

Lösungen

1

Lösung: Siehe Skript Analysis von R. Wirz
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2

```
Remove["Global`*"]
```

```
f[{x_,y_]:=1-x^2-2(y+1)^2;  
f[x,y]
```

$$1 - x^2 - 2(1 + y)^2$$

```
f[{x_,y_}]:=f[x,y];  
p={0,2};
```

a

```
grad[f_,x_,y_]:=D[f,x],D[f,y];  
grad[f[x,y],x,y]
```

$$\{-2x, -4(1+y)\}$$

```
Solve[Evaluate[grad[f[x,y],x,y]==0],{x,y}]
```

$$\{\{x \rightarrow 0, y \rightarrow -1\}\}$$

b

```
e30={Cos[30 Degree],Sin[30 Degree]}
```

$$\left\{\frac{\sqrt{3}}{2}, \frac{1}{2}\right\}$$

```
grad[f[x,y],x,y].e30
```

$$-\sqrt{3}x - 2(1+y)$$

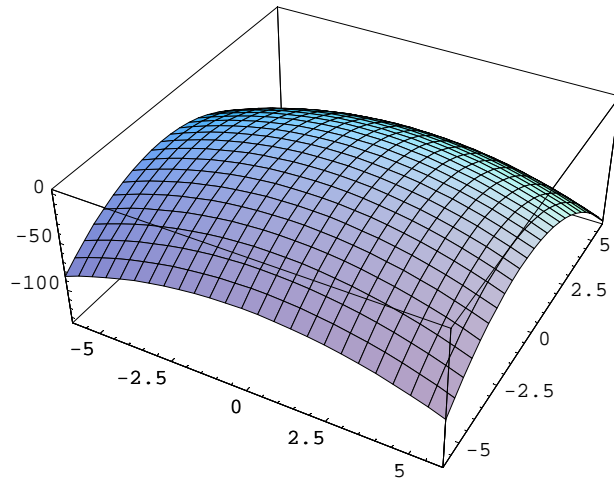
```
grad[f[x,y],x,y].e30 /. {x->0,y->2}
```

$$-6$$

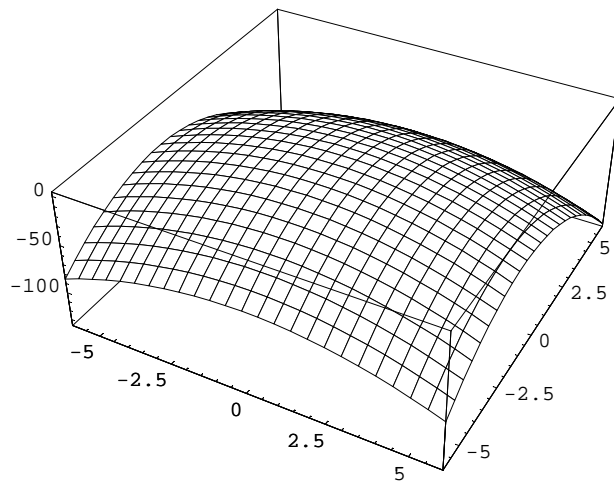
c`ArcTan[-6] // N``-1.40565``ArcTan[-6]/Degree // N``-80.5377`**d**`Norm[grad[f[x,y],x,y]]`
$$\sqrt{4 \text{Abs}[x]^2 + 16 \text{Abs}[1+y]^2}$$
`Norm[grad[f[x,y],x,y]] /. {x->0,y->2}``12``ArcTan[12] // N``1.48766``ArcTan[12]/Degree // N``85.2364`**e**`gradP = grad[f[x,y],x,y] /. {x->0,y->2}``{0, -12}``ϕ[x_, y_] := f[p] + gradP[[1]] (x - p[[1]]) + gradP[[2]] (y - p[[2]]);``ϕ[x, y]``-17 - 12 (-2 + y)``Solve[{ϕ[x,y]==0,y==0},{x}]``{}``%// N``{}`

f

```
p11=Plot3D[f[x,y],{x,-6,6},{y,-6,6}];
```



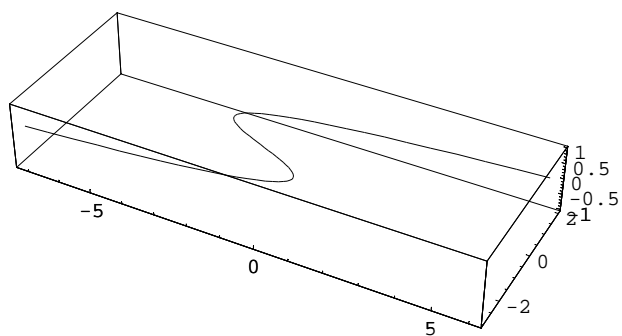
```
p11=Plot3D[f[x,y],{x,-6,6},{y,-6,6},Shading -> False];
```



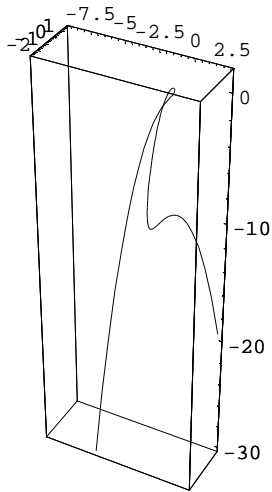
```
v[t_]:= {Expand[t(t-1)(t+2)-2],t}; v[t]
```

```
{-2-2t+t^2+t^3, t}
```

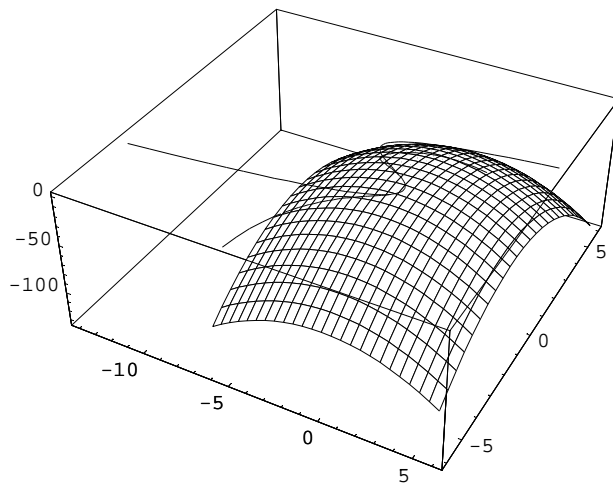
```
p12=ParametricPlot3D[{v[t][[1]],v[t][[2]],0},{t,-3,2}];
```



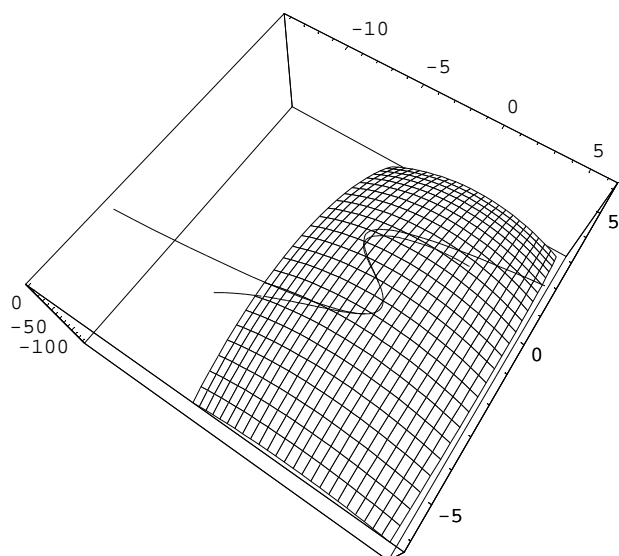
```
p13=ParametricPlot3D[{v[t][[1]],v[t][[2]],f[v[t]]},{t,-2.7,1.7};
```



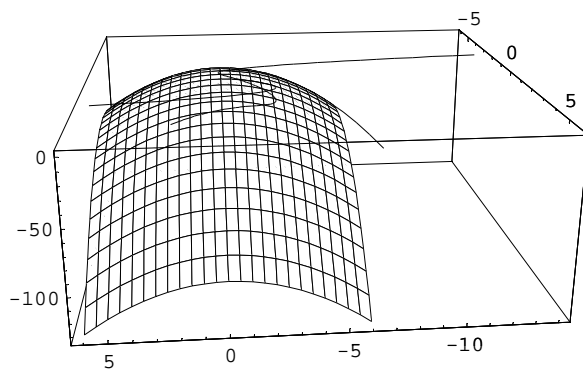
```
Show[p11,p13,p12];
```



```
Show[p11,p13,p12,ViewPoint->{0.424, -0.630, 1.583}];
```



```
Show[p11,p13,p12,ViewPoint->{0.210, 2.200, 0.760}];
```



g

```
v[t_]:= {Expand[t(t-1)(t+2)-2],t}; v[t]
```

```
{-2-2 t+t2+t3, t}
```

```
g[x_,y_]:= Evaluate[v[t][[1]] - x /.t->y];
```

```
g[x,y] ==0
```

```
-2 - x - 2 y + y2 + y3 == 0
```

```
grad[f[x,y],x,y]== λ grad[g[x,y],x,y]
```

```
{-2 x, -4 (1+y)} == {-λ, (-2 + 2 y + 3 y2) λ}
```

```
NSolve[{grad[f[x,y],x,y]== λ grad[g[x,y],x,y], g[x,y]==0}, {λ,x,y}]
```

```
{{λ → -4.92498, x → -2.46249, y → 0.790255}, {λ → -1.66997, x → -0.834987, y → 1.27807},  
{λ → 0.778958 - 0.50778 i, x → 0.389479 - 0.25389 i, y → -1.3675 - 0.33188 i},  
{λ → 0.778958 + 0.50778 i, x → 0.389479 + 0.25389 i, y → -1.3675 + 0.33188 i},  
{λ → 0., x → 0., y → -1.}}
```

```

f[v[t]]
1 - 2 (1 + t)2 - (-2 - 2 t + t2 + t3)2

solv1 = Solve[Evaluate[D[f[v[t]],t]==0},{t]} //N // Flatten
{t → -1., t → -1.3675 - 0.33188 i, t → -1.3675 + 0.33188 i, t → 0.790255, t → 1.27807}

v[t] /. solv1[[1]]
{0., -1.}

f[v[t]] /. solv1[[1]]
1.

v[t] /. solv1[[4]]
{-2.46249, 0.790255}

f[v[t]] /. solv1[[4]]
-11.4739

v[t] /. solv1[[5]]
{-0.834987, 1.27807}

f[v[t]] /. solv1[[5]]
-10.0764

```

==> Punkt {0,-1,1}

h

```

D[f[v[t]],t]
-4 (1 + t) - 2 (-2 + 2 t + 3 t2) (-2 - 2 t + t2 + t3)

D[f[v[t]],t] // Expand
-12 - 4 t + 24 t2 + 12 t3 - 10 t4 - 6 t5

```

3

```

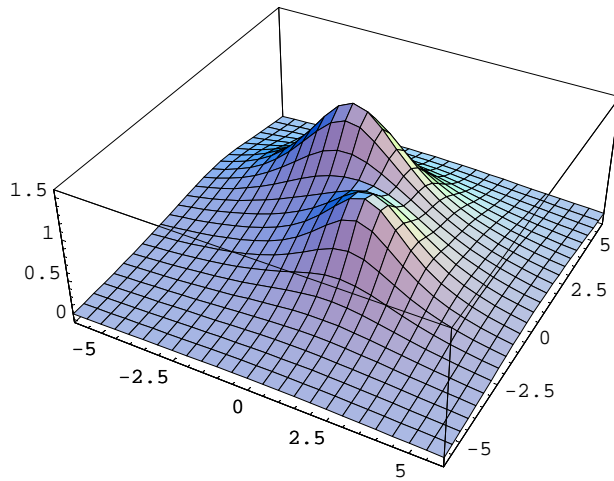
Remove["Global`*"]

f[x_,y_]:=10/(3 (x-1)^2+5(y+1)^2+10)+ 14/(2 (x+1)^2+5(y-2)^2+10)

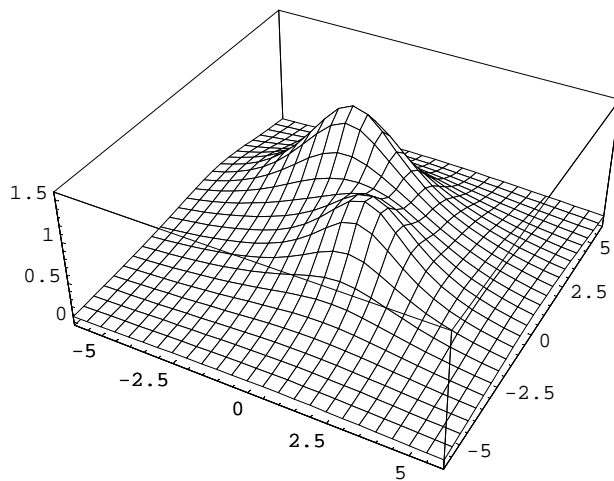
```

a

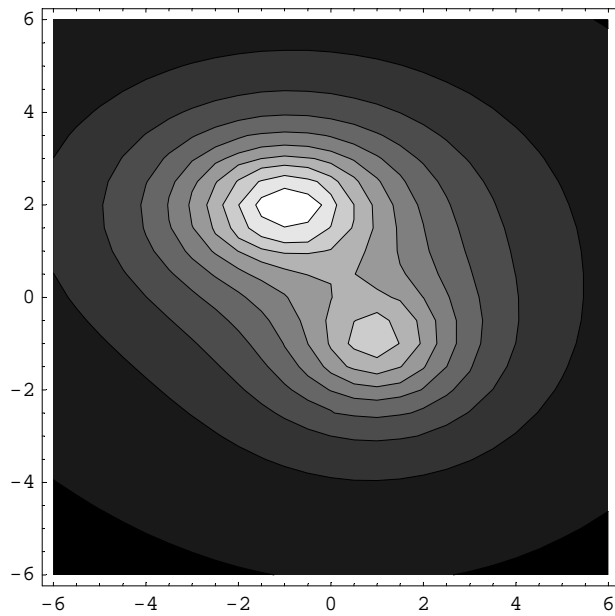
```
pl1=Plot3D[f[x,y],{x,-6,6},{y,-6,6}];
```



```
pl1=Plot3D[f[x,y],{x,-6,6},{y,-6,6},Shading -> False];
```



```
ContourPlot[f[x,y],{x,-6,6},{y,-6,6}];
```



b

```
grad[f_,x_,y_]:= {D[f,x],D[f,y]};
grad[f[x,y],x,y]
```

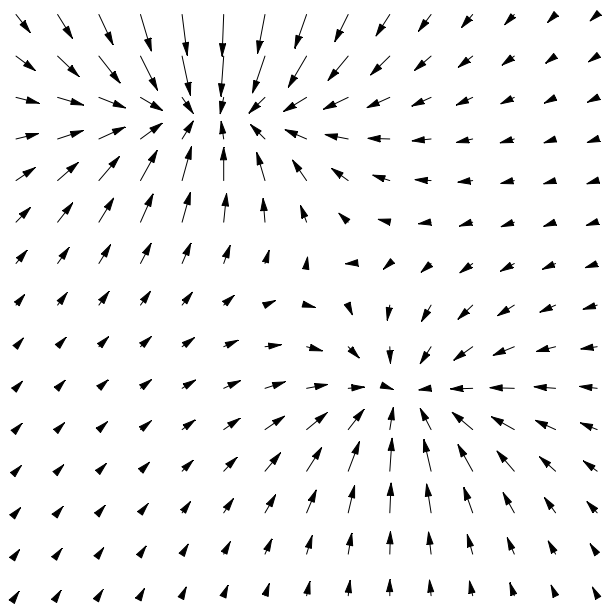
$$\left\{ \begin{aligned} &-\frac{56(1+x)}{(10+2(1+x)^2+5(-2+y)^2)^2} - \frac{60(-1+x)}{(10+3(-1+x)^2+5(1+y)^2)^2}, \\ &-\frac{140(-2+y)}{(10+2(1+x)^2+5(-2+y)^2)^2} - \frac{100(1+y)}{(10+3(-1+x)^2+5(1+y)^2)^2} \end{aligned} \right\}$$

```
grad[f[x,y],x,y] // Simplify
```

$$\left\{ \begin{aligned} &-\frac{56(1+x)}{(10+2(1+x)^2+5(-2+y)^2)^2} - \frac{60(-1+x)}{(10+3(-1+x)^2+5(1+y)^2)^2}, \\ &20 \left(-\frac{7(-2+y)}{(10+2(1+x)^2+5(-2+y)^2)^2} - \frac{5(1+y)}{(10+3(-1+x)^2+5(1+y)^2)^2} \right) \end{aligned} \right\}$$

```
<<Graphics`PlotField`
```

```
PlotGradientField[f[x,y],
  {x, -3, 3}, {y, -3, 3}];
```



```
NSolve[Evaluate[grad[f[x,y],x,y]==0],{x,y}]
```

```
{x -> 3.66929 - 11.2219 i, y -> 7.62486 + 3.16354 i},
{x -> 3.66929 + 11.2219 i, y -> 7.62486 - 3.16354 i},
{x -> 5.7046 - 10.9257 i, y -> 8.21161 + 3.28133 i},
{x -> 5.7046 + 10.9257 i, y -> 8.21161 - 3.28133 i},
{x -> 1.01289 - 1.67828 i, y -> 0.329888 + 3.22855 i},
{x -> 1.01289 + 1.67828 i, y -> 0.329888 - 3.22855 i}, {x -> -0.950244, y -> 1.94983},
{x -> 0.947227, y -> -0.882807}, {x -> 0.293963, y -> 0.350251}}
```

C

```
e30={Cos[30 Degree],Sin[30 Degree]}
```

$$\left\{ \frac{\sqrt{3}}{2}, \frac{1}{2} \right\}$$

```
grad[f[x,y],x,y].e30
```

$$\frac{1}{2} \sqrt{3} \left(-\frac{56(1+x)}{(10+2(1+x)^2+5(-2+y)^2)^2} - \frac{60(-1+x)}{(10+3(-1+x)^2+5(1+y)^2)^2} \right) +$$

$$\frac{1}{2} \left(-\frac{140(-2+y)}{(10+2(1+x)^2+5(-2+y)^2)^2} - \frac{100(1+y)}{(10+3(-1+x)^2+5(1+y)^2)^2} \right)$$

```
%//N
```

$$0.866025 \left(-\frac{56.(1.+x)}{(10.+2.(1.+x)^2+5.(-2.+y)^2)^2} - \frac{60.(-1.+x)}{(10.+3.(-1.+x)^2+5.(1.+y)^2)^2} \right) +$$

$$0.5 \left(-\frac{140.(-2.+y)}{(10.+2.(1.+x)^2+5.(-2.+y)^2)^2} - \frac{100.(1.+y)}{(10.+3.(-1.+x)^2+5.(1.+y)^2)^2} \right)$$

```
grad[f[x,y],x,y].e30 /. {x->0,y->2}
```

$$-\frac{75}{1682} - \frac{5617}{10092\sqrt{3}}$$

```
%//N
```

```
-0.365931
```

d

i

```
f[{x_,y_}]:=f[x,y];
p={0,2};
```

```
gradP = grad[f[x,y],x,y] /. {x->0,y->2}
```

$$\left\{-\frac{5617}{15138}, -\frac{75}{841}\right\}$$

```
(gradP = grad[f[x,y],x,y] /. {x->0,y->2} ) //N
```

```
{-0.371053, -0.0891795}
```

```
ϕ[x_, y_] := f[p] + gradP[[1]] (x - p[[1]]) + gradP[[2]] (y - p[[2]]);
```

```
ϕ[x, y]
```

$$\frac{233}{174} - \frac{5617x}{15138} - \frac{75}{841}(-2+y)$$

```
%//N //Simplify
```

```
1.51744 - 0.371053 x - 0.0891795 y
```

```
Solve[{ϕ[x,y]==z,x==0,y==0},{z}]
```

$$\left\{\left\{z \rightarrow \frac{7657}{5046}\right\}\right\}$$

```
%// N
```

```
{z -> 1.51744}
```

ii

```
Norm[grad[f[x,y],x,y]]
```

$$\sqrt{\left(\text{Abs}\left[-\frac{56(1+x)}{(10+2(1+x)^2+5(-2+y)^2)} - \frac{60(-1+x)}{(10+3(-1+x)^2+5(1+y)^2)}\right]^2 + \text{Abs}\left[-\frac{140(-2+y)}{(10+2(1+x)^2+5(-2+y)^2)} - \frac{100(1+y)}{(10+3(-1+x)^2+5(1+y)^2)}\right]^2\right)}$$

```
Norm[grad[f[x,y],x,y]] /. {x->0,y->2}
```

$$\frac{\sqrt{33373189}}{15138}$$

```
%/N
```

```
0.381619
```

```
ArcTan[%] // N
```

```
0.364561
```

```
ArcTan[%]/Degree // N
```

```
20.8878
```

iii

```
g[x_,y_]:= y-x^2
```

```
solv2= NSolve[{grad[f[x,y],x,y]== λ grad[g[x,y],x,y], g[x,y]==0}, {λ,x,y}]
```

```
{λ → 0.000167092 - 0.00649497 i, x → -1.33529 + 1.36828 i, y → -0.0891819 - 3.65412 i},
{λ → 0.000167092 + 0.00649497 i, x → -1.33529 - 1.36828 i, y → -0.0891819 + 3.65412 i},
{λ → 0.0823729, x → -1.37488, y → 1.89028},
{λ → -0.0128902 - 0.00987242 i, x → 1.20431 - 1.29879 i, y → -0.236493 - 3.12829 i},
{λ → -0.0128902 + 0.00987242 i, x → 1.20431 + 1.29879 i, y → -0.236493 + 3.12829 i},
{λ → -0.0830247 + 0.273961 i, x → -0.170884 + 1.16901 i, y → -1.33739 - 0.399531 i},
{λ → -0.0830247 - 0.273961 i, x → -0.170884 - 1.16901 i, y → -1.33739 + 0.399531 i},
{λ → 0.130264, x → -0.441818, y → 0.195204},
{λ → 0.118471 - 0.0209213 i, x → 1.05079 - 0.244279 i, y → 1.04449 - 0.513372 i},
{λ → 0.118471 + 0.0209213 i, x → 1.05079 + 0.244279 i, y → 1.04449 + 0.513372 i},
{λ → -0.0934765, x → 0.318855, y → 0.101668}}
```

```
Table[{k,solv2[[k]]},{k,1,Length[solv2]}//MatrixForm
```

```
{ 1 {λ → 0.000167092 - 0.00649497 i, x → -1.33529 + 1.36828 i, y → -0.0891819 - 3.65412 i}
 2 {λ → 0.000167092 + 0.00649497 i, x → -1.33529 - 1.36828 i, y → -0.0891819 + 3.65412 i}
 3 {λ → 0.0823729, x → -1.37488, y → 1.89028}
 4 {λ → -0.0128902 - 0.00987242 i, x → 1.20431 - 1.29879 i, y → -0.236493 - 3.12829 i}
 5 {λ → -0.0128902 + 0.00987242 i, x → 1.20431 + 1.29879 i, y → -0.236493 + 3.12829 i}
 6 {λ → -0.0830247 + 0.273961 i, x → -0.170884 + 1.16901 i, y → -1.33739 - 0.399531 i}
 7 {λ → -0.0830247 - 0.273961 i, x → -0.170884 - 1.16901 i, y → -1.33739 + 0.399531 i}
 8 {λ → 0.130264, x → -0.441818, y → 0.195204}
 9 {λ → 0.118471 - 0.0209213 i, x → 1.05079 - 0.244279 i, y → 1.04449 - 0.513372 i}
10 {λ → 0.118471 + 0.0209213 i, x → 1.05079 + 0.244279 i, y → 1.04449 + 0.513372 i}
11 {λ → -0.0934765, x → 0.318855, y → 0.101668}}
```

```
{solv2[[3]],solv2[[8]],solv2[[11]]}
```

```
{λ → 0.0823729, x → -1.37488, y → 1.89028}, {λ → 0.130264, x → -0.441818, y → 0.195204},
{λ → -0.0934765, x → 0.318855, y → 0.101668}}
```

```
f[x,y]/.solv2[[3]]
```

```
1.49939
```

```
f[x,y]/.solv2[[8]]
```

```
0.947994
```

```
f[x,y]/.solv2[[11]]
```

```
1.01722
```

Punkt { -1.37488, 1.89028, 1.49939 }

4

a

```
Series[E^(-x^2), {x, 0, 8}]
```

$$1 - x^2 + \frac{x^4}{2} - \frac{x^6}{6} + \frac{x^8}{24} + O[x]^9$$

```
N[%]
```

$$1. - (x + 0.)^2 + 0.5 (x + 0.)^4 - 0.166667 (x + 0.)^6 + 0.0416667 (x + 0.)^8 + O[x + 0.]^9$$

b

```
Evaluate[Normal[Series[E^(-x^2), {x, 0, 8}]]]
```

$$1 - x^2 + \frac{x^4}{2} - \frac{x^6}{6} + \frac{x^8}{24}$$

```
N[%]
```

$$1. - 1. x^2 + 0.5 x^4 - 0.166667 x^6 + 0.0416667 x^8$$

```
Integrate[Evaluate[Normal[Series[E^(-x^2), {x, 0, 8}]]], {x, -2, 2}]
```

$$\frac{3508}{945}$$

```
%/N
```

$$3.71217$$

```
Integrate[Evaluate[Normal[Series[E^(-x^2), {x, 0, 100}]]], {x, -2, 2}]/N
```

$$1.76416$$

```
Integrate[E^(-x^2), {x, -2, 2}]/N
```

$$1.76416$$

c

```
Evaluate[Normal[Series[Cos[x^2]+E^(x^2), {x, 0, 8}]]]
```

$$2 + x^2 + \frac{x^6}{6} + \frac{x^8}{12}$$

```
N[%]
```

$$2. + x^2 + 0.166667 x^6 + 0.0833333 x^8$$

```
Integrate[Evaluate[Normal[Series[E^(-x^2),{x,0,8}]]],{x,-2,2}]
```

$$\frac{3508}{945}$$

```
%//N
```

```
3.71217
```

```
Integrate[E^(-x^2),{x,-2,2}]/N
```

```
1.76416
```

```
Integrate[Evaluate[Normal[Series[E^(-x^2),{x,0,100}]]],{x,-2,2}]/N
```

```
1.76416
```

d

```
Evaluate[Normal[Series[Sqrt[x],{x,1,6}]]]
```

$$1 + \frac{1}{2}(-1+x) - \frac{1}{8}(-1+x)^2 + \frac{1}{16}(-1+x)^3 - \frac{5}{128}(-1+x)^4 + \frac{7}{256}(-1+x)^5 - \frac{21}{1024}(-1+x)^6$$

```
N[%]
```

$$1. + 0.5(-1. + x) - 0.125(-1. + x)^2 + 0.0625(-1. + x)^3 - 0.0390625(-1. + x)^4 + 0.0273438(-1. + x)^5 - 0.0205078(-1. + x)^6$$

e

```
r=1
```

```
1
```

d

```
Evaluate[Normal[Series[Sqrt[x],{x,1,6}]]]
```

$$1 + \frac{1}{2}(-1+x) - \frac{1}{8}(-1+x)^2 + \frac{1}{16}(-1+x)^3 - \frac{5}{128}(-1+x)^4 + \frac{7}{256}(-1+x)^5 - \frac{21}{1024}(-1+x)^6$$

```
N[%]
```

$$1. + 0.5(-1. + x) - 0.125(-1. + x)^2 + 0.0625(-1. + x)^3 - 0.0390625(-1. + x)^4 + 0.0273438(-1. + x)^5 - 0.0205078(-1. + x)^6$$

e

```
r=1
```

```
1
```

f**Evaluate[Normal[Series[Log[x]-Sin[x],{x,1,6}]]]**

$$(-1+x)(1-\cos[1]) + (-1+x)^5 \left(\frac{1}{5} - \frac{\cos[1]}{120} \right) + (-1+x)^3 \left(\frac{1}{3} + \frac{\cos[1]}{6} \right) +$$

$$(-1+x)^4 \left(-\frac{1}{4} - \frac{\sin[1]}{24} \right) + (-1+x)^6 \left(-\frac{1}{6} + \frac{\sin[1]}{720} \right) + (-1+x)^2 \left(-\frac{1}{2} + \frac{\sin[1]}{2} \right) - \sin[1]$$

N[%]

$$-0.841471 + 0.459698 (-1. + x) - 0.0792645 (-1. + x)^2 + 0.423384 (-1. + x)^3 -$$

$$0.285061 (-1. + x)^4 + 0.195497 (-1. + x)^5 - 0.165498 (-1. + x)^6$$

g**1-1/(1-1/2)+1/(1-1/3)**

$$\frac{1}{2}$$

N[%]

$$0.5$$

h**Sum[1/k,{k,1,n}]**

HarmonicNumber[n]

Limit[Sum[1/k,{k,1,n}],{n->Infinity}]

{∞}

Limit[Sum[1/k,{k,1,n}]-Log[n],{n->Infinity}]

{EulerGamma}

%/N

{0.577216}

\$Aborted