

Lösungen Laplace-Rücktransformationen

?*Laplace*

System`

InverseLaplaceTransform LaplaceTransform

?Laplace*

LaplaceTransform[expr, t, s] gives the Laplace transform of expr. LaplaceTransform[expr, {t1, t2, ... }, {s1, s2, ... }] gives the multidimensional Laplace transform of expr. Mehr...

?InverseLaplace*

InverseLaplaceTransform[expr, s, t] gives the inverse Laplace transform of expr. InverseLaplaceTransform[expr, {s1, s2, ... }, {t1, t2, ... }] gives the multidimensional inverse Laplace transform of expr. Mehr...

?Limit

Limit[expr, x->x0] finds the limiting value of expr when x approaches x0. Mehr...

?Direction

Direction is an option for Limit. Limit[expr, x -> x0, Direction -> 1] computes the limit as x approaches x0 from smaller values. Limit[expr, x -> x0, Direction -> -1] computes the limit as x approaches x0 from larger values. Direction -> Automatic uses Direction -> -1 except for limits at Infinity, where it is equivalent to Direction -> 1.

Remove["Global`*"]

1

a

f[t_]:= t Sin[t]

transf= LaplaceTransform[f[t],t,s]

$$\frac{2s}{(1+s^2)^2}$$

Limit[f[t],t->Infinity]

Interval[{-∞, ∞}]

Limit[s transf,s->0,Direction->-1]

0

!! Ungleiches Resultat - Wieso? !!

```
Limit[f[t],t->0,Direction->-1]
```

0

```
Limit[s transf,s->Infinity]
```

0

b

```
f[t_]:= 1/t Sin[t]
```

```
transf= LaplaceTransform[f[t],t,s]
```

$$\text{ArcTan}\left[\frac{1}{s}\right]$$

```
Limit[f[t],t->Infinity]
```

0

```
Limit[s transf,s->0,Direction->-1]
```

0

```
Limit[f[t],t->0,Direction->-1]
```

1

```
Limit[s transf,s->Infinity]
```

1

c

```
f[t_]:= Integrate[1/τ Sin[τ],{τ,0,t}]
```

```
transf= LaplaceTransform[Evaluate[f[t]],t,s]
```

$$\frac{\text{ArcTan}\left[\frac{1}{s}\right]}{s}$$

```
Limit[f[t],t->Infinity]
```

$$\frac{\pi}{2}$$

```
Limit[s transf,s->0,Direction->-1]
```

$$\frac{\pi}{2}$$

```
Limit[f[t],t->0,Direction->-1]
```

0

```
Limit[s transf,s->Infinity]
```

```
0
```

2

a

```
InverseLaplaceTransform[(6 s + 2)/(2 s^2+4 s+8),s,t]
```

$$\frac{1}{3} e^{-t} (9 \operatorname{Cos}[\sqrt{3} t] - 2 \sqrt{3} \operatorname{Sin}[\sqrt{3} t])$$

b

```
InverseLaplaceTransform[2/(2 s^2+4 s+8),s,t]
```

$$\frac{e^{-t} \operatorname{Sin}[\sqrt{3} t]}{\sqrt{3}}$$

c

```
InverseLaplaceTransform[(6 s - 2)/(2 s^2-4 s-8),s,t]
```

$$\frac{1}{10} e^{t-\sqrt{5} t} (15 - 2 \sqrt{5} + (15 + 2 \sqrt{5}) e^{2 \sqrt{5} t})$$

d

```
InverseLaplaceTransform[(6 s - 2)/(2 s^2+4 s+8)^2,s,t]
```

$$\frac{1}{36} e^{-t} (12 t \operatorname{Cos}[\sqrt{3} t] + \sqrt{3} (-4 + 9 t) \operatorname{Sin}[\sqrt{3} t])$$

3

a

```
InverseLaplaceTransform[(3 s + 4)/(s^2+2 s-3),s,t]
```

$$\frac{5 e^{-3 t}}{4} + \frac{7 e^t}{4}$$

b

`InverseLaplaceTransform[2/(s+2)-2s/(s^2+16)+3/(s^2+15),s,t]`

$$2 e^{-2t} - 2 \cos[4t] + \sqrt{\frac{3}{5}} \sin[\sqrt{15}t]$$

c

`InverseLaplaceTransform[1/(s^2+4s+3),s,t]`

$$\frac{1}{2} e^{-3t} (-1 + e^{2t})$$

d

`InverseLaplaceTransform[e^{(-Pi/4)s} s/(s^2-4),s,t]`

$$\text{InverseLaplaceTransform}\left[\frac{e^{-\frac{\pi s}{4}} s}{-4 + s^2}, s, t\right]$$

(Handrechnung, Verschiebung der Originalfunktion um, $f(t)=0$ für $t < \pi/4$, sonst um $\pi/4$ verschobene Rücktransformierte von $s/(s^2-4)$). Diese ist von der Verschiebung:

`InverseLaplaceTransform[s/(s^2-4),s,t]`

$$\frac{1}{2} e^{-2t} (1 + e^{4t})$$

Verschiebung:

$$\frac{1}{2} e^{-2t} (1 + e^{4t}) \quad / . \quad t \rightarrow (t - \pi/4)$$

$$\frac{1}{2} e^{-2(-\pi/4+t)} (1 + e^{4(-\pi/4+t)})$$

e

`InverseLaplaceTransform[(2s-6)/(s^2+4s-20),s,t]`

$$\frac{1}{12} e^{-2(1+\sqrt{6})t} (12 + 5\sqrt{6}) + (12 - 5\sqrt{6}) e^{4\sqrt{6}t}$$

f

`InverseLaplaceTransform[1/(s(s^2-\omega^2)),s,t]`

$$\frac{e^{-t\omega} (-1 + e^{t\omega})^2}{2\omega^2}$$

g`InverseLaplaceTransform[1/(s^2-2s),s,t]`

$$\frac{1}{2} (-1 + e^{2t})$$