Lösungen

1

```
Remove["Global`*"]
```

Plot von fFourier[t]:

```
<<Calculus`FourierTransform`
```

```
?*FourierTrig*
```

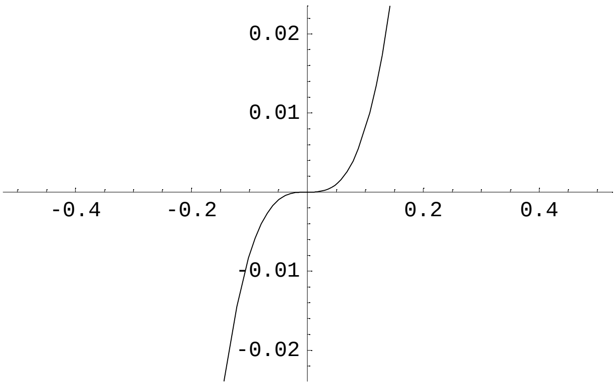
Calculus`FourierTransform`

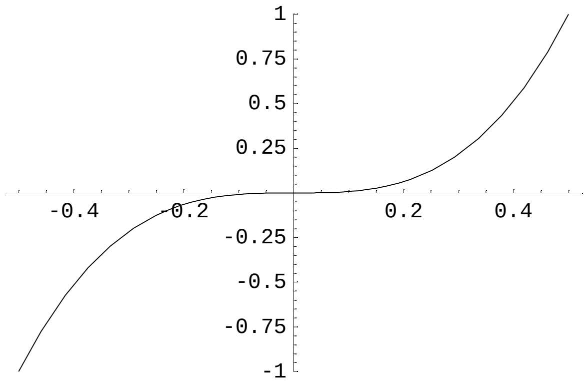
FourierTrigSeries NFourierTrigSeries

```
?FourierTrigSeries
```

FourierTrigSeries[expr, t, k] gives the kth order Fourier trigonometric series approximation to the periodic function of t that is equal to expr for -1/2 <= t <= 1/2, and has a period of 1. FourierTrigSeries[expr, t, k, FourierParameters -> {a, b}] gives the kth order Fourier trigonometric series approximation to the periodic function of t that is equal to expr for -1/(2 Abs[b]) <= t <= 1/(2 Abs[b]), and has a period of 1/Abs[b]. Mehr...

```
f[t_]:= (2 t)^3;
Plot[f[t],{t,-1/2,1/2}];
Plot[f[t],{t,-1/2,1/2},PlotRange->{-1,1}];
```

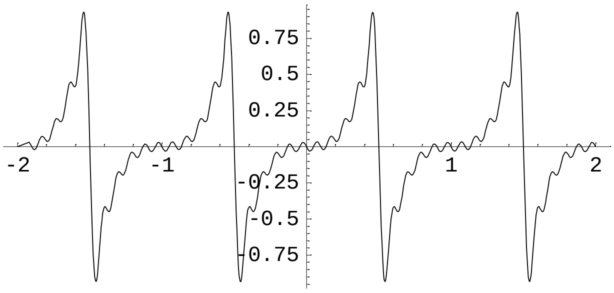




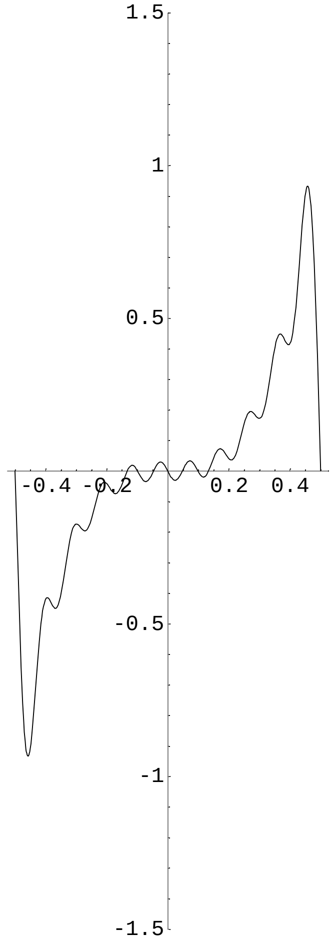
FourierTrigSeries[f[t], t, 10]

$$\frac{2\,(-6+\pi^2)\,\text{Sin}[2\,\pi\,t]}{\pi^3}+\frac{(3-2\,\pi^2)\,\text{Sin}[4\,\pi\,t]}{2\,\pi^3}+\frac{2\,(-2+3\,\pi^2)\,\text{Sin}[6\,\pi\,t]}{9\,\pi^3}+\frac{(3-8\,\pi^2)\,\text{Sin}[8\,\pi\,t]}{16\,\pi^3}+\frac{2\,(-6+25\,\pi^2)\,\text{Sin}[10\,\pi\,t]}{125\,\pi^3}+\frac{(1-6\,\pi^2)\,\text{Sin}[12\,\pi\,t]}{18\,\pi^3}+\frac{2\,(-6+49\,\pi^2)\,\text{Sin}[14\,\pi\,t]}{343\,\pi^3}+\frac{(3-32\,\pi^2)\,\text{Sin}[16\,\pi\,t]}{128\,\pi^3}+\frac{2\,(-2+27\,\pi^2)\,\text{Sin}[18\,\pi\,t]}{243\,\pi^3}+\frac{(3-50\,\pi^2)\,\text{Sin}[20\,\pi\,t]}{250\,\pi^3}$$

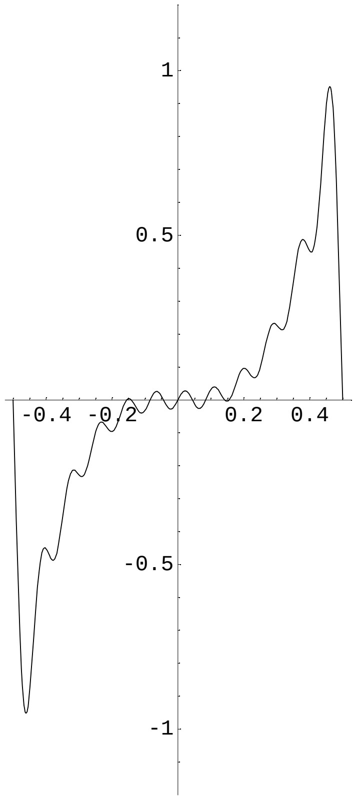
Plot[Evaluate[FourierTrigSeries[f[t], t, 10]],{t,-2,2},AspectRatio->Automatic];



```
Plot[Evaluate[FourierTrigSeries[f[t], t,
10]],{t,-1/2,1/2},AspectRatio->Automatic,PlotRange->{-1.2,1.5}];
```

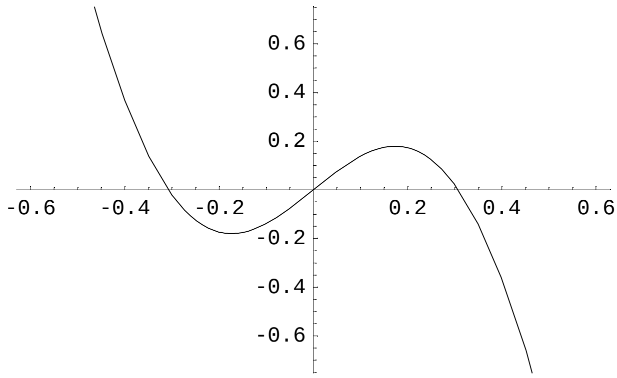


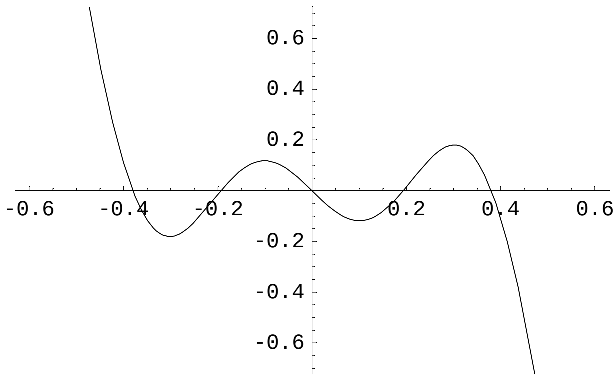
```
Plot[Evaluate[FourierTrigSeries[f[t], t,
11]],{t,-1/2,1/2},AspectRatio->Automatic,PlotRange->{-1.2,1.2}];
```

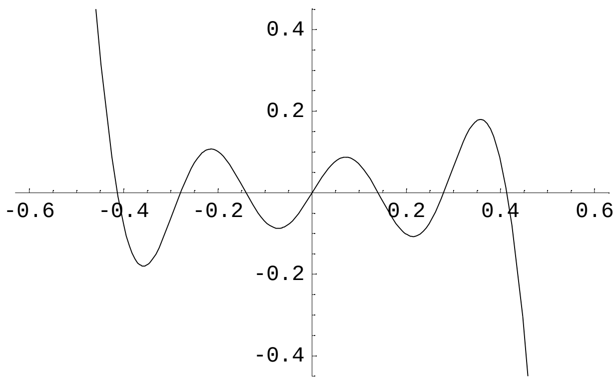


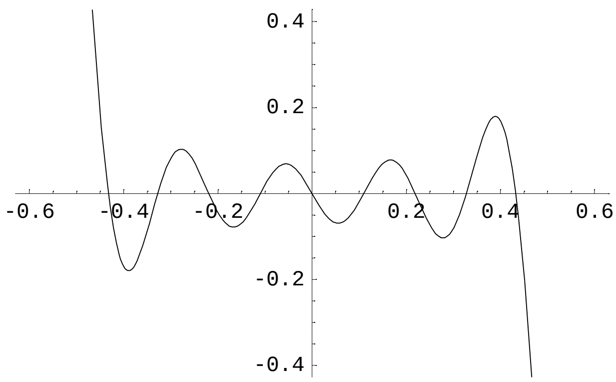
Plot von fFourier[t]-f[t]:

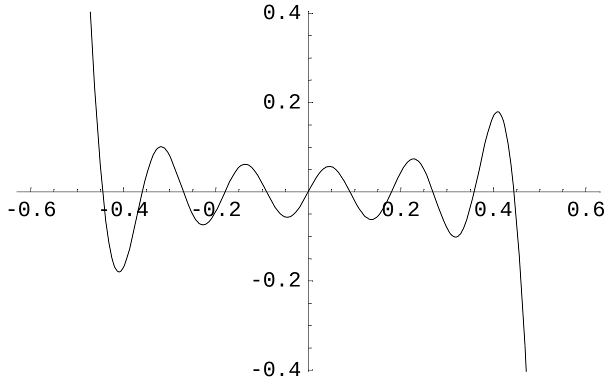
```
Table[Plot[Evaluate[FourierTrigSeries[f[t], t, n]-f[t]],{t,-0.5,0.5}],{n,1,10}];
```

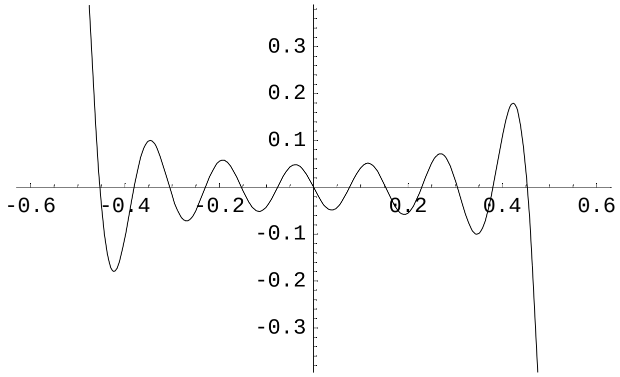


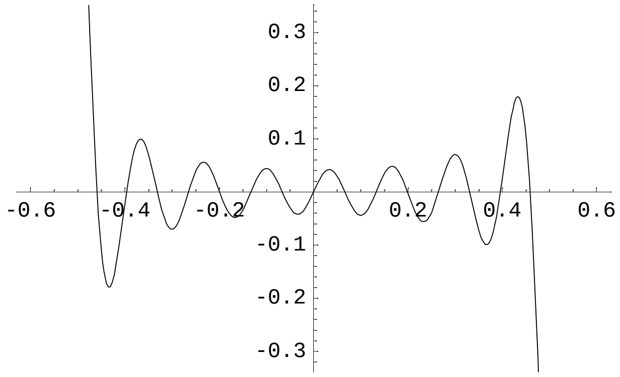

0.6
0.4
0.2
-0.6 -0.4 -0.2 0.2 0.4 0.6
-0.2
-0.4
-0.6

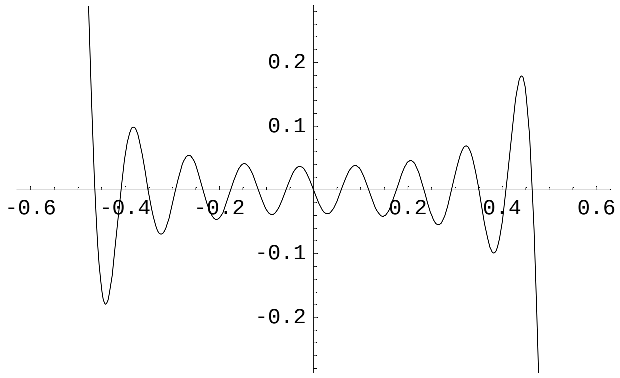

0.4
0.2
-0.6 -0.4 -0.2 0.2 0.4 0.6
-0.2
-0.4

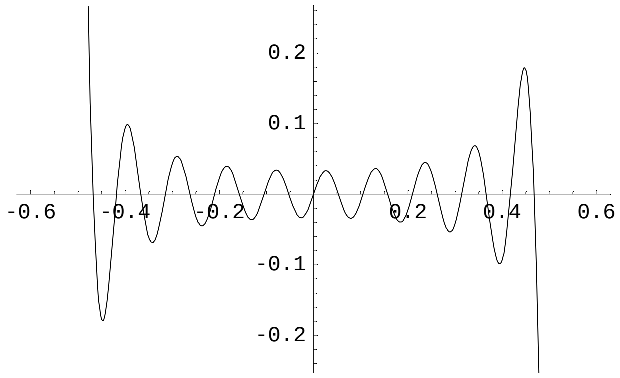

0.4
0.2
-0.6 -0.4 -0.2 0.2 0.4 0.6
-0.2
-0.4

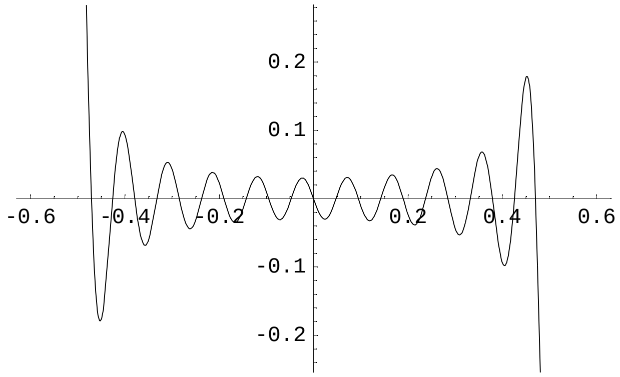

0.4
0.2
-0.6 -0.4 -0.2 0.2 0.4 0.6
-0.2
-0.4


0.3
0.2
0.1
-0.6
-0.4
-0.2
0.2
0.4
0.6
-0.1
-0.2
-0.3


0.3
0.2
0.1
-0.6
-0.4
-0.2
0.2
0.4
0.6
-0.1
-0.2
-0.3


0.2
0.1
-0.6
-0.4
-0.2
0.2
0.4
0.6
-0.1
-0.2


0.2
0.1
-0.6
-0.4
-0.2
0.2
0.4
0.6
-0.1
-0.2



Overshoot heuristisch (Grösse von | fFourier[t]-f[t] |):

```
FindMaximum[Evaluate[(FourierTrigSeries[f[t], t, 10]-f[t]/.t->u)],{u,0.45}]
```

{0.179089, {u → 0.452411}}

Ca. 17.9 % Overshoot

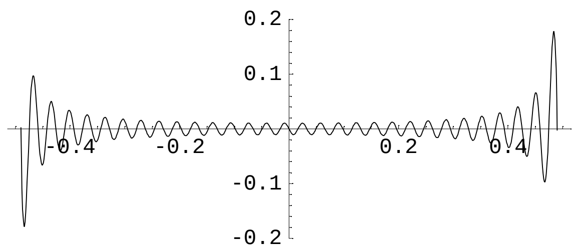
```
FindMaximum[Evaluate[(FourierTrigSeries[f[t], t, 20]-f[t]/.t->u)],{u,0.47}]
```

{0.179009, {u → 0.475614}}

```
FindMaximum[Evaluate[(FourierTrigSeries[f[t], t, 30]-f[t]/.t->u)],{u,0.48}]
```

{0.178993, {u → 0.483608}}

```
Plot[Evaluate[FourierTrigSeries[f[t], t,
30]-f[t]],{t,-1/2+0.01,1/2-0.01},AspectRatio->Automatic,PlotRange->{-0.2,0.2}];
```



Da hier Die Funktion f über die zulässigen Grenzen hinaus definiert ist, kommt es exakt auf den Grenzen beim Plot zu einem Fehler. Man darf daher nicht bis exakt an die Grenzen plotten.

```
?*Max*
```

System`

```
ConstrainedMax      MaxPlotPoints    $MaxExtraPrecision
FindMaximum         MaxPoints        $MaxLicenseProcesses
IndentMaxFraction   MaxRecursion     $MaxMachineNumber
Max                 MaxStepFraction  $MaxNumber
MaxBend             MaxSteps         $MaxPiecewiseCases
Maximize            MaxStepSize      $MaxPrecision
MaxIterations       NMaximize        $MaxRootDegree
MaxMemoryUsed       SpanMaxSize
```

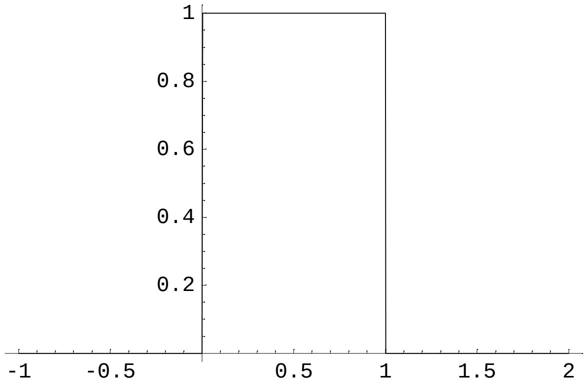
```
FindMaximum[f, {x, x0}] searches for a local maximum in f,
   starting from the point x = x0. FindMaximum[f, {{x, x0}, {y, y0}, ... }]
   searches for a local maximum in a function of several variables. Mehr…
```

2

```
fFour[Ω_,f_]:=1/(2 π) Integrate[Evaluate[f[λ] E^(-I λ Ω)],{λ,-Infinity,Infinity}]
```

a

```
f[λ_ /; λ<0]:=0;
f[λ_ /; 0<=λ && λ<1]:= 1;
f[λ_ /; λ>=0]:=0;
Plot[f[λ],{λ,-1,2}];
fFour[Ω,f]
```



Integrate::idiv : Integral of $e^{-i\lambda\Omega}\lambda^3$ does not converge on $\{-\infty, \infty\}$. Mehr…

$$\frac{\int_{-\infty}^{\infty} 8\, e^{-i\lambda\Omega}\lambda^3\, d\lambda}{2\pi}$$

Das System kann nicht mit uneigentlichen komplexen Integralen dieser Art problemlos umgehen. Es bedarf weiterer Interventionen! Problem: Sinus und Cosinus sind an der Stelle unendlich nicht definiert!

```
1/(2 π) Integrate[Evaluate[f[λ] E^(-I λ Ω)],{λ,0,1}]
```

$$\frac{4\,(6 + e^{-i\Omega}(-6 - 6\,i\,\Omega + 3\,\Omega^2 + i\,\Omega^3))}{\pi\,\Omega^4}$$

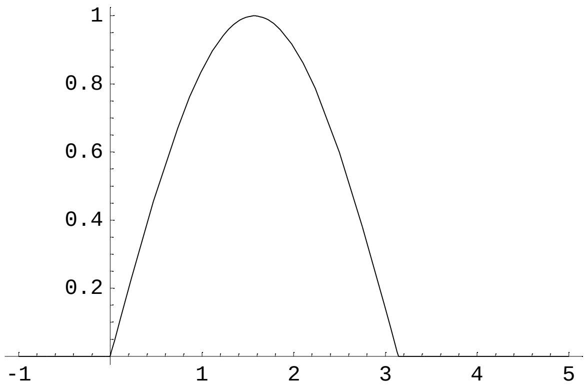
```
Limit[1/(2 π) Integrate[Evaluate[f[λ] E^(-I λ Ω)],{λ,-n,n}],{n->Infinity}]
```

$$\left\{\text{Limit}\left[\frac{8\, i\, (n\,\Omega\,(-6+n^2\,\Omega^2)\,\text{Cos}[n\,\Omega]-3\,(-2+n^2\,\Omega^2)\,\text{Sin}[n\,\Omega])}{\pi\,\Omega^4},\ n\to\infty\right]\right\}$$

b

```
Remove["Global`*"]

f[λ_ /; λ<0]:=0;
f[λ_ /; 0<=λ && λ<Pi]:= Sin[λ];
f[λ_ /; λ>=Pi]:=0;
Plot[f[λ],{λ,-1,5}];
```




```
1/(2 π) Integrate[Evaluate[f[λ] E^(-I λ Ω)],{λ,0,Pi}]
```

$$\frac{\int_0^{\pi} e^{-i\,\lambda\,\Omega}\, f[\lambda]\, d\lambda}{2\,\pi}$$

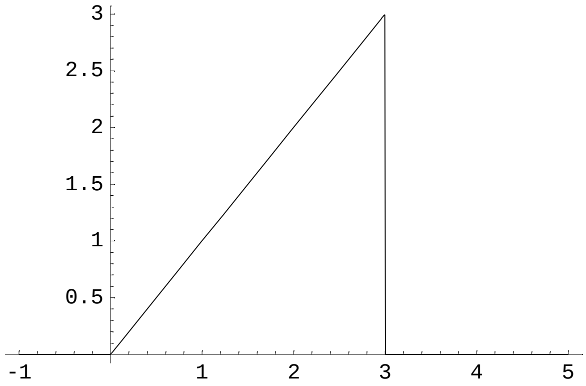
```
1/(2 π) Integrate[Evaluate[Sin[λ] (Cos[λ Ω]+I Sin[λ])],{λ,0,Pi}]
```

$$\frac{\frac{i\,\pi}{2}+\frac{1+\text{Cos}[\pi\,\Omega]}{1-\Omega^2}}{2\,\pi}$$

c

```
Remove["Global`*"]
```

```
f[λ_ /; λ<0]:=0;
f[λ_ /; 0<=λ && λ<3]:= λ;
f[λ_ /; λ>=3]:=0;
Plot[f[λ],{λ,-1,5}];
```




```
1/(2 π) Integrate[Evaluate[λ E^(-I λ Ω)],{λ,0,3}]
```

$$\frac{-1 + e^{-3 i \Omega} (1 + 3 i \Omega)}{2 \pi \Omega^2}$$