Lösungen

1

```
Remove["Global`*"]
```

■ a Resultierende Kraft oder -Gegenkraft

■ 1. Rechnerisch, alles in SI-Einheiten, 3-dim., damit das Vektorprodukt verwendet werden kann (Programmiertechnik)

```
s[1] = {0, 0, 0}; s[2] = {0.10, 0, 0}; s[3] = {0.25, 0, 0};
s[4] = {0.35, 0, 0}; s[5] = {s5x, s5y, 0};
F[1] = {0, 100, 0}; F[2] = {0, -80, 0}; F[3] = {0, 180, 0};
F[4] = 120 {1, 1, 0} / Norm[{1, 1, 0}]; F[5] = {F5x, F5y, 0};
M[k_] := Cross[s[k], F[k]];
```

```
Table[M[k], {k, 1, 5}]
```

```
{{0, 0, 0}, {0, 0., -8.}, {0, 0., 45.}, {0, 0., 29.6985}, {0, 0, F5y s5x - F5x s5y}}
```

■ Gleichgewichtsbedingungen

```
Sum[F[k], {k, 1, 5}] == {0, 0, 0}
```

$\left\{60\sqrt{2} + F5x,\ 200 + 60\sqrt{2} + F5y,\ 0\right\} == \{0, 0, 0\}$

```
Sum[M[k], {k, 1, 5}] == {0, 0, 0}
```

```
{0, 0., 66.6985 + F5y s5x - F5x s5y} == {0, 0, 0}
```

```
solv = Solve[{Sum[F[k], {k, 1, 5}] == {0, 0, 0}, Sum[M[k], {k, 1, 5}] == {0, 0, 0}},
    {s5x, s5y, F5x, F5y}] // Flatten // ExpandAll
```

```
Solve::svars : Equations may not give solutions for all "solve" variables. Mehr…
```

```
{s5x → 0.234151 + 0.297883 s5y, F5x → -84.8528, F5y → -284.853}
```

==> F[5] = { -84.8528 N, -129.853 N }

```
sx[sy_] := (s5x /. solv[[1]]) /. s5y → sy; sx[xy]
```

```
0.234151 + 0.297883 xy
```

==> Gerade (Wirkungslinie) . Für sy = 0 (d.h. auf dem Balken, y = 0) wird:

```
sx[0]
```

```
0.234151
```

==> Ca. 23.4 cm

```
steigung = sx[1] - sx[0]
```

```
0.297883
```

0.297883 ist die Steigung (Tangens): Winkel zum Balken:

```
ArcTan[steigung]
```

```
0.289513
```

In Alt-Grad zur positiven x-Achse:

```
ArcTan[steigung] / Degree
```

```
16.5879
```

■ 2. Graphisch, Zusatzaufgabe (optional)

Parallele Kräfte werden addiert, indem man zu den parallelen Kräften senkrechte Hilfskräfte vernünftiger Grösse hinzufügt, deren Summe 0 ergibt. Man hat daher im Total nichts zugefügt. Nun kann man mit den entstehenden Kräfteparallelogrammen wie gewohnt arbeiten und zeichnerisch die Resultierende graphisch sukzessiv bestimmen. Das Vorgehen sowie der damit befolgte Weg hängt vom Gutdünken des jeweiligen Autors ab. Daher gibt es keinen allgemein verbindlichen Lösungsweg. Für das eindeutige Resultat kann man a) heranziehen.

■ b Schwerpunkt

Totale Masse:

```
Anzahl = Sum[k, {k, 1, 7}]
```

```
28
```

```
TotaleMasse = 1.3 Sum[k, {k, 1, 7}]
```

```
36.4
```

```
SchwerpunktX = Sum[k * (k * 1.5 + 1.5 / 2) * 1.3 , {k, 1, 7}] / TotaleMasse
```

```
8.25
```

(Das 1.3 kürzt sich raus.)

```
SchwerpunktY = Sum[(7 - (k - 1)) * ((k - 1) * 1.0 + 1.0 / 2) * 1.3 , {k, 1, 7}] / TotaleMasse
```

```
2.5
```

```
{sX, sY} = {8.25, 2.5}
```

```
{8.25, 2.5}
```

2 Alles in SI-Einheiten

```
Remove["Global`*"]
```

```
m1 = 1.4; v11 = 1.6; s1 = 0.15;
m2 = 0.6; m3 = 0.2; r = 0.55; t1 = 1; g = 9.81;
```

■ 1

```
Ekin1 = 1 / 2 m1 v11^2
```

$$\frac{\text{m1 v11}^2}{2}$$

■ 2

```
Ekin1 == 1 / 2 k s1^2
```

$$\frac{\text{m1 v11}^2}{2} == \frac{\text{k s1}^2}{2}$$

```
Solve[Ekin1 == 1 / 2 k s1^2, {k}]
```

$$\left\{\left\{k \to \frac{\text{m1 v11}^2}{\text{s1}^2}\right\}\right\}$$

■ 3

```
Impuls = m1 v11
```

m1 v11

```
gleich1 = ( Ekin1 == 1 / 2 (m1 v12^2 + m2 v22^2) )
```

$$\frac{\text{m1 v11}^2}{2} == \frac{1}{2}\left(\text{m1 v12}^2 + \text{m2 v22}^2\right)$$

```
gleich2 = ( Impuls == m1 v12 + m2 v22 )
```

m1 v11 == m1 v12 + m2 v22

```
Solve[{gleich1, gleich2}, {v12, v22}] // Chop
```

$$\left\{\{\text{v12} \to \text{v11}, \text{v22} \to 0\}, \left\{\text{v12} \to \text{v11} - \frac{\text{2 m2 v11}}{\text{m1} + \text{m2}}, \text{v22} \to \frac{\text{2 m1 v11}}{\text{m1} + \text{m2}}\right\}\right\}$$

```
v22 = 2.24`;
```

```
v12 = 0.64`;
```

```
Ekin12 = 1 / 2 m1 v12^2
```

0.2048 m1

■ 4

```
Solve[v22 == ω r, {ω}]
```

$\{\{\omega \to \frac{2.24}{r}\}\}$

```
ω = 4.072727272727273`;
```

■ 5

```
Erot = 1 / 2 m2 v22^2
```

2.5088 m2

```
TraeghMom = m2 r^2
```

$m2\, r^2$

```
Erot = 1 / 2 TraeghMom ω^2
```

$8.29355\, m2\, r^2$

```
Ekin12 + Erot == Ekin1
```

$0.2048\, m1 + 8.29355\, m2\, r^2 == \frac{m1\, v11^2}{2}$

■ 6

```
Drehimpuls = TraeghMom ω
```

$4.07273\, m2\, r^2$

```
Drehimpuls = m2 r^2 ω
```

$4.07273\, m2\, r^2$

```
Impuls2 = m2 v22
```

2.24 m2

```
Drehimpuls = Impuls2 r
```

2.24 m2 r

■ 7

```
MittleresMoment = (Drehimpuls - 0) / (t1 - 0)
```

$\frac{2.24\, m2\, r}{t1}$

■ 8

```
Erot == m3 g h
```

$8.29355\ m2\ r^2 == g\ h\ m3$

```
Solve[Erot == m3 g h, {h}]
```

$\left\{\left\{h \to \frac{8.29355\ m2\ r^2}{g\ m3}\right\}\right\}$