

Lösungen

1

```
Remove["Global`*"]
```

```
Dreh[φ_] := {{Cos[φ], -Sin[φ]}, {Sin[φ], Cos[φ]}}; Dreh[φ] // MatrixForm
```

```

$$\begin{pmatrix} \cos[\varphi] & -\sin[\varphi] \\ \sin[\varphi] & \cos[\varphi] \end{pmatrix}$$

```

```
P1 = {2, 1}; P2 = {3, 2}; P3 = {1, 3};
```

```
φ = 71 Degree;
```

```
Q1 = Dreh[φ].P1 // N
```

```
{-0.294382, 2.21661}
```

```
Q2 = Dreh[φ].P2 // N
```

```
{-0.914333, 3.48769}
```

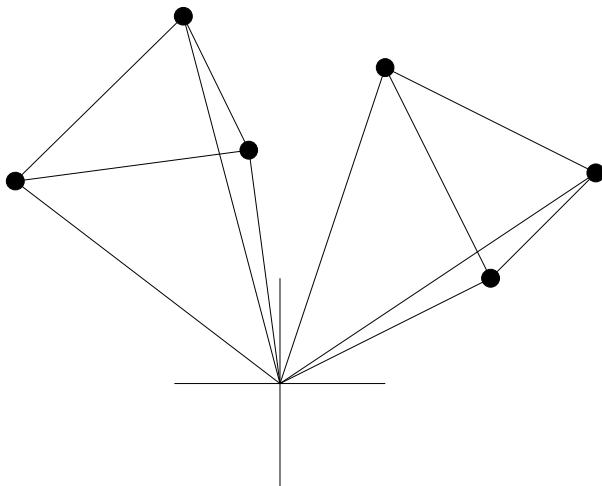
```
Q3 = Dreh[φ].P3 // N
```

```
{-2.51099, 1.92222}
```

```

o = {0, 0};
Show[Graphics[
  {
    Line[{{-1, 0}, {1, 0}}, Line[{{0, -1}, {0, 1}},
    Line[{P1, P2, P3, P1}], Line[{Q1, Q2, Q3, Q1}],
    Line[{o, P1}], Line[{o, P2}], Line[{o, P3}],
    Line[{o, Q1}], Line[{o, Q2}], Line[{o, Q3}],
    PointSize[0.03], Point[P1],
    Point[P2], Point[P3], Point[Q1], Point[Q2], Point[Q3]
  }
], AspectRatio -> Automatic];

```



2

```

α[x_] := ArcTan[x[[2]] / x[[1]]]

```

```

x1 = {4, 1}; α[x1]

```

```

ArcTan[ $\frac{1}{4}$ ]

```

```

% // N

```

```

0.244979

```

```

S[0] = {{1, 0}, {0, -1}}; S[0] // MatrixForm

```

```


$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$


```

```

S[α_] := Dreh[α].S[0].Dreh[-α]; S[α] // Simplify // MatrixForm

```

```


$$\begin{pmatrix} \cos[2\alpha] & \sin[2\alpha] \\ \sin[2\alpha] & -\cos[2\alpha] \end{pmatrix}$$


```

```

s1 = S[α[x1]].P1

```

```


$$\left\{ \frac{38}{17}, \frac{1}{17} \right\}$$


```

```

S1 // N
{2.23529, 0.0588235}

S2 = S[α[x1]] . P2
{
   $\frac{61}{17}$ ,  $-\frac{6}{17}$ 
}

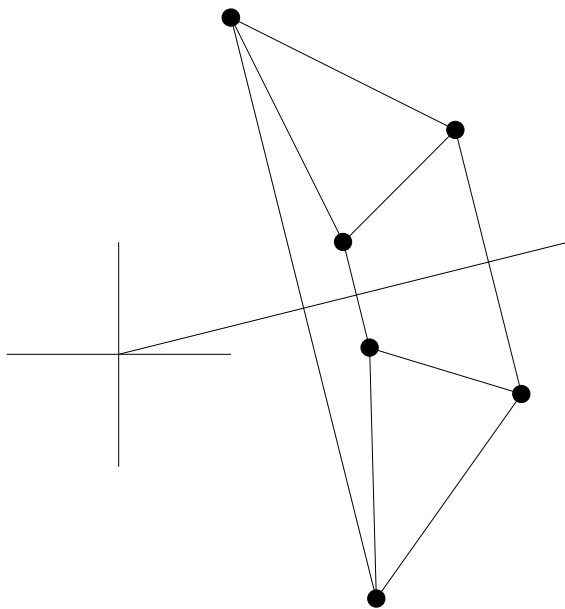
S2 // N
{3.58824, -0.352941}

S3 = S[α[x1]] . P3
{
   $\frac{39}{17}$ ,  $-\frac{37}{17}$ 
}

S2 // N
{3.58824, -0.352941}

Show[Graphics[
  {
    Line[{{-1, 0}, {1, 0}}], Line[{{0, -1}, {0, 1}}],
    Line[{P1, P2, P3, P1}], Line[{S1, S2, S3, S1}],
    Line[{o, x1}],
    Line[{P1, S1}], Line[{P2, S2}], Line[{P3, S3}],
    PointSize[0.03], Point[P1],
    Point[P2], Point[P3], Point[S1], Point[S2], Point[S3]
  }
], AspectRatio → Automatic];

```



3

```
Eigensystem[S[α[x1]]]
```

```
{{-1, 1}, {{-1/4, 1}, {4, 1}}}
```

{4, 1} ist x_1 gestreckt (Richtung der Spiegelungsgerade). $\left\{ \begin{smallmatrix} 1 \\ 4 \end{smallmatrix}, 1 \right\}$ ist darauf senkrecht. Der Eigenwert 1 bewirkt die Spiegelung, der Eigenwert -1 das Fixhalten der Abbildung senkrecht zur Spiegelungsachse.

4

```
Remove["Global`*"]
```

```
vec0 = {0, 0, 0};
```

```
a = {3, 1, 2}; b = {-1, 2, -2}; u = {2, 3, -1}; M = Transpose[{a, b, u}]; Det[M]
```

```
-7
```

a, b, u sind linear unabhängig, da die Determinante nicht 0 ist.

$\text{Proj} \cdot \text{Transpose}[\{a, b, u\}] = \text{Transpose}[\{a, b, 0\}] \Rightarrow \text{Proj} = \text{Transpose}[\{a, b, 0\}] \cdot \text{Inverse}[M]$

```
M // MatrixForm
```

$$\begin{pmatrix} 3 & -1 & 2 \\ 1 & 2 & 3 \\ 2 & -2 & -1 \end{pmatrix}$$

```
Transpose[{a, b, vec0}] // MatrixForm
```

$$\begin{pmatrix} 3 & -1 & 0 \\ 1 & 2 & 0 \\ 2 & -2 & 0 \end{pmatrix}$$

```
vec0 = {0, 0, 0};
```

```
Proj = Transpose[{a, b, vec0}] \cdot Inverse[M]; Proj // MatrixForm
```

$$\begin{pmatrix} -\frac{5}{7} & \frac{8}{7} & 2 \\ -\frac{18}{7} & \frac{19}{7} & 3 \\ \frac{6}{7} & -\frac{4}{7} & 0 \end{pmatrix}$$

```
% // N // MatrixForm
```

$$\begin{pmatrix} -0.714286 & 1.14286 & 2. \\ -2.57143 & 2.71429 & 3. \\ 0.857143 & -0.571429 & 0. \end{pmatrix}$$

```
T1 = {0, 2, 3}; T2 = {1, 1, 0}; T3 = {2, 0, 2};
```

```
N1 = Proj \cdot T1
```

$$\left\{ \frac{58}{7}, \frac{101}{7}, -\frac{8}{7} \right\}$$

```

% // N
{8.28571, 14.4286, -1.14286}

N2 = Proj.T2
{ $\frac{3}{7}$ ,  $\frac{1}{7}$ ,  $\frac{2}{7}$ }

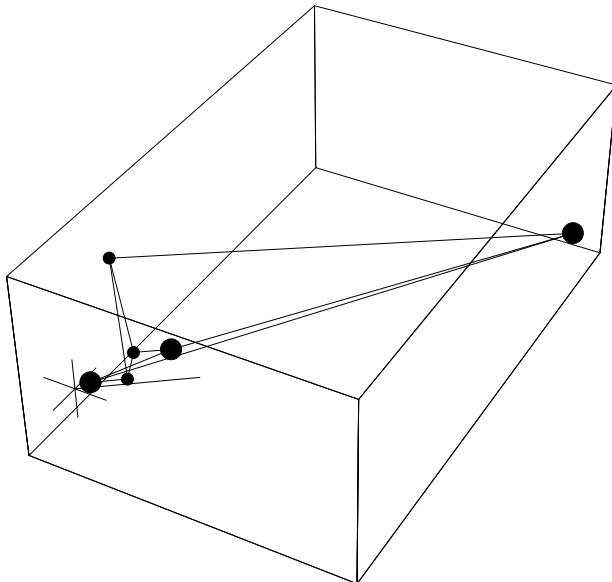
% // N
{0.428571, 0.142857, 0.285714}

N3 = Proj.T3
{ $\frac{18}{7}$ ,  $\frac{6}{7}$ ,  $\frac{12}{7}$ }

% // N
{2.57143, 0.857143, 1.71429}

o = {0, 0, 0};
Show[Graphics3D[
  {
    Line[{{-1, 0, 0}, {1, 0, 0}}],
    Line[{{0, -1, 0}, {0, 1, 0}}], Line[{{0, 0, -1}, {0, 0, 1}}],
    Line[{N1, N2, N3, N1}], Line[{T1, T2, T3, T1}],
    Line[{o, u}],
    Line[{T1, N1}], Line[{T2, N2}], Line[{T3, N3}],
    PointSize[0.02], Point[T1], Point[T2],
    Point[T3], PointSize[0.035], Point[N1], Point[N2], Point[N3]
  }
], AspectRatio -> Automatic, PlotRange -> {{-1, 9}, {-1, 15}, {-2, 4}}];

```



5

$$\mathbf{M1} = (\mathbf{T1} + \mathbf{N1}) / 2$$

$$\left\{ \frac{29}{7}, \frac{115}{14}, \frac{13}{14} \right\}$$

N[%]

$$\{4.14286, 8.21429, 0.928571\}$$

$$\mathbf{M2} = (\mathbf{T2} + \mathbf{N2}) / 2$$

$$\left\{ \frac{5}{7}, \frac{4}{7}, \frac{1}{7} \right\}$$

N[%]

$$\{0.714286, 0.571429, 0.142857\}$$

$$\mathbf{M3} = (\mathbf{T3} + \mathbf{N3}) / 2$$

$$\left\{ \frac{16}{7}, \frac{3}{7}, \frac{13}{7} \right\}$$

N[%]

$$\{2.28571, 0.428571, 1.85714\}$$

Matr = Transpose[{M1, M2, M3}].Inverse[Transpose[{T1, T2, T3}]]; Matr // MatrixForm

$$\begin{pmatrix} \frac{1}{7} & \frac{4}{7} & 1 \\ -\frac{9}{7} & \frac{13}{7} & \frac{3}{2} \\ \frac{3}{7} & -\frac{2}{7} & \frac{1}{2} \end{pmatrix}$$

N[%] // MatrixForm

$$\begin{pmatrix} 0.142857 & 0.571429 & 1. \\ -1.28571 & 1.85714 & 1.5 \\ 0.428571 & -0.285714 & 0.5 \end{pmatrix}$$

Eigensystem[Matr] // N

$$\{\{1., 1., 0.5\}, \{1.16667, 0., 1.\}, \{0.666667, 1., 0.\}, \{-2., -3., 1.\}\}$$

a={3,1,2}; b={-1,2,-2}; u={2,3,-1};

Auf den Ersten Blick fällt auf, dass unter den Eigenvektoren -u und daher u (gestreckt) vorkommt. Der dazugehörige Eigenwert ist 0.5, während die andern beiden 1 sind (keine Streckung in diese Richtungen).

Matr.T1

$$\left\{ \frac{29}{7}, \frac{115}{14}, \frac{13}{14} \right\}$$

Richtig

Matr.T2

$$\left\{ \frac{5}{7}, \frac{4}{7}, \frac{1}{7} \right\}$$

Richtig

Matr.T3

$$\left\{ \frac{16}{7}, \frac{3}{7}, \frac{13}{7} \right\}$$

Richtig

6**Remove["Global`*"]****a = {3, 1, 2};****b1 = a / Norm[a]**

$$\left\{ \frac{3}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \sqrt{\frac{2}{7}} \right\}$$

{1, y1, 0}.a == 0

$$3 + y1 == 0$$

Solve[{1, y1, 0}.a == 0, {y1}]

$$\{\{y1 \rightarrow -3\}\}$$

b2work = {1, -3, 0}

$$\{1, -3, 0\}$$

b2 = b2work / Norm[b2work]

$$\left\{ \frac{1}{\sqrt{10}}, -\frac{3}{\sqrt{10}}, 0 \right\}$$

b3 = Cross[b1, b2]

$$\left\{ \frac{3}{\sqrt{35}}, \frac{1}{\sqrt{35}}, -\sqrt{\frac{5}{7}} \right\}$$

U = Transpose[{b1, b2, b3}]; U // MatrixForm

$$\begin{pmatrix} \frac{3}{\sqrt{14}} & \frac{1}{\sqrt{10}} & \frac{3}{\sqrt{35}} \\ \frac{1}{\sqrt{14}} & -\frac{3}{\sqrt{10}} & \frac{1}{\sqrt{35}} \\ \sqrt{\frac{2}{7}} & 0 & -\sqrt{\frac{5}{7}} \end{pmatrix}$$

```
Dreh0[φ_] := {{1, 0, 0}, {0, Cos[φ], -Sin[φ]}, {0, Sin[φ], Cos[φ]}};  
Dreh0[φ] // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos[\varphi] & -\sin[\varphi] \\ 0 & \sin[\varphi] & \cos[\varphi] \end{pmatrix}$$

```
DrehAchse[φ_] := U.Dreh0[φ].Inverse[U]
```

```
Dreh = DrehAchse[56 Degree] // N; Dreh // MatrixForm
```

$$\begin{pmatrix} 0.842569 & -0.348681 & 0.410487 \\ 0.537598 & 0.590679 & -0.601736 \\ -0.0326523 & 0.727681 & 0.685138 \end{pmatrix}$$

```
T1 = {0, 2, 3}; T2 = {1, 1, 0}; T3 = {2, 0, 2};
```

```
R1 = Dreh.T1
```

```
{0.5341, -0.623851, 3.51078}
```

```
R2 = Dreh.T2
```

```
{0.493888, 1.12828, 0.695029}
```

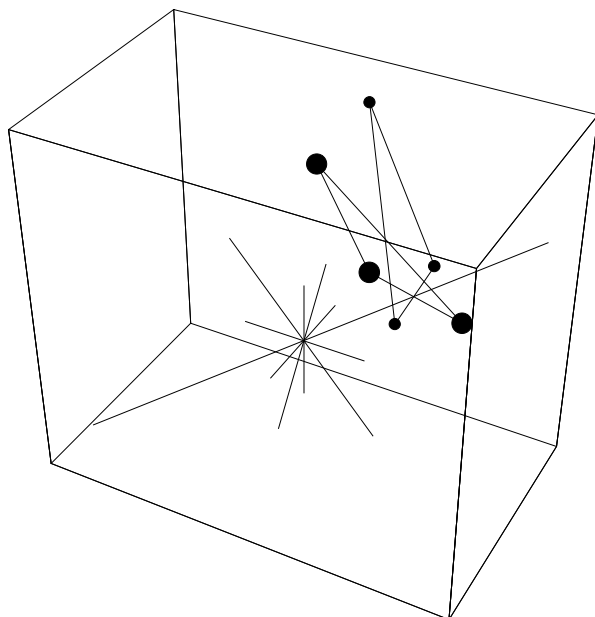
```
R3 = Dreh.T3
```

```
{2.50611, -0.128277, 1.30497}
```

```

o = {0, 0, 0};
Show[Graphics3D[
  {
    Line[{{-1, 0, 0}, {1, 0, 0}}],
    Line[{{0, -1, 0}, {0, 1, 0}}, Line[{{0, 0, -1}, {0, 0, 1}}],
    Line[{-4 b1, 4 b1}], Line[{-2 b2, 2 b2}], Line[{-2 b3, 2 b3}],
    Line[{R1, R2, R3, R1}], Line[{T1, T2, T3, T1}],
    PointSize[0.02], Point[T1], Point[T2],
    Point[T3], PointSize[0.035], Point[R1], Point[R2], Point[R3]
  }
], AspectRatio -> Automatic];

```



7

a Berechnung der Punkte

```
Remove["Global`*"]
```

```
a = {3, 1, 2};
```

```
b = {-1, 3, 0};
```

```
c = Cross[a, b]
```

```
{-6, -2, 10}
```

```
B = Transpose[{a, b, c}]; B // MatrixForm
```

```

( 3  -1  -6 )
( 1  3  -2 )
( 2  0  10 )

```

$D\lambda = \{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, -1\}\}; D\lambda // \text{MatrixForm}$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$A = B.D\lambda.\text{Inverse}[B]; A // \text{MatrixForm}$

$$\begin{pmatrix} \frac{17}{35} & -\frac{6}{35} & \frac{6}{7} \\ -\frac{6}{35} & \frac{33}{35} & \frac{2}{7} \\ \frac{6}{7} & \frac{2}{7} & -\frac{3}{7} \end{pmatrix}$$

$T1 = \{0, 2, 3\}; T2 = \{1, 1, 0\}; T3 = \{2, 0, 2\};$

$S1 = A.T1$

$$\left\{ \frac{78}{35}, \frac{96}{35}, -\frac{5}{7} \right\}$$

$S2 = A.T2$

$$\left\{ \frac{11}{35}, \frac{27}{35}, \frac{8}{7} \right\}$$

$S3 = A.T3$

$$\left\{ \frac{94}{35}, \frac{8}{35}, \frac{6}{7} \right\}$$

$M1 = (T1 + S1) / 2$

$$\left\{ \frac{39}{35}, \frac{83}{35}, \frac{8}{7} \right\}$$

$M2 = (T2 + S2) / 2$

$$\left\{ \frac{23}{35}, \frac{31}{35}, \frac{4}{7} \right\}$$

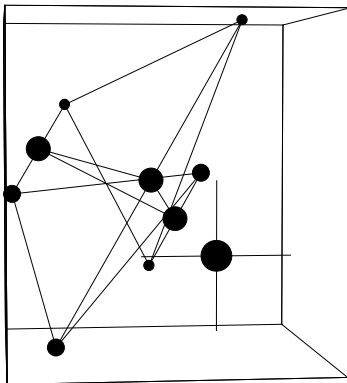
$M3 = (T3 + S3) / 2$

$$\left\{ \frac{82}{35}, \frac{4}{35}, \frac{10}{7} \right\}$$

```

o = {0, 0, 0};
Show[Graphics3D[
  {
    Line[{{-1, 0, 0}, {1, 0, 0}}],
    Line[{{0, -1, 0}, {0, 1, 0}}, Line[{{0, 0, -1}, {0, 0, 1}}],
    Line[{S1, S2, S3, S1}], Line[{T1, T2, T3, T1}], Line[{M1, M2, M3, M1}],
    Line[{S1, T1}], Line[{S2, T2}], Line[{S3, T3}],
    PointSize[0.03], Point[T1], Point[T2], Point[T3],
    PointSize[0.05], Point[S1], Point[S2], Point[S3], PointSize[0.07],
    Point[M1], Point[M2], Point[M3], PointSize[0.09], Point[o]
  }
], AspectRatio -> Automatic, ViewPoint -> {0.478, 4.341, 0.259}];

```



b Kontrolle: Liegen die Punkte M_k in der Ebene ?

```
Solve[ $\lambda \mathbf{a} + \mu \mathbf{b} + \nu \mathbf{c} == \mathbf{M1}$ , { $\lambda$ ,  $\mu$ ,  $\nu$ }]
```

```
{{ $\lambda \rightarrow \frac{4}{7}$ ,  $\mu \rightarrow \frac{3}{5}$ ,  $\nu \rightarrow 0$ }}
```

```
Solve[ $\lambda \mathbf{a} + \mu \mathbf{b} + \nu \mathbf{c} == \mathbf{M2}$ , { $\lambda$ ,  $\mu$ ,  $\nu$ }]
```

```
{{ $\lambda \rightarrow \frac{2}{7}$ ,  $\mu \rightarrow \frac{1}{5}$ ,  $\nu \rightarrow 0$ }}
```

```
Solve[ $\lambda \mathbf{a} + \mu \mathbf{b} + \nu \mathbf{c} == \mathbf{M3}$ , { $\lambda$ ,  $\mu$ ,  $\nu$ }]
```

```
{{ $\lambda \rightarrow \frac{5}{7}$ ,  $\mu \rightarrow -\frac{1}{5}$ ,  $\nu \rightarrow 0$ }}
```

In Ordnung, denn der Koeffizient für die Richtung \mathbf{c} ist immer 0.