

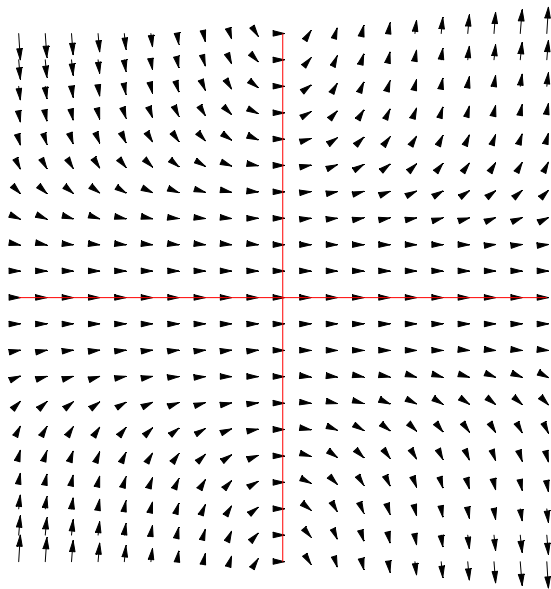
Lösungen

1

a

```
<<Graphics`PlotField`
```

```
g1=PlotVectorField[{1, x y^3},{x,-2,2,0.2},{y,-2,2,0.2},Epilog→  
{Hue[1],Line[{{-2,0},{2,0}}],Line[{{0,-2},{0,2}}]},AspectRatio→Automatic];
```



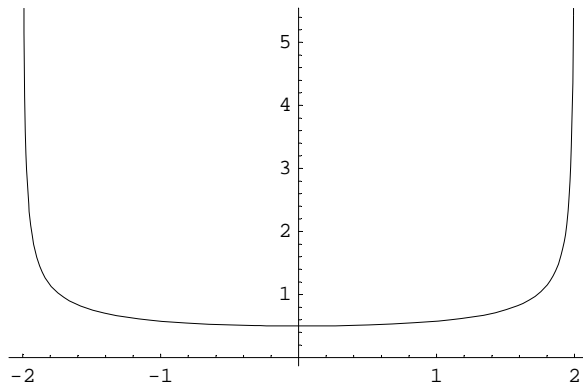
b

```
DSolve[y'[x]==x y[x]^3,y[x],x ]
```

```
{{y[x] -> -\frac{1}{\sqrt{-x^2 - 2 C[1]}}, {y[x] -> \frac{1}{\sqrt{-x^2 - 2 C[1]}}}}
```

c

```
g3=Plot[1/Sqrt[-x^2+4],{x,-2,2}]; (* Eine Lösungskurve *)
```



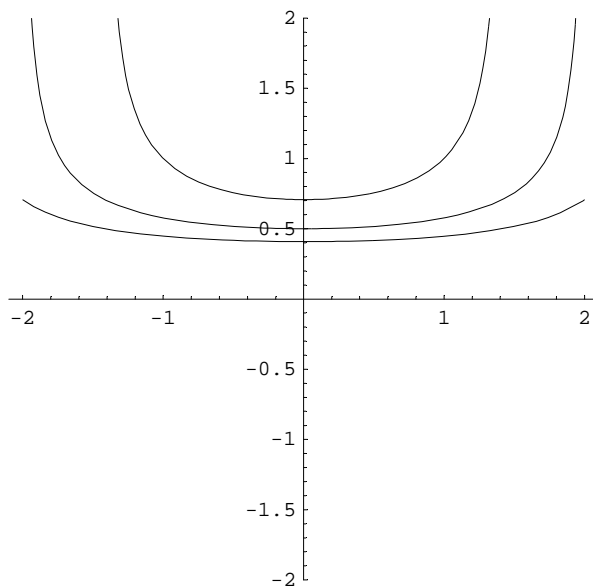
```
g3=ParametricPlot[Evaluate[Table[{x,1/Sqrt[-x^2-2
c]},{c,-3,-1}]],{x,-2,2},AspectRatio->Automatic,PlotRange->{-2,2}];
```

ParametricPlot::pptr : {x, $\frac{1}{\sqrt{2-x^2}}$ } does not evaluate to a pair of real numbers at x = -2.. Mehr...

ParametricPlot::pptr : {x, $\frac{1}{\sqrt{2-x^2}}$ } does not evaluate to a pair of real numbers at x = -1.83773. Mehr...

ParametricPlot::pptr : {x, $\frac{1}{\sqrt{2-x^2}}$ } does not evaluate to a pair of real numbers at x = -1.66076. Mehr...

General::stop : Further output of ParametricPlot::pptr will be suppressed during this calculation. Mehr...



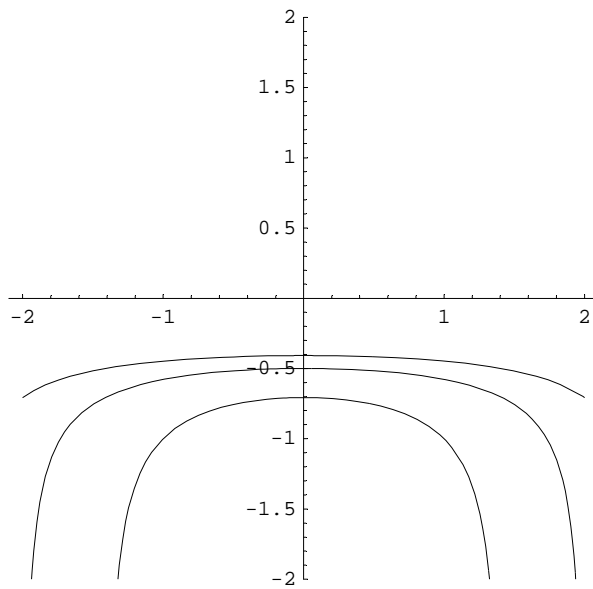
```
g4=ParametricPlot[Evaluate[Table[{x,-1/Sqrt[-x^2-2
c]},{c,-3,-1}],{x,-2,2},AspectRatio->Automatic,PlotRange->{-2,2}];
```

ParametricPlot::pptr : $\{x, -\frac{1}{\sqrt{2-x^2}}\}$ does not evaluate to a pair of real numbers at $x = -2$. Mehr...

ParametricPlot::pptr : $\{x, -\frac{1}{\sqrt{2-x^2}}\}$ does not evaluate to a pair of real numbers at $x = -1.83773$. Mehr...

ParametricPlot::pptr : $\{x, -\frac{1}{\sqrt{2-x^2}}\}$ does not evaluate to a pair of real numbers at $x = -1.66076$. Mehr...

General::stop : Further output of ParametricPlot::pptr will be suppressed during this calculation. Mehr...



d

```
DSolve[{y'[x] == x y[x]^3, y[1] == 1}, y[x], x ]
```

DSolve::bvnul : For some branches of the general solution, the given boundary conditions lead to an empty solution. Mehr...

$$\left\{ \left\{ y[x] \rightarrow \frac{1}{\sqrt{2-x^2}} \right\} \right\}$$

```
DSolve[{y'[x]==x y[x]^3,y[1]==1},y,x ]
```

DSolve::bvnul : For some branches of the general solution, the given boundary conditions lead to an empty solution. Mehr...

$$\left\{ y \rightarrow \text{Function}\left[\{x\}, \frac{1}{\sqrt{2-x^2}} \right] \right\}$$

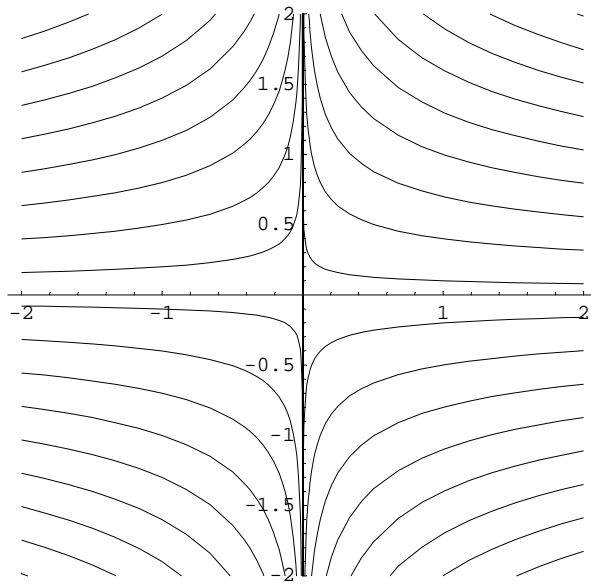
e

```
u1[x_, c_] := Re[PowerExpand[(c / x) ^ (1 / 3)]];
Print[u1[x, c]];
u[x_, c_] := Abs[(c / x) ^ (1 / 3) Sign[x c]];
u[x, c]

 $\text{Re}\left[\frac{c^{1/3}}{x^{1/3}}\right]$ 

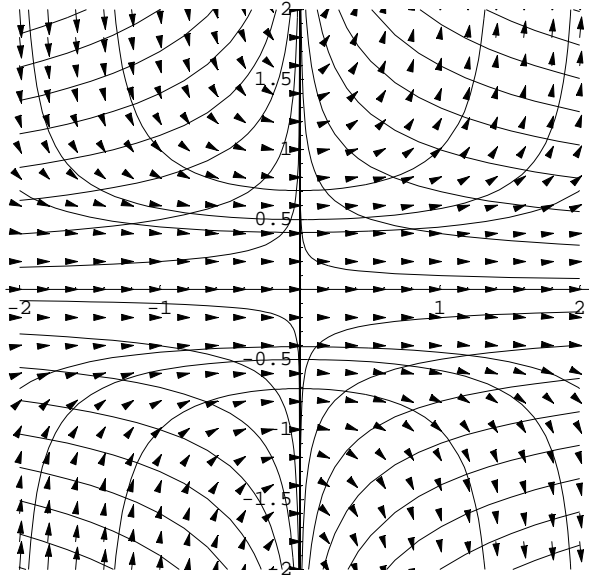
 $\text{Abs}\left[\frac{c}{x}\right]^{1/3} \text{Sign}[c x]$ 

g2 = Plot[Evaluate[Table[u[x, c^3], {c, -5, 5, 0.3}]],
  {x, -2, 2}, PlotRange -> {-2, 2}, AspectRatio -> Automatic];
```



f

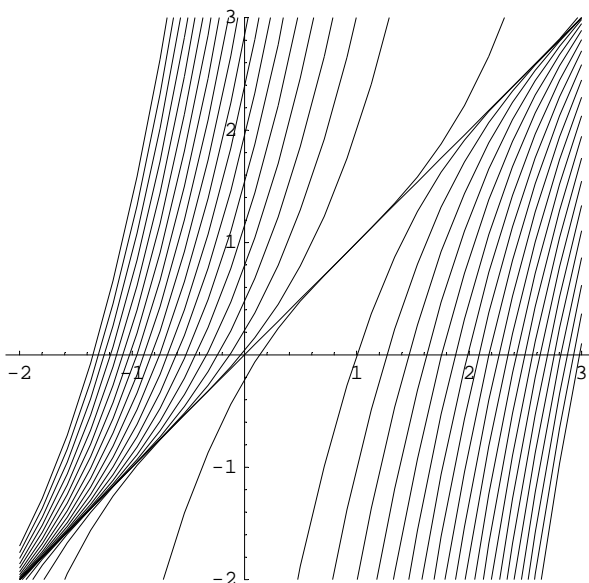
```
Show[g2,g1,g3,g4,AspectRatio->Automatic];
```



2

Wo existiert eine eindeutige Lösung der Differentialgleichung?

```
solution=Evaluate[Table[NDSolve[{y'[x]-1==2 Re[(y[x]-x)^2]^(1/3),y[1.]==
a},y,{x,-3,3}],{a,-20,20}]];
solution[[1]];
tab=Table[
Re[y[x]]/.solution[[k]],{k,1,Length[solution]}
];
Plot[Evaluate[Union[{{x}},tab]],{x,-2,3},PlotRange->{-2,3},AspectRatio->Automatic];
```



```
Remove["Global`*"]
```

```
f[x_,y_]:=1+(2 (y-x)^2)^(1/3);
(Normal[Series[f[x,y],{y,x0,1}]]//Expand
```

$$1 + \frac{2 \cdot 2^{1/3} x x_0}{3 (x^2 - 2 x x_0 + x_0^2)^{2/3}} - \frac{2 \cdot 2^{1/3} x_0^2}{3 (x^2 - 2 x x_0 + x_0^2)^{2/3}} +$$

$$2^{1/3} (x^2 - 2 x x_0 + x_0^2)^{1/3} - \frac{2 \cdot 2^{1/3} x y}{3 (x^2 - 2 x x_0 + x_0^2)^{2/3}} + \frac{2 \cdot 2^{1/3} x_0 y}{3 (x^2 - 2 x x_0 + x_0^2)^{2/3}}$$

```
(Normal[Series[f[x,y],{y,x0,1}]]/.x0->x)//Expand
```

```
Power::infy : Infinite expression  $\frac{1}{0^{2/3}}$  encountered. Mehr...
```

```
Power::infy : Infinite expression  $\frac{1}{0^{2/3}}$  encountered. Mehr...
```

```
∞::indet : Indeterminate expression ComplexInfinity+ComplexInfinity encountered. Mehr...
```

```
Indeterminate
```

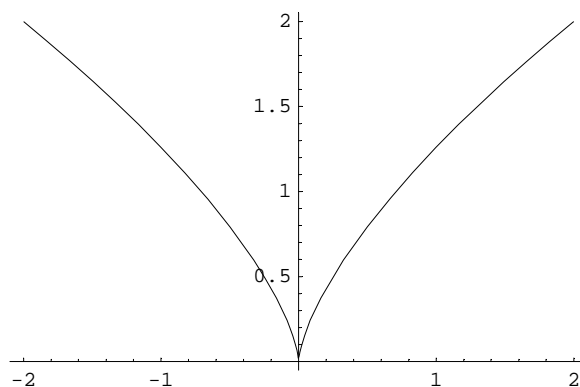
```
D[f[x,y],y]
```

$$\frac{2 \cdot 2^{1/3} (-x + y)}{3 ((-x + y)^2)^{2/3}}$$

Entwickelt man die Funktion auf der rechten Seite in eine Potenzreihe mit der Variablen y und dem Zentrum $y_0=x$, dass ein Pol entsteht. Die Ableitung nach y (Steigung Richtung y) hat bei $y = x$ einen Pol. Bei $y = x$ kann daher eine Differenz zweier Funktionswerte $|f(x,y_1)-f(x,y_2)|$ (welche wegen dem Pol nicht definiert sind) nicht kleiner als $L \cdot |y_1-y_2|$ sein

Beispiel $x = 0$

```
Plot[f[0,y]-f[0,0],{y,-2,2}];
```



Für x ungleich y und kleine Differenzen $|y_1-y_2|$ ist Lipschitz erfüllbar.

3**a****Lösung**

```
DSolve[y'[x]==x y[x],y[x],x ]
```

```
{{y[x] -> ex2/2 C[1]}}
```

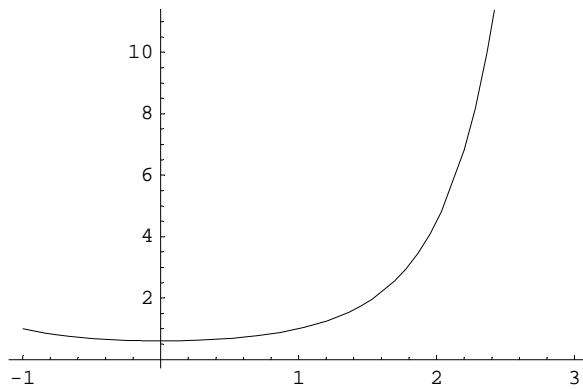
```
DSolve[{y'[x] == x y[x], y[1] == 1}, y[x], x ]
```

```
{{y[x] -> e-1/2 + x2/2}}
```

Plot der exakten Lösung der Differentialgleichung in einem Schritt

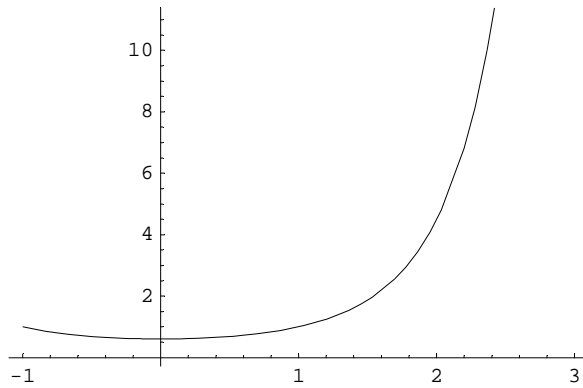
```
Remove["Global`*"];  
solv = Flatten[DSolve[{y'[x]==x y[x],y[1]==1},y,x]];  
y = y/.solv;  
Print[Simplify[y[x]]];  
Plot[y[x],{x,-1,3}];
```

$e^{\frac{1}{2}(-1+x^2)}$



Plot der numerischen Lösung einer Differentialgleichung in einem Schritt

```
Remove["Global`*"];
solution = NDSolve[{y'[x] == x y[x], y[1] == 1}, y, {x, -1, 3}, WorkingPrecision -> 24];
Plot[y[x] /. solution, {x, -1, 3}];
```



b

Lösung

```
DSolve[y'[x]==Sin[x]/Cos[y[x]],y[x],x ]
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. Mehr...

```
{{y[x] -> ArcSin[C[1] - Cos[x]]}}
```

```
DSolve[{y'[x] == Sin[x] / Cos[y[x]], y[0] == 1}, y[x], x ]
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. Mehr...

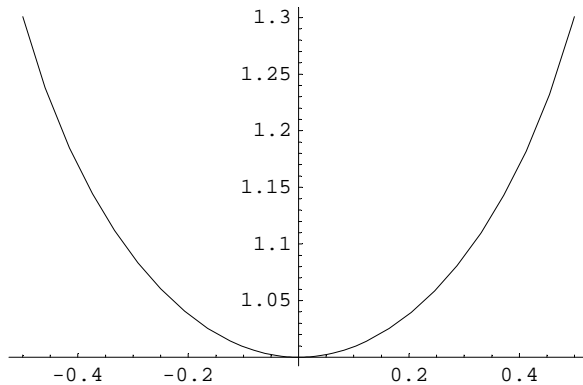
```
{{y[x] -> ArcSin[1 - Cos[x] + Sin[1]]}}
```

Plot der exakten Lösung der Differentialgleichung in einem Schritt

```
Remove["Global`*"];
solv = Flatten[DSolve[{y'[x]==Sin[x]/Cos[y[x]],y[0]==1},y,x]];
y = y/.solv;
Print[Simplify[y[x]]];
Plot[y[x],{x,-0.5,0.5}];

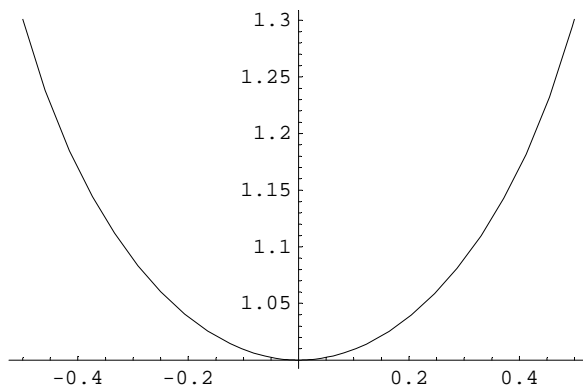
Solve::ifun : Inverse functions are being used by Solve, so some
solutions may not be found; use Reduce for complete solution information. Mehr...

ArcSin[1 - Cos[x] + Sin[1]]
```



Plot der numerischen Lösung einer Differentialgleichung in einem Schritt

```
Remove["Global`*"];
solution = NDSolve[{y'[x] == Sin[x] / Cos[y[x]], y[0] == 1}, y, {x, -0.5, 0.5},
WorkingPrecision -> 24]; Plot[y[x] /. solution, {x, -0.5, 0.5}];
```



3

a

Lösung

```
Remove["Global`*"];
DSolve[y'[x]==x y[x],y[x],x ]
```

$$\left\{ \left\{ y[x] \rightarrow e^{\frac{x^2}{2}} C[1] \right\} \right\}$$

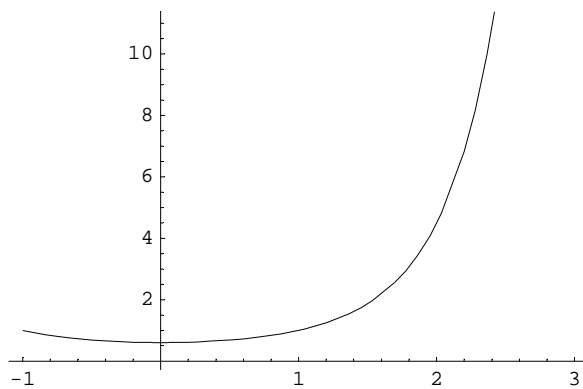
```
DSolve[{y'[x] == x y[x], y[1] == 1}, y[x], x ]
```

$$\left\{ \left\{ y[x] \rightarrow e^{-\frac{1}{2} + \frac{x^2}{2}} \right\} \right\}$$

Plot der exakten Lösung der Differentialgleichung in einem Schritt

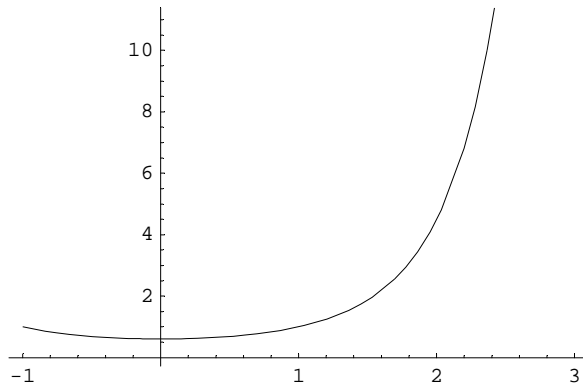
```
Remove["Global`*"];
solv = Flatten[DSolve[{y'[x]==x y[x],y[1]==1},y,x]];
y = y/.solv;
Print[Simplify[y[x]]];
Plot[y[x],{x,-1,3}];
```

$$e^{\frac{1}{2}(-1+x^2)}$$



Plot der numerischen Lösung einer Differentialgleichung in einem Schritt

```
Remove["Global`*"];
solution = NDSolve[{y'[x] == x y[x], y[1] == 1}, y, {x, -1, 3}, WorkingPrecision -> 24];
Plot[y[x] /. solution, {x, -1, 3}];
```



x

Lösung

```
Remove["Global`*"];
DSolve[y'[x]==x^4/y[x],y[x],x ]
```

$$\left\{ \left\{ y[x] \rightarrow -\sqrt{\frac{2}{5}} \sqrt{x^5 + 5 C[1]} \right\}, \left\{ y[x] \rightarrow \sqrt{\frac{2}{5}} \sqrt{x^5 + 5 C[1]} \right\} \right\}$$

```
DSolve[{y'[x] == x^4 / y[x], y[1] == 2}, y[x], x ]
```

DSolve::bvnul : For some branches of the general solution, the given boundary conditions lead to an empty solution. Mehr...

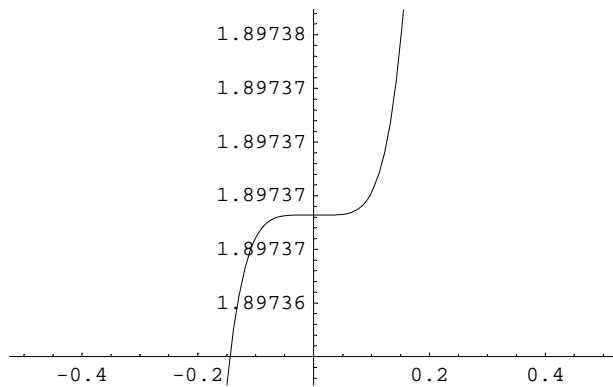
$$\left\{ \left\{ y[x] \rightarrow \sqrt{\frac{2}{5}} \sqrt{9 + x^5} \right\} \right\}$$

Plot der exakten Lösung der Differentialgleichung in einem Schritt

```
Remove["Global`*"];
solv = Flatten[DSolve[{y'[x]==x^4/y[x],y[1]==2},y,x]];
y = y/.solv;
Print[Simplify[y[x]]];
Plot[y[x],{x,-0.5,0.5}];
```

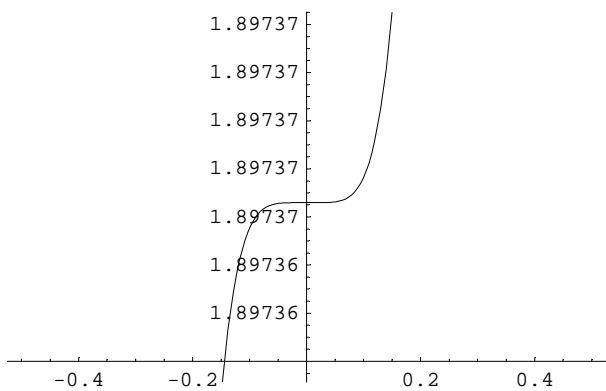
DSolve::bvnul : For some branches of the general solution, the given boundary conditions lead to an empty solution. Mehr...

$$\sqrt{\frac{2}{5}} \sqrt{9+x^5}$$



Plot der numerischen Lösung einer Differentialgleichung in einem Schritt

```
Remove["Global`*"];
solution =
NDSolve[{y'[x] == x^4/y[x], y[1] == 2}, y, {x, -0.5, 0.5}, WorkingPrecision -> 24];
Plot[y[x] /. solution, {x, -0.5, 0.5}];
```



4

a

Lösung

```
DSolve[y'[x]== 4x + 7 y[x]+3,y[x],x ]
```

$$\left\{ \left\{ y[x] \rightarrow -\frac{25}{49} - \frac{4x}{7} + e^{7x} C[1] \right\} \right\}$$

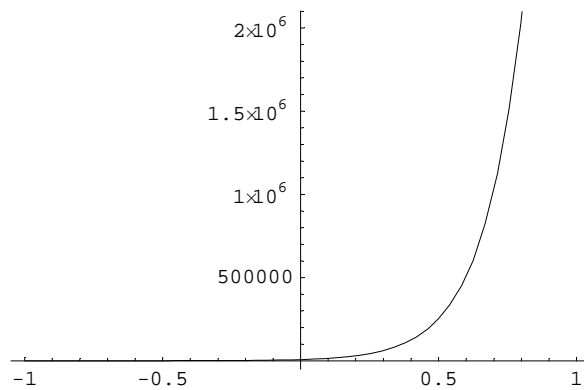
```
DSolve[{y'[x] == 4x + 7 y[x] + 3, y[-1] == 7}, y[x], x ]
```

$$\left\{ \left\{ y[x] \rightarrow \frac{1}{49} (-25 + 340 e^{7+7x} - 28x) \right\} \right\}$$

Plot der exakten Lösung der Differentialgleichung in einem Schritt

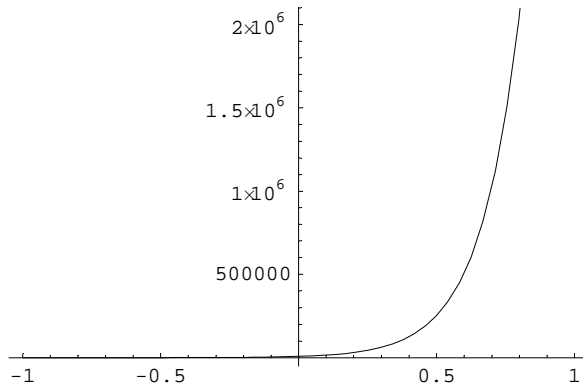
```
Remove["Global`*"];
solv = Flatten[DSolve[{y'[x]==4x+7 y[x]+3,y[-1]==7},y,x]];
y = y/.solv;
Print[Simplify[y[x]]];
Plot[y[x],{x,-1,1}];
```

$$\frac{1}{49} (-25 + 340 e^{7+7x} - 28x)$$



Plot der numerischen Lösung einer Differentialgleichung in einem Schritt

```
Remove["Global`*"];
solution = NDSolve[{y'[x] == 4 x + 7 y[x] + 3, y[-1] == 7}, y,
  {x, -1, 3}, WorkingPrecision -> 24]; Plot[y[x] /. solution, {x, -1, 1}];
```



b

Lösung

```
DSolve[y'[x] == (y[x]+1)/x+3, y[x], x ]
```

```
{{y[x] -> x C[1] + x (-1/x + 3 Log[x])}}
```

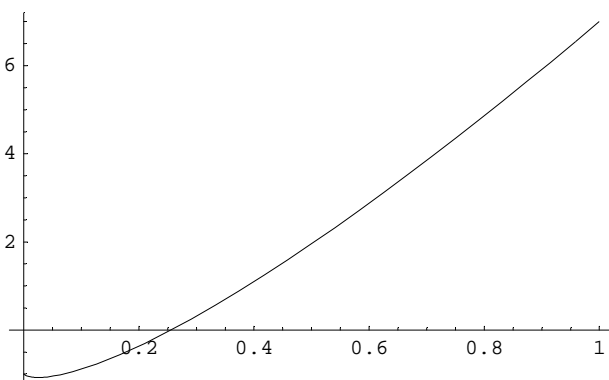
```
DSolve[{y'[x] == (y[x] + 1) / x + 3, y[1] == 7}, y[x], x ]
```

```
{{y[x] -> -1 + 8 x + 3 x Log[x]}}
```

Plot der exakten Lösung der Differentialgleichung in einem Schritt

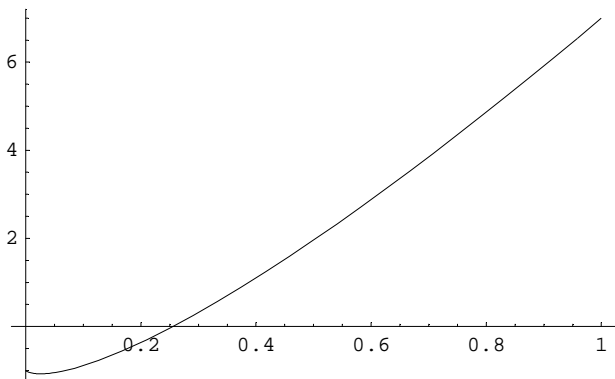
```
Remove["Global`*"];
solv = Flatten[DSolve[{y'[x]==(y[x]+1)/x+3,y[1]==7},y,x]];
y = y/.solv;
Print[Simplify[y[x]]];
Plot[y[x],{x,0,1}];
```

```
-1 + 8 x + 3 x Log[x]
```



Plot der numerischen Lösung einer Differentialgleichung in einem Schritt

```
Remove["Global`*"];
solution = NDSolve[{y'[x] == (y[x] + 1) / x + 3, y[1] == 7}, y,
  {x, -1, 3}, WorkingPrecision -> 24]; Plot[y[x] /. solution, {x, 0, 1}];
```



C

Lösung

```
D[x+Sin[x y+x],x] dx + D[x+Sin[x y+x],y] dy
```

```
dy x Cos[x + x y] + dx (1 + (1 + y) Cos[x + x y])
```

```
D[x+Sin[x y+x],x] dx + D[x+Sin[x y+x],y] dy //InputForm
```

```
dy*x*Cos[x + x*y] + dx*(1 + (1 + y)*Cos[x + x*y])
```

```
(1 + (1 + y)*Cos[x + x*y])/(x*Cos[x + x*y])
```

$$\frac{(1 + (1 + y) \cos[x + x y]) \operatorname{Sec}[x + x y]}{x}$$

```
DSolve[y'[x]==-(1 + (1 + y[x])*Cos[x + x*y[x]])/(x*Cos[x + x*y[x]]), y[x], x ]
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. Mehr...

$$\left\{ \left\{ y[x] \rightarrow \frac{-x - \operatorname{ArcSin}[x - C[1]]}{x} \right\} \right\}$$

```
DSolve[
```

```
{y'[x] == -(1 + (1 + y[x]) * Cos[x + x * y[x]]) / (x * Cos[x + x * y[x]]), y[1/2] == 1}, y[x], x ]
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. Mehr...

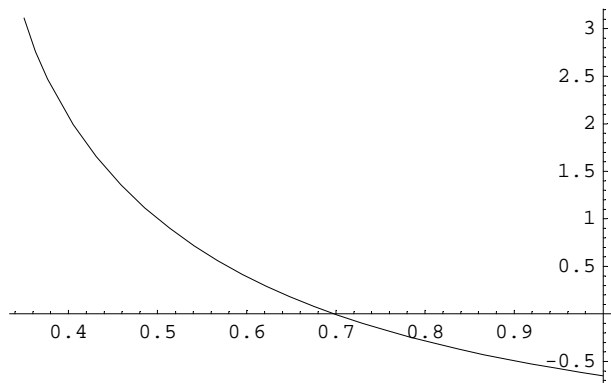
$$\left\{ \left\{ y[x] \rightarrow \frac{-x - \operatorname{ArcSin}\left[x + \frac{1}{2}(-1 - 2 \operatorname{Sin}[1])\right]}{x} \right\} \right\}$$

Plot der exakten Lösung der Differentialgleichung in einem Schritt

```
Remove["Global`*"];  
solv = Flatten[DSolve[{y'[x]==-(1+(1+y[x])*Cos[x+x*y[x]])/(x*Cos[x+x*y[x]]),y[1/2]=  
1},y,x]];  
y = y/.solv;  
Print[Simplify[y[x]]];  
Plot[y[x],{x,0.35,1}];
```

Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information. Mehr...

$$-1 + \frac{\text{ArcSin}\left[\frac{1}{2} - x + \text{Sin}[1]\right]}{x}$$



Plot der numerischen Lösung einer Differentialgleichung in einem Schritt

```

Remove["Global`*"];
solution =
NDSolve[{y'[x] == -(1 + (1 + y[x]) * Cos[x + x * y[x]]) / (x * Cos[x + x * y[x]]), y[1/2] == 1},
y, {x, -1, 3}, WorkingPrecision -> 24]; Plot[y[x] /. solution, {x, 35, 1}];

NDSolve::npsz : At x == 0.34147098483443720960209635789296234262`24.,
step size is effectively zero; singularity or stiff system suspected. Mehr...

NDSolve::npsz : At x == 2.34147098465845268764366342549167823426`24.,
step size is effectively zero; singularity or stiff system suspected. Mehr...

InterpolatingFunction::dmval : Input value {2.37928} lies outside the
range of data in the interpolating function. Extrapolation will be used. Mehr...

InterpolatingFunction::dprec : The precision of input value
{2.37928} and/or the interpolation grid is insufficient to compute the value. Mehr...

Plot::plnr : y[x] /. solution is not a machine-size real number at x = 2.379277713479137`. Mehr...

InterpolatingFunction::dmval : Input value {3.8835} lies outside the
range of data in the interpolating function. Extrapolation will be used. Mehr...

InterpolatingFunction::dprec : The precision of input value
{3.8835} and/or the interpolation grid is insufficient to compute the value. Mehr...

Plot::plnr : y[x] /. solution is not a machine-size real number at x = 3.8834991952187057`. Mehr...

InterpolatingFunction::dmval : Input value {2.35643} lies outside the
range of data in the interpolating function. Extrapolation will be used. Mehr...

General::stop :
Further output of InterpolatingFunction::dmval will be suppressed during this calculation. Mehr...

InterpolatingFunction::dprec : The precision of input value
{2.35643} and/or the interpolation grid is insufficient to compute the value. Mehr...

General::stop :
Further output of InterpolatingFunction::dprec will be suppressed during this calculation. Mehr...

Plot::plnr : y[x] /. solution is not a machine-size real number at x = 2.356434054419371`. Mehr...

General::stop : Further output of Plot::plnr will be suppressed during this calculation. Mehr...

```

