Lösungen

1

a

```
f[x_]:=4x^4+5x^5-6x^6+x^0-2/x^2
```

```
f[x]
```

$$1-\frac{2}{x^2}+4x^4+5x^5-6x^6$$

i

```
Integrate[f[x],x]+c
```

$$c+\frac{2}{x}+x+\frac{4x^5}{5}+\frac{5x^6}{6}-\frac{6x^7}{7}$$

```
N[%]
```

$$c+\frac{2.}{x}+x+0.8x^5+0.833333x^6-0.857143x^7$$

ii

```
Integrate[f[x],{x,1,2}]
```

$$-\frac{2209}{70}$$

```
N[%]
```

$$-31.5571$$

iii

```
Integrate[f[x],{x,-1,1}]
```

```
Integrate::idiv : Integral of 1-2/x^2+4x^4+5x^5-6x^6 does not converge on {-1, 1}. Mehr…
```

$$\int_{-1}^{1}\left(1-\frac{2}{x^2}+4x^4+5x^5-6x^6\right)\,dx$$

iv

D[Evaluate[Integrate[f[x],{x,0,t}]],t]

Integrate::idiv : Integral of $1-\frac{2}{x^2}+4x^4+5x^5-6x^6$ does not converge on $\{0, t\}$. Mehr…

$$1-\frac{2}{t^2}+4t^4+5t^5-6t^6$$

b

f[x_]:=1/Sin[x/3-3]^2

f[x]

$$\text{Csc}\left[3-\frac{x}{3}\right]^2$$

i

Integrate[f[x],x]+c

$$c+3\,\text{Cot}\left[3-\frac{x}{3}\right]$$

N[%]

$$c+3.\,\text{Cot}[3.-0.333333\,x]$$

ii

Integrate[f[x],{x,2,t}]

$$(-2+t)\,\text{If}\left[\text{Re}[t]\le 9\,||\,\text{Im}[t]\neq 0,\ \frac{3\,(-\text{Cot}[\frac{7}{3}]+\text{Cot}[3-\frac{t}{3}])}{-2+t},\ \text{Integrate}\left[\text{Csc}\left[\frac{1}{3}\,(7-(-2+t)\,x)\right]^2,\{x,0,1\},\text{Assumptions}\to \text{Im}[t]==0\,\&\&\,\text{Re}[t]>9\right]\right]$$

Assuming[t ∈ Reals, Integrate[f[x], {x, 2, t}]]

$$\text{If}\left[2<t\le 9,\ -3\,\text{Cot}\left[\frac{7}{3}\right]+3\,\text{Cot}\left[3-\frac{t}{3}\right],\ \text{Integrate}\left[\text{Csc}\left[3-\frac{x}{3}\right]^2,\{x,2,t\},\text{Assumptions}\to t\le 2\,||\,t>9\right]\right]$$

Integrate[f[x],{x,2,t},GenerateConditions→False]

$$-3\,\text{Cot}\left[\frac{7}{3}\right]+3\,\text{Cot}\left[3-\frac{t}{3}\right]$$

D[Evaluate[Integrate[f[x],{x,2,t},GenerateConditions→False]],t]

$$\text{Csc}\left[3-\frac{t}{3}\right]^2$$

Integrate[Evaluate[Evaluate[D[Evaluate[Integrate[f[x],{x,2,t},GenerateConditions→False]],t]]],{t,1,u},GenerateConditions→False]

$$-3\,\text{Cot}\left[\frac{8}{3}\right]+3\,\text{Cot}\left[3-\frac{u}{3}\right]$$

c

f[x_]:=1/(x(x+2)(x-2))

f[x]

$$\frac{1}{(-2+x)\,x\,(2+x)}$$

Apart[f[x]]

$$\frac{1}{8\,(-2+x)}-\frac{1}{4\,x}+\frac{1}{8\,(2+x)}$$

i

Integrate[f[x],x]+c

$$c-\frac{\text{Log}[x]}{4}+\frac{1}{8}\,\text{Log}[-4+x^2]$$

Integrate[f[x],{x,3,5}]

$$\frac{1}{8}\,\text{Log}\left[\frac{189}{125}\right]$$

N[%]

```
0.0516792
```

ii

Integrate[f[x],{x,3,Infinity}]

$$\frac{1}{8}\,\text{Log}\left[\frac{9}{5}\right]$$

N[%]

```
0.0734733
```

d

f[x_]:=7 Log[x]-(E^x-E^(-x))/E^(3x)

f[x]

$$-e^{-3x}\,(-e^{-x}+e^{x})+7\,\text{Log}[x]$$

i

Integrate[f[x],{x,1,E}]

$$\frac{1}{4}\left(28+\frac{1}{e^4}-\frac{2}{e^2}-e^{-4e}+2\,e^{-2e}\right)$$

N[%]

6.93908

e

f[x_]:=x Log[x]^2+x

f[x]

$$x + x \, \text{Log}[x]^2$$

i

Integrate[f[x],x]+c

$$c + \frac{3 x^2}{4} - \frac{1}{2} x^2 \, \text{Log}[x] + \frac{1}{2} x^2 \, \text{Log}[x]^2$$

N[%]

$$c + 0.75 x^2 - 0.5 x^2 \, \text{Log}[x] + 0.5 x^2 \, \text{Log}[x]^2$$

ii

Integrate[f[x],{x,t,2t}]

$$\frac{1}{4} t^2 \, (9 + 8 \, \text{Log}[2]^2 - 4 \, \text{Log}[4] + 2 \, \text{Log}[t] \, (-3 + \text{Log}[256] + 3 \, \text{Log}[t]))$$

N[%]

$$0.25 t^2 \, (7.29845 + 2. \, \text{Log}[t] \, (2.54518 + 3. \, \text{Log}[t]))$$

2

Remove["Global`*"]

a

f[x_]:=Sin[x]; x0=2 Pi; n=8;

Series[f[x],{x,x0,n}]

$$(x - 2\pi) - \frac{1}{6} (x - 2\pi)^3 + \frac{1}{120} (x - 2\pi)^5 - \frac{(x - 2\pi)^7}{5040} + O[x - 2\pi]^9$$

f[x_]:=Sin[2x];Series[f[x],{x,x0,n}]

$$2 (x - 2\pi) - \frac{4}{3} (x - 2\pi)^3 + \frac{4}{15} (x - 2\pi)^5 - \frac{8}{315} (x - 2\pi)^7 + O[x - 2\pi]^9$$

N[%]

$$2.\,(x-6.28319)-1.33333\,(x-6.28319)^3+0.266667\,(x-6.28319)^5-0.0253968\,(x-6.28319)^7+O[x-6.28319]^9$$

b

f[x_]:=Cos[x^2]+E^(-x^2); x0=0; n=8;

Series[f[x],{x,x0,n}]

$$2-x^2-\frac{x^6}{6}+\frac{x^8}{12}+O[x]^9$$

N[%]

$$2.-(x+0.)^2-0.166667\,(x+0.)^6+0.0833333\,(x+0.)^8+O[x+0.]^9$$

c

Integrate[Normal[Series[f[x],{x,x0,n}]],{x,-2,2}]

$$\frac{1144}{189}$$

N[%]

6.05291

n=100;Integrate[Normal[Series[f[x],{x,x0,n}]],{x,-2,2}]

$$\frac{955172222549638550919235434889421329778742044280747488681718633410 48024}{355467717415096296432904067772125497824921840054300271666644666503906 25}$$

N[%]

2.68709

NIntegrate[f[x],{x,-2,2}]

2.68709

d

Konvergenzradius von e^x und von cos (x) ist unendlich (wegen den Fakultäten im Nenner, folgt aus der Formel für den Konvergenzradius).Wird auch für x^2 u.s.w übernommen.

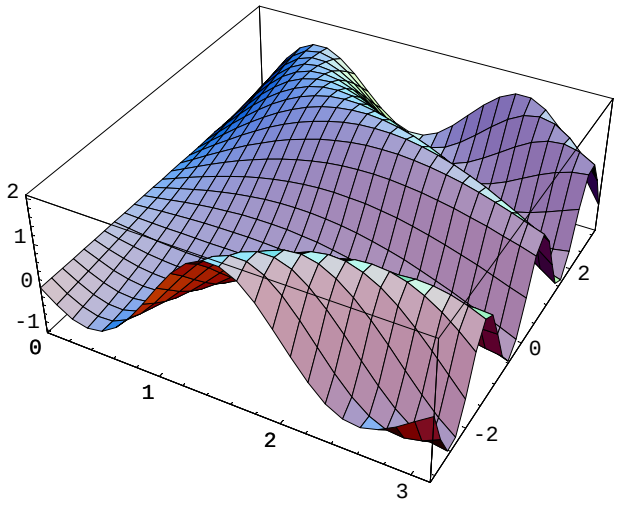
3

Remove["Global`*"]

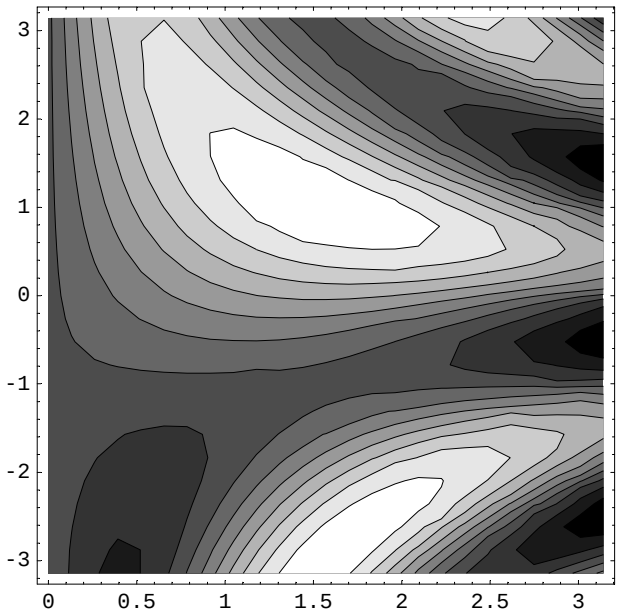
```
f[x_,y_]:=Sin[x y]+Sin[x];
f[{x_,y_}]:=f[x,y];
```

a

```
Plot3D[f[x,y],{x,0,Pi},{y,-Pi,Pi}];
```




```
ContourPlot[f[x,y],{x,0,Pi},{y,-Pi,Pi}];
```



b

```
grad[f_,x_,y_]:={D[f,x],D[f,y]};
grad[f[x,y],x,y]
```

```
{Cos[x] + y Cos[x y], x Cos[x y]}
```

Solve[Evaluate[grad[f[x,y],x,y]=={0,0}],{x,y}]

```
Solve::incnst : Inconsistent or redundant transcendental
   equation. After reduction, the bad equation is -ArcCos[Cos[x]] == 0. Mehr…

Solve::incnst : Inconsistent or redundant transcendental
   equation. After reduction, the bad equation is ArcCos[Cos[x y]] == 0. Mehr…

Solve::ifun : Inverse functions are being used by Solve, so some
   solutions may not be found; use Reduce for complete solution information. Mehr…

Solve::svars : Equations may not give solutions for all "solve" variables. Mehr…
```

$$\left\{\{y \to -1,\ x \to 0\},\ \left\{y \to 1,\ x \to -\frac{\pi}{2}\right\},\ \left\{y \to 1,\ x \to \frac{\pi}{2}\right\}\right\}$$

NSolve[Evaluate[grad[f[x,y],x,y]=={0,0}],{x,y}]

```
Solve::incnst : Inconsistent or redundant transcendental
   equation. After reduction, the bad equation is -ArcCos[Cos[x]] == 0. Mehr…

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   equation. After reduction, the bad equation is ArcCos[Cos[x y]] == 0. Mehr…

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   solutions may not be found; use Reduce for complete solution information. Mehr…

Solve::svars : Equations may not give solutions for all "solve" variables. Mehr…
```

```
{{y → 1., x → -1.5708}, {y → 1., x → 1.5708}}
```

FindRoot[Evaluate[grad[f[x,y],x,y]=={0,0}],{x,Pi/2},{y,0}]

```
{x → 0., y → -1.}
```

f[0,-1]

```
0
```

FindRoot[Evaluate[grad[f[x,y],x,y]=={0,0}],{x,Pi/2},{y,0.5}]

```
{x → 1.5708, y → 1.}
```

f[Pi/2,1]

```
2
```

FindRoot[Evaluate[grad[f[x,y],x,y]=={0,0}],{x,Pi/2},{y,3}]

```
{x → 1.5708, y → 3.}
```

f[Pi/2,3]

```
0
```

Rand

Evaluate[(D[f[x,y],y]==0)/.x->0]

```
True
```

Alles Lösungen

Evaluate[(D[f[x,y],y]==0)/.x->Pi]

$\pi \cos[\pi y] == 0$

Solve[Evaluate[(D[f[x,y],y]==0)/.x->Pi],{y}]

Solve::ifun : Inverse functions are being used by Solve, so some
solutions may not be found; use Reduce for complete solution information. Mehr…

$\left\{\left\{y \to -\frac{1}{2}\right\}, \left\{y \to \frac{1}{2}\right\}\right\}$

Evaluate[(D[f[x,y],x]==0)/.y->-Pi]

$\cos[x] - \pi \cos[\pi x] == 0$

Solve[Evaluate[(D[f[x,y],x]==0)/.y->-Pi],{x}]

Solve::tdep : The equations appear to involve the
variables to be solved for in an essentially non-algebraic way. Mehr…

$\text{Solve}[\cos[x] - \pi \cos[\pi x] == 0, \{x\}]$

FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->-Pi],{x,0}]

FindRoot::jsing :
Encountered a singular Jacobian at the point {x} = {0.}. Try perturbing the initial point(s). Mehr…

$\{x \to 0.\}$

FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->-Pi],{x,0.1}]

$\{x \to 0.405515\}$

FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->-Pi],{x,1}]

$\{x \to 5.57793\}$

FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->-Pi],{x,1.1}]

$\{x \to 1.50651\}$

FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->-Pi],{x,2}]

$\{x \to -1.50651\}$

FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->-Pi],{x,2.1}]

$\{x \to 3.40044\}$

Evaluate[(D[f[x,y],x]==0)/.y->Pi]

$\cos[x] + \pi \cos[\pi x] == 0$

FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->Pi],{x,0}]

FindRoot::jsing :
Encountered a singular Jacobian at the point {x} = {0.}. Try perturbing the initial point(s). Mehr…

$\{x \to 0.\}$

FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->Pi],{x,0.05}]

$\{x \to 2.42298\}$

```
FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->Pi],{x,0.1}]
```

$\{x \to 1.49203\}$

```
FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->Pi],{x,1}]
```

$\{x \to -2.42298\}$

```
FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->Pi],{x,0.5}]
```

$\{x \to 0.58547\}$

```
FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->Pi],{x,2}]
```

$\{x \to 3.59249\}$

```
FindRoot[Evaluate[(D[f[x,y],x]==0)/.y->Pi],{x,2.1}]
```

$\{x \to 2.42298\}$

c

```
grad[f[x,y],x,y].{1,2} 1/Norm[{1,2}]/.{x->1,y->1}
```

$$\frac{4\,\text{Cos}[1]}{\sqrt{5}}$$

```
N[%]
```

0.966522

d

```
ArcTan[%]
```

0.768376

```
%/Degree
```

44.0247

e

```
g[x_,y_]:=y^2-x
```

```
Solve[Evaluate[{grad[f[x,y],x,y]== λ grad[g[x,y],x,y],g[x,y]==0}],{x,y,λ}]
```

```
Solve::tdep : The equations appear to involve the
   variables to be solved for in an essentially non-algebraic way. Mehr…
```

$\text{Solve}[\{\{\text{Cos}[x] + y\,\text{Cos}[x\,y],\ x\,\text{Cos}[x\,y]\} == \{-\lambda,\ 2\,y\,\lambda\},\ -x + y^2 == 0\},\ \{x,\ y,\ \lambda\}]$

```
Evaluate[{grad[f[x,y],x,y]== λ grad[g[x,y],x,y]}]/.x->y^2
```

$\{\{\text{Cos}[y^2] + y\,\text{Cos}[y^3],\ y^2\,\text{Cos}[y^3]\} == \{-\lambda,\ 2\,y\,\lambda\}\}$

```
Evaluate[grad[f[x,y],x,y][[2]]== λ grad[g[x,y],x,y][[2]]/.
λ->-grad[f[x,y],x,y][[1]]] /.x->y^2
```

$$y^2 \text{Cos}[y^3] == 2\, y\, (-\text{Cos}[y^2] - y \text{Cos}[y^3])$$

```
InputForm[%]
```

```
y^2*Cos[y^3] == 2*y*(-Cos[y^2] - y*Cos[y^3])
```

```
tabl1=Table[FindRoot[y^2*Cos[y^3] == 2*y*(-Cos[y^2] - y*Cos[y^3]),{y,k
Pi}],{k,-1,1,0.0125}]//Flatten;tabl1=Union[tabl1//Chop];
tabl1//MatrixForm
```

$$\begin{pmatrix}
y \to -3.93852 \\
y \to -3.09442 \\
y \to -2.9965 \\
y \to -2.86363 \\
y \to -2.72898 \\
y \to -2.59678 \\
y \to -2.40392 \\
y \to -2.40392 \\
y \to -2.40392 \\
y \to -2.22876 \\
y \to -2.22876 \\
y \to -2.00558 \\
y \to -1.63135 \\
y \to -1.63135 \\
y \to -1.11101 \\
y \to -1.11101 \\
y \to -1.11101 \\
y \to -0.634007 \\
y \to -0.634007 \\
y \to -0.634007 \\
y \to 0 \\
y \to 1.18504 \\
y \to 1.18504 \\
y \to 1.18504 \\
y \to 1.18504 \\
y \to 1.72169 \\
y \to 1.96582 \\
y \to 1.96582 \\
y \to 2.21938 \\
y \to 2.21938 \\
y \to 2.43266 \\
y \to 2.43266 \\
y \to 2.57288 \\
y \to 2.57288 \\
y \to 2.73715 \\
y \to 2.87038 \\
y \to 2.87038 \\
y \to 2.98174 \\
y \to 2.98174 \\
y \to 2.98174 \\
y \to 3.10912
\end{pmatrix}$$

```
tabl2=Union[Round[100000
Table[{u=y/.tabl1[[k]],u^2},{k,1,Length[tabl1]}]]/100000]//N;
tabl3=Map[Reverse,tabl2]  ;
tabl4 = Select[tabl3,#[[1]]<=Pi&]; tabl4//MatrixForm
```

$$\begin{pmatrix} 2.6613 & -1.63135 \\ 1.23435 & -1.11101 \\ 0.40196 & -0.63401 \\ 0. & 0. \\ 1.40431 & 1.18504 \\ 2.9642 & 1.72169 \end{pmatrix}$$

Extrema

$$\begin{pmatrix} 2.6613 & -1.63135 \\ 1.23435 & -1.11101 \\ 0.40196 & -0.63401 \\ 0. & 0. \\ 1.40431 & 1.18504 \\ 2.9642 & 1.72169 \end{pmatrix}$$

Funktionswerte in den Extrema

```
Map[f,tabl4]//MatrixForm
```

$$\begin{pmatrix} 1.39405 \\ -0.0362476 \\ 0.139126 \\ 0. \\ 1.98182 \\ -0.748048 \end{pmatrix}$$