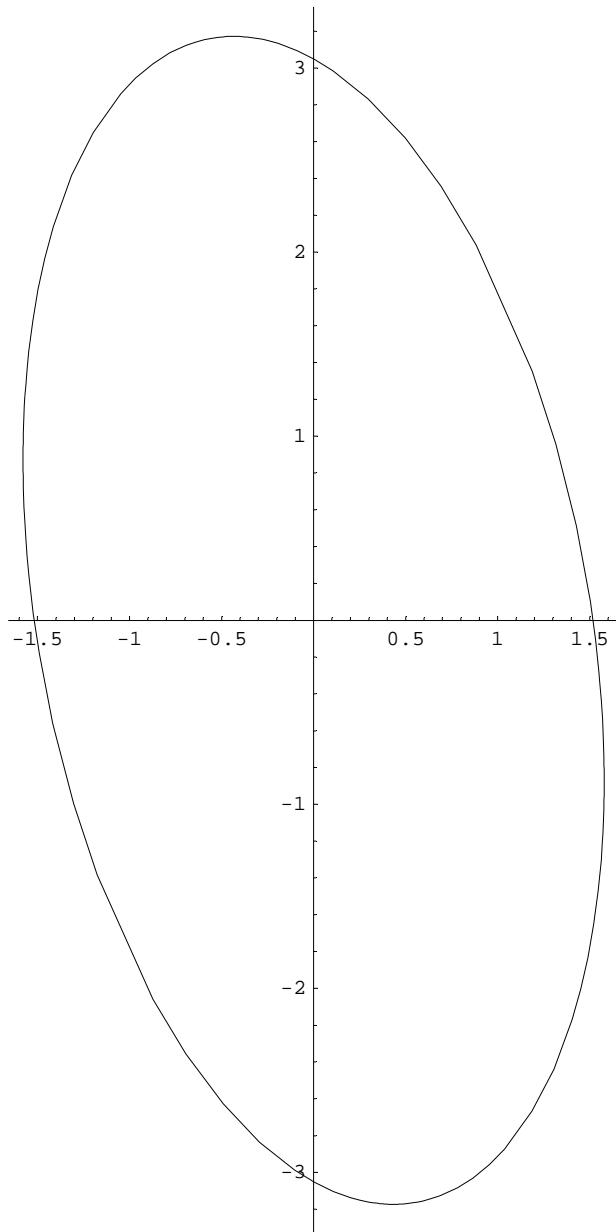


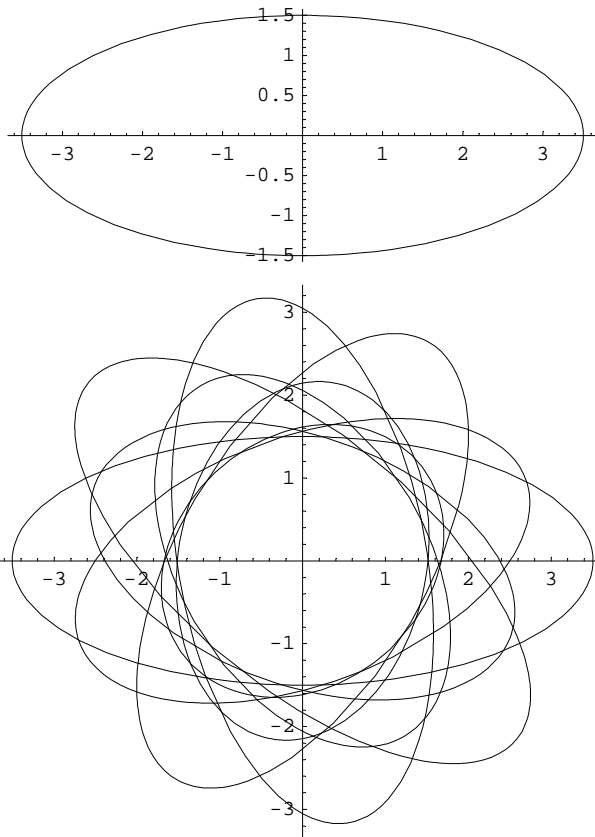
```
Show[Table[p12[φ, 1 + Sqrt[φ]], {φ, 2 Pi / 9, 2 Pi, 2 Pi / 9}]];
```











Die eingehüllte Kurve ist der zentrische Kreis mit dem Radius $\lambda_2 = 1.5$.