

# Uebungen

## D'Gl. 2. Ordnung

```
Remove["Global`*"]
```

```
links = LaplaceTransform[y''[t] + y'[t] + y[t], t, s] /.
  {LaplaceTransform[y[t], t, s] → Y[s], y[0] → 1, y'[0] → 1}
```

```
-2 - s + Y[s] + s Y[s] + s2 Y[s]
```

```
rechts = LaplaceTransform[Sin[t], t, s]
```

$$\frac{1}{1 + s^2}$$

```
solv = Solve[links == rechts, {Y[s]}] // Flatten
```

$$\left\{ Y[s] \rightarrow \frac{3 + s + 2 s^2 + s^3}{(1 + s^2) (1 + s + s^2)} \right\}$$

```
U[s] := Y[s] /. solv; U[s]
```

$$\frac{3 + s + 2 s^2 + s^3}{(1 + s^2) (1 + s + s^2)}$$

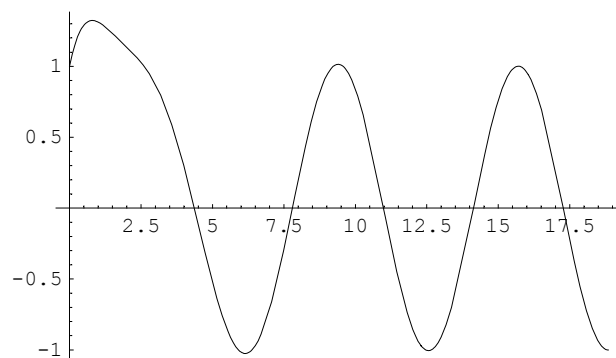
```
U[s] // Apart
```

$$-\frac{s}{1 + s^2} + \frac{3 + 2 s}{1 + s + s^2}$$

```
u0[t_] := InverseLaplaceTransform[U[s], s, t] // Simplify; u0[t]
```

$$-\text{Cos}[t] + \frac{2}{3} e^{-t/2} \left( 3 \text{Cos}\left[\frac{\sqrt{3} t}{2}\right] + 2 \sqrt{3} \text{Sin}\left[\frac{\sqrt{3} t}{2}\right] \right)$$

```
Plot[Evaluate[{u0[t]}], {t, 0, 6 Pi};
```



## System

```
Remove["Global`*"]
```

```
links1 = LaplaceTransform[y'[t] + z[t], t, s] /. {LaplaceTransform[y[t], t, s] -> Y[s],
  y[0] -> 1, y'[0] -> 0, LaplaceTransform[z[t], t, s] -> Z[s], z[0] -> 1, z'[0] -> 0}
-s + s^2 Y[s] + Z[s]
```

```
links2 = LaplaceTransform[y[t] - z'[t], t, s] /. {LaplaceTransform[y[t], t, s] -> Y[s],
  y[0] -> 1, y'[0] -> 0, LaplaceTransform[z[t], t, s] -> Z[s], z[0] -> 1, z'[0] -> 0}
1 + Y[s] - s Z[s]
```

```
rechts1 = LaplaceTransform[Sin[t], t, s]
```

$$\frac{1}{1 + s^2}$$

```
rechts2 = LaplaceTransform[Cos[t], t, s]
```

$$\frac{s}{1 + s^2}$$

```
solv = Solve[{links1 == rechts1, links2 == rechts2}, {Y[s], Z[s]}] // Flatten
```

$$\left\{ Y[s] \rightarrow -\frac{1 - 2s - s^4}{(1 + s^2)(1 + s^3)}, Z[s] \rightarrow -\frac{-1 - s - s^2 - s^4}{(1 + s^2)(1 + s^3)} \right\}$$

```
U[s] := Y[s] /. solv[[1]]; U[s]
```

$$-\frac{1 - 2s - s^4}{(1 + s^2)(1 + s^3)}$$

```
U[s] // Apart
```

$$-\frac{1}{3(1 + s)} + \frac{-1 + s}{1 + s^2} + \frac{1 + s}{3(1 - s + s^2)}$$

```
V[s] := Z[s] /. solv[[2]]; V[s]
```

$$-\frac{-1 - s - s^2 - s^4}{(1 + s^2)(1 + s^3)}$$

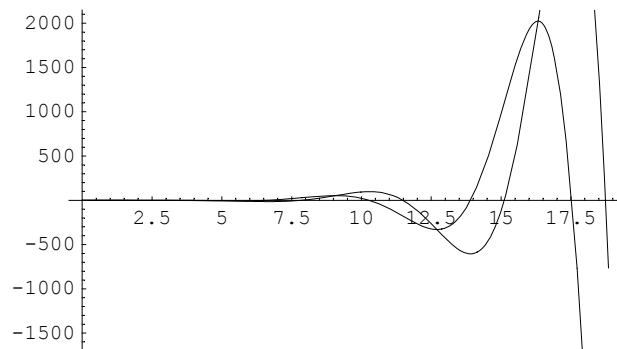
```
u[t_] := InverseLaplaceTransform[U[s], s, t] // Simplify; u[t]
```

$$-\frac{e^{-t}}{3} + \cos[t] - \sin[t] + \frac{1}{3} e^{t/2} \left( \cos\left[\frac{\sqrt{3}t}{2}\right] + \sqrt{3} \sin\left[\frac{\sqrt{3}t}{2}\right] \right)$$

```
v[t_] := InverseLaplaceTransform[V[s], s, t] // Simplify; v[t]
```

$$\frac{1}{3} \left( e^{-t} + 3 \cos[t] + e^{t/2} \left( -\cos\left[\frac{\sqrt{3}t}{2}\right] + \sqrt{3} \sin\left[\frac{\sqrt{3}t}{2}\right] \right) \right)$$

```
Plot[Evaluate[{u[t], v[t]}], {t, 0, 6 Pi}];
```



## Faltung

```
Remove["Global`*"]
```

### ■ Rechnung gewöhnlich

```
links = LaplaceTransform[y''[t] + 3 y'[t] + 4 y[t], t, s] /.
  {LaplaceTransform[y[t], t, s] → Y[s], y[0] → 0, y'[0] → 0}
```

```
General::spell1 :
```

Possible spelling error: new symbol name "links" is similar to existing symbol "Links". Mehr...

```
4 Y[s] + 3 s Y[s] + s^2 Y[s]
```

```
rechts = LaplaceTransform[Sin[t], t, s]
```

$$\frac{1}{1 + s^2}$$

```
solv = Solve[links == rechts, {Y[s]}] // Flatten
```

```
{Y[s] →  $\frac{1}{4 + 3 s + 5 s^2 + 3 s^3 + s^4}$ }
```

```
U[s] := Y[s] /. solv; U[s]
```

$$\frac{1}{4 + 3 s + 5 s^2 + 3 s^3 + s^4}$$

```
U[s] // Apart
```

$$\frac{1 - s}{6 (1 + s^2)} + \frac{2 + s}{6 (4 + 3 s + s^2)}$$

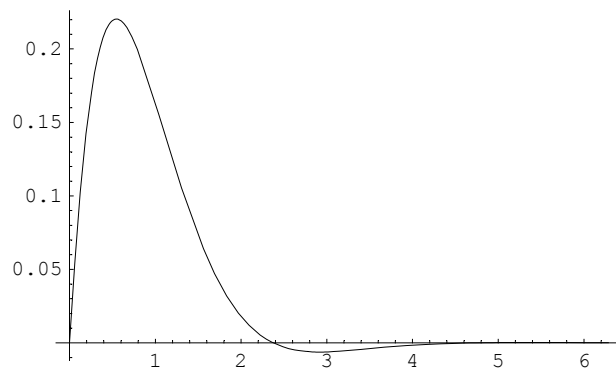
```
InverseLaplaceTransform[U[s], s, t] // Simplify
```

$$\frac{1}{42} \left( 7 (-\cos[t] + \sin[t]) + e^{-3t/2} \left( 7 \cos\left[\frac{\sqrt{7} t}{2}\right] + \sqrt{7} \sin\left[\frac{\sqrt{7} t}{2}\right] \right) \right)$$

```
g[t_] := InverseLaplaceTransform[1 / (s^2 + 3 s + 4), s, t] // Simplify; g[t]
```

$$\frac{2 e^{-3t/2} \sin\left[\frac{\sqrt{7} t}{2}\right]}{\sqrt{7}}$$

```
Plot[Evaluate[{g[t]}], {t, 0, 2 Pi};
```



## ■ Rechnung mit Faltung

```
faltung[u_, v_, t_] := Integrate[u[λ] v[t-λ], {λ, 0, t}];
```

```
u[t_] := g[t]; v[t_] := Sin[t];
```

```
faltung[u, v, t]
```

$$\frac{1}{42} \left( -7 \cos[t] + 7 \sin[t] + e^{-3t/2} \left( 7 \cos\left[\frac{\sqrt{7}t}{2}\right] + \sqrt{7} \sin\left[\frac{\sqrt{7}t}{2}\right] \right) \right)$$

## ■ Faltung allgemein

```
Remove["Global`*"]
```

```
(* Zur Information *)LaplaceTransform[a y''[t] + b y'[t] + c y[t], t, s] /.
```

```
{LaplaceTransform[y[t], t, s] → Y[s], y[0] → y0, y'[0] → y1}
```

```
c Y[s] + b (-y0 + s Y[s]) + a (-s y0 - y1 + s2 Y[s])
```

### ■ Beispiel 1

```

a = 1; b = 1; c = 1;
y0 = 1; y1 = 1;
f[t_] := Sin[t];

mo[k_] := Module[{}, r[t_] :=
  InverseLaplaceTransform[(a s y0 + a y1 + b y0) / (a s^2 + b s + c), s, t] // Simplify;
  Print["r[t] = ", r[t]];
  g[t_] := InverseLaplaceTransform[1 / (a s^2 + b s + c), s, t] // Simplify;
  Print["g[t] = ", g[t]];
  faltung[f_, g_, t_] := Integrate[f[λ] g[t - λ], {λ, 0, t},
    GenerateConditions → False] // Simplify;
  Print["y[t] = ", faltung[f, g, t]];
];
mo[k]

```

$$r[t] = e^{-t/2} \left( \cos\left[\frac{\sqrt{3}t}{2}\right] + \sqrt{3} \sin\left[\frac{\sqrt{3}t}{2}\right] \right)$$

$$g[t] = \frac{2 e^{-t/2} \sin\left[\frac{\sqrt{3}t}{2}\right]}{\sqrt{3}}$$

$$y[t] = -\cos[t] + \frac{1}{3} e^{-t/2} \left( 3 \cos\left[\frac{\sqrt{3}t}{2}\right] + \sqrt{3} \sin\left[\frac{\sqrt{3}t}{2}\right] \right)$$

### ■ Beispiel 2

```

a = 1; b = 1; c = 1;
y0 = 1; y1 = 1;
f[t_] := Cos[t];

mo[k_] := Module[{}, r[t_] :=
  InverseLaplaceTransform[(a s y0 + a y1 + b y0) / (a s^2 + b s + c), s, t] // Simplify;
  Print["r[t] = ", r[t]];
  g[t_] := InverseLaplaceTransform[1 / (a s^2 + b s + c), s, t] // Simplify;
  Print["g[t] = ", g[t]];
  faltung[f_, g_, t_] := Integrate[f[λ] g[t - λ], {λ, 0, t},
    GenerateConditions → False] // Simplify;
  Print["y[t] = ", faltung[f, g, t]];
];
mo[k]

```

$$r[t] = e^{-t/2} \left( \cos\left[\frac{\sqrt{3}t}{2}\right] + \sqrt{3} \sin\left[\frac{\sqrt{3}t}{2}\right] \right)$$

$$g[t] = \frac{2 e^{-t/2} \sin\left[\frac{\sqrt{3}t}{2}\right]}{\sqrt{3}}$$

$$y[t] = \sin[t] - \frac{2 e^{-t/2} \sin\left[\frac{\sqrt{3}t}{2}\right]}{\sqrt{3}}$$

### ■ Beispiel 3

```

a = 1; b = 1; c = 1;
y0 = 1; y1 = 1;
f[t_] := E^(-t);

mo[k_] := Module[{}, r[t_] :=
  InverseLaplaceTransform[(a s y0 + a y1 + b y0) / (a s^2 + b s + c), s, t] // Simplify;
  Print["r[t] = ", r[t]];
  g[t_] := InverseLaplaceTransform[1 / (a s^2 + b s + c), s, t] // Simplify;
  Print["g[t] = ", g[t]];
  faltung[f_, g_, t_] := Integrate[f[λ] g[t - λ], {λ, 0, t},
    GenerateConditions -> False] // Simplify;
  Print["y[t] = ", faltung[f, g, t]];
];
mo[k]

```

$$r[t] = e^{-t/2} \left( \cos\left[\frac{\sqrt{3}}{2} t\right] + \sqrt{3} \sin\left[\frac{\sqrt{3}}{2} t\right] \right)$$

$$g[t] = \frac{2 e^{-t/2} \sin\left[\frac{\sqrt{3}}{2} t\right]}{\sqrt{3}}$$

$$y[t] = \frac{1}{3} e^{-t} \left( 3 + e^{t/2} \left( -3 \cos\left[\frac{\sqrt{3}}{2} t\right] + \sqrt{3} \sin\left[\frac{\sqrt{3}}{2} t\right] \right) \right)$$

### ■ Beispiel 4

```

a = 1; b = 1; c = 1;
y0 = 1; y1 = 1;
f[t_] := t;

mo[k_] := Module[{}, r[t_] :=
  InverseLaplaceTransform[(a s y0 + a y1 + b y0) / (a s^2 + b s + c), s, t] // Simplify;
  Print["r[t] = ", r[t]];
  g[t_] := InverseLaplaceTransform[1 / (a s^2 + b s + c), s, t] // Simplify;
  Print["g[t] = ", g[t]];
  faltung[f_, g_, t_] := Integrate[f[λ] g[t - λ], {λ, 0, t},
    GenerateConditions -> False] // Simplify;
  Print["y[t] = ", faltung[f, g, t] // Simplify];
];
mo[k]

```

$$r[t] = e^{-t/2} \left( \cos\left[\frac{\sqrt{3}}{2} t\right] + \sqrt{3} \sin\left[\frac{\sqrt{3}}{2} t\right] \right)$$

$$g[t] = \frac{2 e^{-t/2} \sin\left[\frac{\sqrt{3}}{2} t\right]}{\sqrt{3}}$$

$$y[t] = -1 + t - \frac{1}{3} e^{-t/2} \left( -3 \cos\left[\frac{\sqrt{3}}{2} t\right] + \sqrt{3} \sin\left[\frac{\sqrt{3}}{2} t\right] \right)$$