

Lösungen

1. Dirachsche Deltafunktion und Einheitssprung

a

$$\int_{-3\pi}^{3\pi} \text{DiracDelta}[x] \, dx$$

1

b

$$\int_{-3\pi}^{3\pi} \text{DiracDelta}[x] \, E^{(x+1)} \, dx$$

e

c

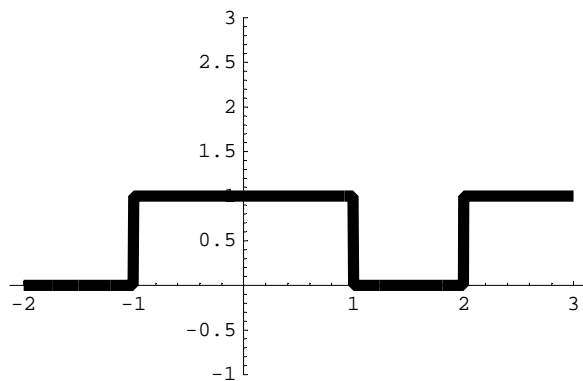
$$\int_{-3\pi}^{3\pi} \text{DiracDelta}\left[x - \frac{\pi}{2}\right] \sin[x] \, dx$$

1

d

```
u[x_]=UnitStep[(x+1)(x-1)(x-2)];
```

```
Plot[u[x],{x,-2,3},PlotRange->{-1,3},AxesOrigin->{0,0},PlotStyle->{Thickness[0.02]}];
```



e

$$\int_{-3}^5 u[x] dx$$

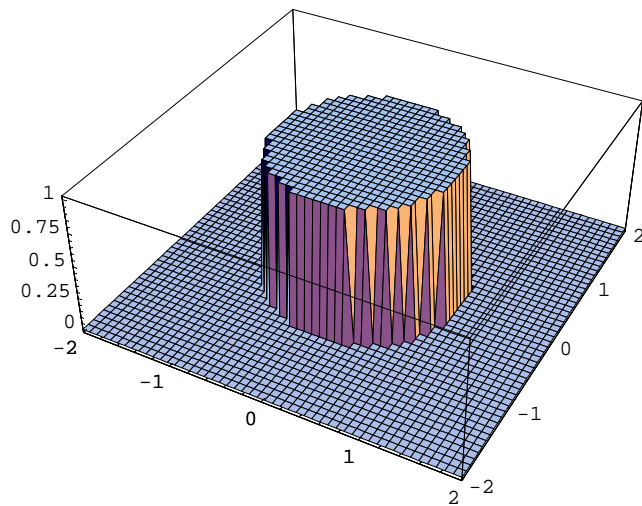
5

```
u[x] // FunctionExpand
```

```
UnitStep[-2 + x] - UnitStep[-1 + x] + UnitStep[1 + x]
```

f

```
Plot3D[UnitStep[1 - x^2 - y^2], {x, -2, 2}, {y, -2, 2}, PlotPoints -> 50];
```



2. D'Gl. 2. Ordnung

a

```
Remove["Global`*"]
```

```
links = LaplaceTransform[y''[t] + y'[t] + y[t], t, s] /.
```

```
{LaplaceTransform[y[t], t, s] -> Y[s], y[0] -> 1, y'[0] -> 1}
```

```
General::spell1 :
```

```
Possible spelling error: new symbol name "links" is similar to existing symbol "Links". Mehr...
```

```
-2 - s + Y[s] + s Y[s] + s^2 Y[s]
```

```
rechts = LaplaceTransform[DiracDelta[t], t, s]
```

1

```
solv = Solve[links == rechts, {Y[s]}] // Flatten
```

$$\{Y[s] \rightarrow \frac{3+s}{1+s+s^2}\}$$

```
U[s] := Y[s] /. solv; U[s]
```

$$\frac{3+s}{1+s+s^2}$$

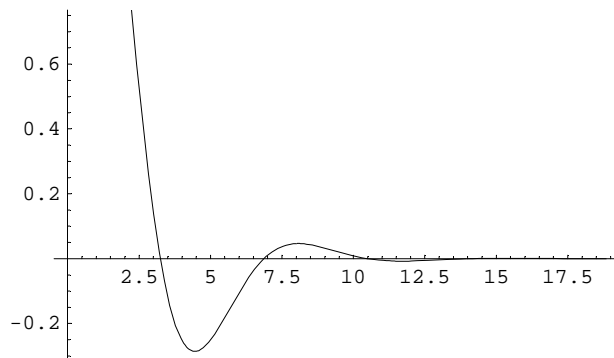
```
U[s] // Apart
```

$$\frac{3+s}{1+s+s^2}$$

```
u0[t_] := InverseLaplaceTransform[U[s], s, t] // Simplify; u0[t]
```

$$\frac{1}{3} e^{-t/2} \left(3 \cos\left[\frac{\sqrt{3}t}{2}\right] + 5\sqrt{3} \sin\left[\frac{\sqrt{3}t}{2}\right] \right)$$

```
Plot[Evaluate[{u0[t]}], {t, 0, 6 Pi}];
```



b

```
Remove["Global`*"]
```

```
links = LaplaceTransform[y''[t] + y'[t] + y[t], t, s] /.
```

```
{LaplaceTransform[y[t], t, s] -> Y[s], y[0] -> 1, y'[0] -> 1}
```

```
General::spell1 :
```

Possible spelling error: new symbol name "links" is similar to existing symbol "Links". Mehr...

$$-2 - s + Y[s] + s Y[s] + s^2 Y[s]$$

```
LaplaceTransform[DiracDelta[t-1], t, s]
```

$$e^{-s}$$

```
rechts = LaplaceTransform[DiracDelta[t] + DiracDelta[t - 1] + DiracDelta[t - 2], t, s]
```

$$1 + e^{-2s} + e^{-s}$$

```
solv = Solve[links == rechts, {Y[s]}] // Flatten
```

$$\{Y[s] \rightarrow \frac{e^{-2s} (1 + e^s + 3 e^{2s} + e^{2s} s)}{1 + s + s^2}\}$$

```
U[s] := Y[s] /. solv; U[s]
```

$$\frac{e^{-2s} (1 + e^s + 3 e^{2s} + e^{2s} s)}{1 + s + s^2}$$

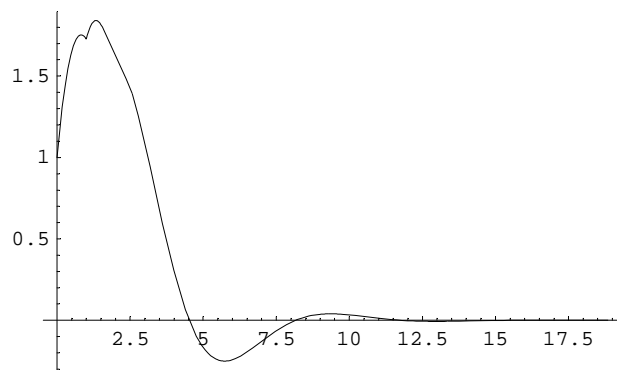
```
U[s] // Apart
```

$$\frac{e^{-2s}}{1 + s + s^2} + \frac{e^{-s}}{1 + s + s^2} + \frac{3 + s}{1 + s + s^2}$$

```
u0[t_] := InverseLaplaceTransform[U[s], s, t] // Simplify; u0[t]
```

$$\frac{1}{3} e^{-t/2} \left(3 \cos\left[\frac{\sqrt{3} t}{2}\right] + 5 \sqrt{3} \sin\left[\frac{\sqrt{3} t}{2}\right] + 2 \sqrt{3} e \sin\left[\frac{1}{2} \sqrt{3} (-2 + t)\right] \text{UnitStep}[-2 + t] + 2 \sqrt{3} e \sin\left[\frac{1}{2} \sqrt{3} (-1 + t)\right] \text{UnitStep}[-1 + t] \right)$$

```
Plot[Evaluate[{u0[t]}], {t, 0, 6 Pi}];
```



C

```
Remove["Global`*"]
```

```
links = LaplaceTransform[y''[t] + y'[t] + y[t], t, s] /.  
{LaplaceTransform[y[t], t, s] -> Y[s], y[0] -> 1, y'[0] -> 1}
```

```
General::spell1 :
```

Possible spelling error: new symbol name "links" is similar to existing symbol "Links". Mehr...

$$-2 - s + Y[s] + s Y[s] + s^2 Y[s]$$

```
rechts=LaplaceTransform[UnitStep[t], t, s]
```

$$\frac{1}{s}$$

```
solv = Solve[links == rechts, {Y[s]}] // Flatten
```

$$\left\{ Y[s] \rightarrow \frac{1 + 2 s + s^2}{s (1 + s + s^2)} \right\}$$

```
U[s] := Y[s] /. solv; U[s]
```

$$\frac{1 + 2 s + s^2}{s (1 + s + s^2)}$$

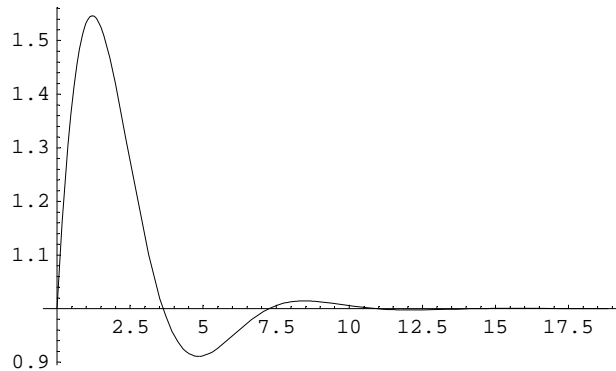
```
U[s] // Apart
```

$$\frac{1}{s} + \frac{1}{1+s+s^2}$$

```
u0[t_] := InverseLaplaceTransform[U[s], s, t] // Simplify; u0[t]
```

$$1 + \frac{2 e^{-t/2} \operatorname{Sin}\left[\frac{\sqrt{3} t}{2}\right]}{\sqrt{3}}$$

```
Plot[Evaluate[{u0[t]}], {t, 0, 6 Pi}];
```



d

```
Remove["Global`*"]
```

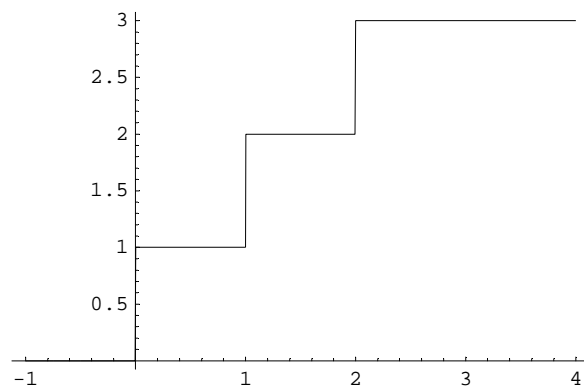
```
links = LaplaceTransform[y''[t] + y'[t] + y[t], t, s] /.
  {LaplaceTransform[y[t], t, s] → Y[s], y[0] → 1, y'[0] → 1}
```

```
General::spell1 :
```

```
Possible spelling error: new symbol name "links" is similar to existing symbol "Links". Mehr...
```

```
-2 - s + Y[s] + s Y[s] + s^2 Y[s]
```

```
Plot[UnitStep[t]+UnitStep[t-1]+UnitStep[t-2],{t,-1,4}];
```



```
rechts=LaplaceTransform[UnitStep[t]+UnitStep[t-1]+UnitStep[t-2] ,t,s]
```

$$\frac{1}{s} + \frac{e^{-2s}}{s} + \frac{e^{-s}}{s}$$

```
solv = Solve[links == rechts, {Y[s]}] // Flatten
```

$$\{Y[s] \rightarrow \frac{e^{-2s} (1 + e^s + e^{2s} + 2 e^{2s} s + e^{2s} s^2)}{s (1 + s + s^2)}\}$$

```
U[s] := Y[s] /. solv; U[s]
```

$$\frac{e^{-2s} (1 + e^s + e^{2s} + 2 e^{2s} s + e^{2s} s^2)}{s (1 + s + s^2)}$$

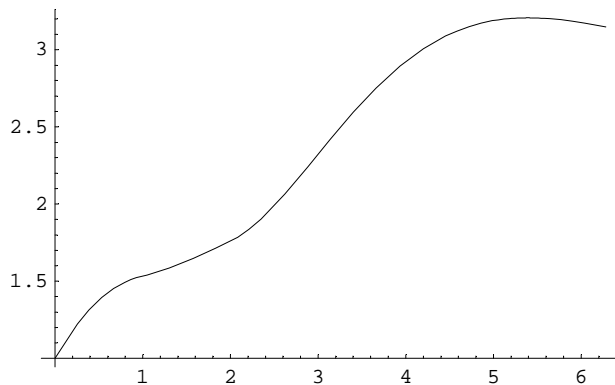
```
U[s] // Apart
```

$$\frac{e^{-2s}}{s (1 + s + s^2)} + \frac{e^{-s}}{s (1 + s + s^2)} + \frac{(1 + s)^2}{s (1 + s + s^2)}$$

```
u0[t_] := InverseLaplaceTransform[U[s], s, t] // Simplify; u0[t]
```

$$\frac{1}{3} e^{-t/2} \left(3 e^{t/2} + 2 \sqrt{3} \operatorname{Sin}\left[\frac{\sqrt{3} t}{2}\right] + \left(3 e^{t/2} - 3 e \operatorname{Cos}\left[\frac{1}{2} \sqrt{3} (-2 + t)\right] - \sqrt{3} e \operatorname{Sin}\left[\frac{1}{2} \sqrt{3} (-2 + t)\right] \right) \operatorname{UnitStep}[-2 + t] + \left(3 e^{t/2} - 3 \sqrt{e} \operatorname{Cos}\left[\frac{1}{2} \sqrt{3} (-1 + t)\right] - \sqrt{3} e \operatorname{Sin}\left[\frac{1}{2} \sqrt{3} (-1 + t)\right] \right) \operatorname{UnitStep}[-1 + t] \right)$$

```
Plot[Evaluate[{u0[t]}], {t, 0, 2Pi};
```



```
Plot[Evaluate[{u0[t]}], {t, 0, 6 Pi};
```

