

Kurs ■ Cours

6. Manipulation von Listen ■ Manipulation de listes

Die Gliederung dieses Kurses folgt in groben Zügen dem Buch von Nancy Blachman: A Practical Approach....
 Hinweis: Kapitel 6 lesen!
 Run mit WIN+*Mathematica* Version 5.2

■ L'articulation de ce cours correspond à peu près à celle du livre de Nancy Blachman: A Practical Approach....
 Indication: Lire le chapitre 6.
 Testé avec *Mathematica* version 5.2+WIN

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6.0. Einleitung

■ Introduction

Listen - was sind Listen - was ist das?

■ Des listes - qu'est - ce?

```
In[1]:= ??List
{e1, e2, ... } is a list of elements. Mehr...
Attributes[List] = {Locked, Protected}
```

Eingabe einer Liste:

■ Entrer une liste:

```
In[2]:= list0 = {5, 4.78, a, 8x + y, Sin[x], D,
Integer, x->E}
Out[2]= {5, 4.78, a, 8x + y, Sin[x], D, Integer, x → e}
```

Oft gibt *Mathematica* auch Listen aus. Beispiel:

■ *Mathematica* sort souvent des listes. Exemple:

```
In[3]:= Solve[{2x + 5y == 19, 3x + 7y == 27}]
```

```
Out[3]= { {x → 2, y → 3} }
```

Hier ist eine Liste mit einer Liste ausgegeben worden.

- Ici on a sorti une liste qui contient une liste.

Ebenso z.B. bei "Options":

- Egalement p.ex. avec "Options":

```
In[4]:= Options[NIntegrate]
```

```
Out[4]= {AccuracyGoal → ∞, Compiled → True, EvaluationMonitor → None, GaussPoints → Automatic,
MaxPoints → Automatic, MaxRecursion → 6, Method → Automatic, MinRecursion → 0,
PrecisionGoal → Automatic, SingularityDepth → 4, WorkingPrecision → MachinePrecision}
```

Hier ist eine Liste ausgegeben worden.

- Ici on a sorti une liste.

6.1. Erzeugung von Listen

■ Générer des listes

6.1.1. Befehle

■ Ordres

Zur Erzeugung von Listen stehen verschiedene Befehle zur Verfügung:

- Pour créer des listes on a plusieurs ordres à disposition:

```
In[5]:= ??Range
```

```
Range[imax] generates the list {1, 2, ..., imax}. Range[imin, imax] generates
the list {imin, ..., imax}. Range[imin, imax, di] uses step di. Mehr...
```

```
Attributes[Range] = {Listable, Protected}
```

```
In[6]:= ??Table
```

```
Table[expr, {imax}] generates a list of imax copies of expr. Table[
expr, {i, imax}] generates a list of the values of expr when i runs from
1 to imax. Table[expr, {i, imin, imax}] starts with i = imin. Table[expr,
{i, imin, imax, di}] uses steps di. Table[expr, {i, imin, imax}, {j, jmin,
jmax}, ... ] gives a nested list. The list associated with i is outermost. Mehr...
```

```
Attributes[Table] = {HoldAll, Protected}
```

```
In[7]:= ??Array
```

```
Array[f, n] generates a list of length n, with elements f[i]. Array[
f, {n1, n2, ... }] generates an n1 by n2 by ... array of nested lists,
with elements f[i1, i2, ... ]. Array[f, {n1, n2, ... }, {r1, r2, ... }]
generates a list using the index origins ri (default 1). Array[f, dims,
origin, h] uses head h, rather than List, for each level of the array. Mehr...
```

```
Attributes[Array] = {Protected}
```

6.1.2. Erzeugung einer Liste mit "Range"

■ Créer une liste avec "Range"

Beispiele: ■ Exemples:

In[8]:= Range[5]

Out[8]= {1, 2, 3, 4, 5}

In[9]:= Range[7.896]

Out[9]= {1, 2, 3, 4, 5, 6, 7}

In[10]:= Range[6,12]

Out[10]= {6, 7, 8, 9, 10, 11, 12}

In[11]:= Range[3.4, 8.7]

Out[11]= {3.4, 4.4, 5.4, 6.4, 7.4, 8.4}

In[12]:= Range[3.4, 8.7, 1.2]

Out[12]= {3.4, 4.6, 5.8, 7., 8.2}

6.1.3. Erzeugung einer Liste mit "Table"

■ Générer des listes avec "Table"

Beispiele: ■ Exemples:

In[13]:= Table[hoi,{6}]

Out[13]= {hoi, hoi, hoi, hoi, hoi, hoi}

In[14]:= Table[i!,{i, 6}]

Out[14]= {1, 2, 6, 24, 120, 720}

In[15]:= Table[3i,{i, 6, 14}]

Out[15]= {18, 21, 24, 27, 30, 33, 36, 39, 42}

In[16]:= Table[i,{i, 4, 40, 5}]

Out[16]= {4, 9, 14, 19, 24, 29, 34, 39}

In[17]:= Table[{i, i^2},{i, 5}]

Out[17]= {{1, 1}, {2, 4}, {3, 9}, {4, 16}, {5, 25}}

In[18]:= Table[a[i] + b[j], {i, 1, 3}, {j, 4, 6}]

Out[18]= {{a[1] + b[4], a[1] + b[5], a[1] + b[6]}, {a[2] + b[4], a[2] + b[5], a[2] + b[6]}, {a[3] + b[4], a[3] + b[5], a[3] + b[6]}}

Das ist eine Liste von Listen, d.h. eine Matrix.

Zur besseren Darstellung verwenden wir "MatrixForm":

■ C'est une liste de listes, c'est-à-dire une matrice.

Pour plus de clarté nous utilisons "MatrixForm"

```
In[19]:= MatrixForm[%]
```

```
Out[19]//MatrixForm=
```

$$\begin{pmatrix} a[1] + b[4] & a[1] + b[5] & a[1] + b[6] \\ a[2] + b[4] & a[2] + b[5] & a[2] + b[6] \\ a[3] + b[4] & a[3] + b[5] & a[3] + b[6] \end{pmatrix}$$

Ein Iterator kann auch von einem andern abhängen:

■ Un itérateur peut aussi dépendre d'un autre:

```
In[20]:= Table[a[i] + b[j], {i, 1, 3}, {j, i, 6}]
```

```
Out[20]= {{a[1] + b[1], a[1] + b[2], a[1] + b[3], a[1] + b[4], a[1] + b[5], a[1] + b[6]},  
          {a[2] + b[2], a[2] + b[3], a[2] + b[4], a[2] + b[5], a[2] + b[6]},  
          {a[3] + b[3], a[3] + b[4], a[3] + b[5], a[3] + b[6]}}
```

```
In[21]:= MatrixForm[%]
```

```
Out[21]//MatrixForm=
```

$$\begin{pmatrix} \{a[1] + b[1], a[1] + b[2], a[1] + b[3], a[1] + b[4], a[1] + b[5], a[1] + b[6]\} \\ \{a[2] + b[2], a[2] + b[3], a[2] + b[4], a[2] + b[5], a[2] + b[6]\} \\ \{a[3] + b[3], a[3] + b[4], a[3] + b[5], a[3] + b[6]\} \end{pmatrix}$$

6.1.4. "Array"

■ "Array"

Mit "Array" kann man eine symbolische Matrix erzeugen:

■ Avec "Array" on peut créer une matrice symbolique:

```
In[22]:= Clear[a]
```

```
In[23]:= Array[a,{2, 3}]
```

```
Out[23]= {{a[1, 1], a[1, 2], a[1, 3]}, {a[2, 1], a[2, 2], a[2, 3]}}
```

```
In[24]:= MatrixForm[%]
```

```
Out[24]//MatrixForm=
```

$$\begin{pmatrix} a[1, 1] & a[1, 2] & a[1, 3] \\ a[2, 1] & a[2, 2] & a[2, 3] \end{pmatrix}$$

```
In[25]:= a = 17; Array[a,{2, 3}] // MatrixForm
```

```
Out[25]//MatrixForm=
```

$$\begin{pmatrix} 17[1, 1] & 17[1, 2] & 17[1, 3] \\ 17[2, 1] & 17[2, 2] & 17[2, 3] \end{pmatrix}$$

6.2. Umordnen von Listen

■ Ranger autrement des listes

6.2.1. Befehle

■ Ordres

In[26]:= ??Sort

```
Sort[list] sorts the elements of list into canonical
order. Sort[list, p] sorts using the ordering function p. Mehr...
Attributes[Sort] = {Protected}
```

In[27]:= ??Reverse

```
Reverse[expr] reverses the order of the elements in expr. Mehr...
Attributes[Reverse] = {Protected}
```

In[28]:= ??RotateLeft

```
RotateLeft[expr, n] cycles the elements in expr n positions to the left.
RotateLeft[expr] cycles one position to the left. RotateLeft[expr, {n1,
n2, ... }] cycles elements at successive levels ni positions to the left. Mehr...
Attributes[RotateLeft] = {Protected}
```

In[29]:= ??RotateRight

```
RotateRight[expr, n] cycles the elements in expr n positions to the right.
RotateRight[expr] cycles one position to the right. RotateRight[expr, {n1,
n2, ... }] cycles elements at successive levels ni positions to the right. Mehr...
Attributes[RotateRight] = {Protected}
```

In[30]:= ??Permutations

```
Permutations[list] generates a list of all possible permutations of the elements in list.
Mehr...
Attributes[Permutations] = {Protected}
```

In[31]:= ??Drop

```
Drop[list, n] gives list with its first n elements dropped. Drop[list,
-n] gives list with its last n elements dropped. Drop[list, {n}] gives list
with its nth element dropped. Drop[list, {m, n}] gives list with elements m
through n dropped. Drop[list, {m, n, s}] gives list with elements m through
n in steps of s dropped. Drop[list, seq1, seq2, ... ] gives a nested list in
which elements specified by seqi have been dropped at level i in list. Mehr...
Attributes[Drop] = {NHoldRest, Protected}
```

In[32]:= ??Take

Take[list, n] gives the first n elements of list. Take[list, -n] gives the last n elements of list. Take[list, {m, n}] gives elements m through n of list. Take[list, {m, n, s}] gives elements m through n in steps of s. Take[list, seq1, seq2, ...] gives a nested list in which elements specified by seqi are taken at level i in list. Mehr...

Attributes[Take] = {NHoldRest, Protected}

In[33]:= ??First

First[expr] gives the first element in expr. Mehr...

Attributes[First] = {Protected}

In[34]:= ??Last

Last[expr] gives the last element in expr. Mehr...

Attributes[Last] = {Protected}

In[35]:= ??Part

expr[[i]] or Part[expr, i] gives the ith part of expr. expr[[-i]] counts from the end. expr[[0]] gives the head of expr. expr[[i, j, ...]] or Part[expr, i, j, ...] is equivalent to expr[[i]] [[j]] expr[{{i1, i2, ...} }] gives a list of the parts i1, i2, ... of expr. Mehr...

Attributes[Part] = {NHoldRest, Protected, ReadProtected}

In[36]:= ??Rest

Rest[expr] gives expr with the first element removed. Mehr...

Attributes[Rest] = {Protected}

In[37]:= ??Select

Select[list, crit] picks out all elements ei of list for which crit[ei] is True. Select[list, crit, n] picks out the first n elements for which crit[ei] is True. Mehr...

Attributes[Select] = {Protected}

6.2.2. Anwendungen

■ Applications

Hier einige Beispiele:

■ Voici quelques exemples:

In[38]:= listA = Table[Random[Integer, {0, 12}], {16}]

Out[38]= {3, 0, 5, 12, 1, 1, 6, 6, 8, 2, 3, 2, 11, 3, 10, 1}

In[39]:= Sort[listA]

Out[39]= {0, 1, 1, 1, 2, 2, 3, 3, 5, 6, 6, 8, 10, 11, 12}

In[40]:= Union[listA]

Out[40]= {0, 1, 2, 3, 5, 6, 8, 10, 11, 12}

"Sort" sortiert die Liste, "Union" sortiert sie auch, eliminiert aber Duplikate von Elementen.

- "Sort" trie la liste, "Union" la trie aussi, mais élimine les doubles des éléments.

```
In[41]:= Reverse[listA]
Out[41]= {1, 10, 3, 11, 2, 3, 2, 8, 6, 6, 1, 1, 12, 5, 0, 3}

In[42]:= RotateLeft[%,3]
Out[42]= {11, 2, 3, 2, 8, 6, 6, 1, 1, 12, 5, 0, 3, 1, 10, 3}

In[43]:= RotateRight[%,4]
Out[43]= {3, 1, 10, 3, 11, 2, 3, 2, 8, 6, 6, 1, 1, 12, 5, 0}

In[44]:= Permutations[{a, b, c}]
Out[44]= {{17, b, c}, {17, c, b}, {b, 17, c}, {b, c, 17}, {c, 17, b}, {c, b, 17}}

In[45]:= Permutations[{a, b, b}]
Out[45]= {{17, b, b}, {b, 17, b}, {b, b, 17}}
```

6.3. Erweiterung und Verkürzung von Listen

- Allonger et raccourcir des listes

6.3.1. Befehle ■ Ordres

Probiere aus: ■ Essaie:

```
In[46]:= ?Append
Append[expr, elem] gives expr with elem appended. Mehr...
In[47]:= ?AppendTo
AppendTo[s, elem] appends elem to the value of s, and resets s to the result. Mehr...
In[48]:= ?Drop
Drop[list, n] gives list with its first n elements dropped. Drop[list,
-n] gives list with its last n elements dropped. Drop[list, {n}] gives list
with its nth element dropped. Drop[list, {m, n}] gives list with elements m
through n dropped. Drop[list, {m, n, s}] gives list with elements m through
n in steps of s dropped. Drop[list, seq1, seq2, ... ] gives a nested list in
which elements specified by seqi have been dropped at level i in list. Mehr...
In[49]:= ?Insert
Insert[list, elem, n] inserts elem at position n in list. If n is negative,
the position is counted from the end. Insert[expr, elem, {i, j, ... }]
inserts elem at position {i, j, ... } in expr. Insert[expr, elem, {{i1,
j1, ... }, {i2, j2, ... }, ... }] inserts elem at several positions. Mehr...
```

In[50]:= ?Prepend

Prepend[expr, elem] gives expr with elem prepended. Mehr...

In[51]:= ?PrependTo

PrependTo[s, elem] prepends elem to the value of s, and resets s to the result. Mehr...

In[52]:= ?Rest

Rest[expr] gives expr with the first element removed. Mehr...

In[53]:= ?Take

Take[list, n] gives the first n elements of list. Take[list, -n] gives the last n elements of list. Take[list, {m, n}] gives elements m through n of list. Take[list, {m, n, s}] gives elements m through n in steps of s. Take[list, seq1, seq2, ...] gives a nested list in which elements specified by seqi are taken at level i in list. Mehr...

In[54]:= ?First

First[expr] gives the first element in expr. Mehr...

In[55]:= ?Last

Last[expr] gives the last element in expr. Mehr...

In[56]:= ?Part

expr[[i]] or Part[expr, i] gives the ith part of expr. expr[[-i]] counts from the end. expr[[0]] gives the head of expr. expr[[i, j, ...]] or Part[expr, i, j, ...] is equivalent to expr[[i]] [[j]] expr[{{i1, i2, ... } }] gives a list of the parts i1, i2, ... of expr. Mehr...

In[57]:= ?Select

Select[list, crit] picks out all elements ei of list for which crit[ei] is True. Select[list, crit, n] picks out the first n elements for which crit[ei] is True. Mehr...

6.3.2. Beispiele

■ Exemples

Probiere aus: ■ Essaie:

In[58]:= Print[listA];
Rest[listA]

{3, 0, 5, 12, 1, 1, 6, 6, 8, 2, 3, 2, 11, 3, 10, 1}

Out[59]= {0, 5, 12, 1, 1, 6, 6, 8, 2, 3, 2, 11, 3, 10, 1}

Was ist passiert?

■ Que s'est-il passé?

In[60]:=
Drop[listA, 10]

Out[60]= {3, 2, 11, 3, 10, 1}

```

In[61]:= Drop[listA, -5]
Out[61]= {3, 0, 5, 12, 1, 1, 6, 6, 8, 2, 3}

In[62]:= Drop[listA, {2}]
Out[62]= {3, 5, 12, 1, 1, 6, 6, 8, 2, 3, 2, 11, 3, 10, 1}

In[63]:= Drop[listA, {1}]
Out[63]= {0, 5, 12, 1, 1, 6, 6, 8, 2, 3, 2, 11, 3, 10, 1}

In[64]:= Drop[listA, {3}]
Out[64]= {3, 0, 12, 1, 1, 6, 6, 8, 2, 3, 2, 11, 3, 10, 1}

In[65]:= Drop[listA, {1,8}]
Out[65]= {8, 2, 3, 2, 11, 3, 10, 1}

In[66]:= Drop[listA, {4,12}]
Out[66]= {3, 0, 5, 11, 3, 10, 1}

In[67]:= Take[listA,4]
Out[67]= {3, 0, 5, 12}

In[68]:= Take[listA,-6]
Out[68]= {3, 2, 11, 3, 10, 1}

In[69]:= Take[listA,{4,6}]
Out[69]= {12, 1, 1}

In[70]:= Append[listA, 99]
Out[70]= {3, 0, 5, 12, 1, 1, 6, 6, 8, 2, 3, 2, 11, 3, 10, 1, 99}

In[71]:= Prepend[listA, 100]
Out[71]= {100, 3, 0, 5, 12, 1, 1, 6, 6, 8, 2, 3, 2, 11, 3, 10, 1}

In[72]:= Insert[listA, 999, 4]
Out[72]= {3, 0, 5, 999, 12, 1, 1, 6, 6, 8, 2, 3, 2, 11, 3, 10, 1}

In[73]:= listA
Out[73]= {3, 0, 5, 12, 1, 1, 6, 6, 8, 2, 3, 2, 11, 3, 10, 1}

```

Bei all diesen Operationen ist "listA" unverändert geblieben. Nicht aber bei den folgenden Operationen:

■ Pendant toutes ces opérations "listA" n'a pas changé. Cela n'est pas le cas pour les opérations suivantes:

```

In[74]:= Print[listA];
AppendTo[listA, 2000]

{3, 0, 5, 12, 1, 1, 6, 6, 8, 2, 3, 2, 11, 3, 10, 1}

Out[75]= {3, 0, 5, 12, 1, 1, 6, 6, 8, 2, 3, 2, 11, 3, 10, 1, 2000}

```

```
In[76]:= PrependTo[listA, 555]
Out[76]= {555, 3, 0, 5, 12, 1, 1, 6, 6, 8, 2, 3, 2, 11, 3, 10, 1, 2000}
```

6.4. Zählung der Elemente einer Liste

■ Compter les éléments d'une liste

6.4.1. Befehle:

■ Ordres:

Probiere aus: ■ Essaie:

```
In[77]:= ?Length
Length[expr] gives the number of elements in expr. Mehr...
```

```
In[78]:= ?Dimensions
Dimensions[expr] gives a list of the dimensions of expr. Dimensions[
expr, n] gives a list of the dimensions of expr down to level n. Mehr...
```

6.4.2. Anwendungen

■ Applications

```
In[79]:= Length[listA]
Out[79]= 18

In[80]:= Length[{a, 4, 2^8, t^2 + 2t - 6, matrix}]
Out[80]= 5

In[81]:= Length[{{a, 4, 2^8, t^2 + 2t - 6, matrix},
listA}]
Out[81]= 2

In[82]:= Dimensions[{{a, 4, 2^8, t^2 + 2t - 6, matrix},
listA}]
Out[82]= {2}

In[83]:= Dimensions[{{a, 4, 2^8, t^2 + 2t - 6, matrix},
listA}, 0]
Out[83]= {}

In[84]:= Dimensions[{{a, 4, 2^8, t^2 + 2t - 6, matrix},
listA}, 1]
Out[84]= {2}
```

```
In[85]:= Dimensions[{{a, 4, 2^8, t^2 + 2t - 6, matrix},
                    listA}, 2]

Out[85]= {2}

In[86]:= MatrixForm[{{a, 4, 2^8, t^2 + 2t - 6, matrix},
                     listA}]

Out[86]//MatrixForm=

$$\begin{pmatrix} \{17, 4, 256, -6 + 2t + t^2, \text{matrix}\} \\ \{555, 3, 0, 5, 12, 1, 1, 6, 6, 8, 2, 3, 2, 11, 3, 10, 1, 2000\} \end{pmatrix}$$


In[87]:= Dimensions[{{a, 4, 2^8, t^2 + 2t - 6},
                     {1, 2, 3, 4}}, 2]

Out[87]= {2, 4}

In[88]:= MatrixForm[{{a, 4, 2^8, t^2 + 2t - 6},
                     {1, 2, 3, 4}}]

Out[88]//MatrixForm=

$$\begin{pmatrix} 17 & 4 & 256 & -6 + 2t + t^2 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

```

6.5. Zusammenfügen von Listen, Beziehungen

zwischen Listen

■ Assemblage de listes, relations entre listes

6.5.1. Befehle

■ Ordres

Probiere aus: ■ Essaie

In[89]:= ?Complement

Complement[eall, e1, e2, ...] gives the elements in eall which are not in any of the ei.
Mehr...

In[90]:= ?Union

Union[list1, list2, ...] gives a sorted list of all the distinct
elements that appear in any of the listi. Union[list] gives a sorted version
of a list, in which all duplicated elements have been dropped. Mehr...

In[91]:= ?Join

Join[list1, list2, ...] concatenates lists together. Join
can be used on any set of expressions that have the same head. Mehr...

In[92]:= ?Intersection

Intersection[list1, list2, ...] gives a sorted list of the elements common to all the listi.
Mehr...

6.5.2. Anwendungen

■ Applications

```
In[93]:= Complement[{1, 2, 3, 4, 5}, {1, 2, 3}]
Out[93]= {4, 5}

In[94]:= Intersection[{a, b, c, d, e, f},
                      {d, e, f, g, h, i, j, k}]
Out[94]= {d, e, f}

In[95]:= Union[{a, b, c, d, e, f},
                  {d, e, f, g, h, i, j, k}]
Out[95]= {17, b, c, d, e, f, g, h, i, j, k}

In[96]:= Union[{1, 1, 1, 1, 2, 1, 3, 3, 2, 4, 2, 4, 4,
                  3, 3, 3}]
Out[96]= {1, 2, 3, 4}

In[97]:= Join[{a, b, c, d, e, f}, {d, e, f, g, h, i, j, k}]
Out[97]= {17, b, c, d, e, f, d, e, f, g, h, i, j, k}

In[98]:= Join[{1, 2, 1, 1, 2, 2}, {1, 2, 2, 1, 3, 2, 1,
                  3, 3}]
Out[98]= {1, 2, 1, 1, 2, 2, 1, 2, 1, 3, 2, 1, 3, 3}

In[99]:= Union[{1, 2, 1, 1, 2, 2}, {1, 2, 2, 1, 3, 2, 1,
                  3, 3}]
Out[99]= {1, 2, 3}
```

6.6. Veränderung der Form einer Liste

■ Changer la forme d'une liste

6.6.1. Befehle

■ Ordres

Probiere aus: ■ Essaie:

```
In[100]:= ?Flatten
Flatten[list] flattens out nested lists. Flatten[list, n] flattens
          to level n. Flatten[list, n, h] flattens subexpressions with head h. Mehr...
```

```
In[101]:= ?Partition

Partition[list, n] partitions list into non-overlapping sublists of length n. Partition[list,
n, d] generates sublists with offset d. Partition[list, {n1, n2, ...}] partitions
a nested list into blocks of size n1 × n2 × ... . Partition[list, {n1, n2, ...},
{d1, d2, ...}] uses offset di at level i in list. Partition[list, n, d, {kL, kR}]
specifies that the first element of list should appear at position kL in the first
sublist, and the last element of list should appear at or after position kR in the
last sublist. If additional elements are needed, Partition fills them in by treating
list as cyclic. Partition[list, n, d, {kL, kR}, x] pads if necessary by repeating
the element x. Partition[list, n, d, {kL, kR}, {x1, x2, ...}] pads if necessary by
cyclically repeating the elements xi. Partition[list, n, d, {kL, kR}, {}] uses no
padding, and so can yield sublists of different lengths. Partition[list, nlist, dlist,
{klistL, klistR}, padlist] specifies alignments and padding in a nested list. Mehr...
```



```
In[102]:= ?Transpose

Transpose[list] transposes the first two levels in list.
Transpose[list, {n1, n2, ...}] transposes list so that the levels 1,
2, ... in list correspond to levels n1, n2, ... in the result. Mehr...
```

6.6.2. Anwendungen

■ Applications

```
In[103]:= vector1 = Range[10]

Out[103]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

In[104]:= matrix1 = Partition[vector1, 5]

Out[104]= {{1, 2, 3, 4, 5}, {6, 7, 8, 9, 10}}
```



```
In[105]:= MatrixForm[matrix1]

Out[105]//MatrixForm= 
$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 6 & 7 & 8 & 9 & 10 \end{pmatrix}$$

```



```
In[106]:= matrix2 = Partition[vector1, 4]

Out[106]= {{1, 2, 3, 4}, {5, 6, 7, 8}}
```



```
In[107]:= matrix3 = Partition[vector1, 3]

Out[107]= {{1, 2, 3}, {4, 5, 6}, {7, 8, 9}}
```

```
In[108]:= matrix4 = Partition[vector1, 2.5]

Partition::ilsmp : Single or list of positive machine-size integers
expected at position 2 of Partition[{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, 2.5]. Mehr...
Out[108]= Partition[{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, 2.5]
```

Was ist passiert?

■ Que s'est-il passé?

```
In[109]:= Partition[{1, 35, 432, 2, 5467, 4, 3, 987, 87},
3]
```

```
Out[109]= {{1, 35, 432}, {2, 5467, 4}, {3, 987, 87}}
```

```
In[110]:= Partition[{1, 2, 3, 4, 5, 6, 7}, 3, 1]
```

```
Out[110]= {{1, 2, 3}, {2, 3, 4}, {3, 4, 5}, {4, 5, 6}, {5, 6, 7}}
```

```
In[111]:= Partition[{{1, 2, 3}, {4, 5, 6}, {7, 8, 9},
{a, b, c}}, 2, 1]
```

```
Out[111]= {{{1, 2, 3}, {4, 5, 6}}, {{4, 5, 6}, {7, 8, 9}}, {{7, 8, 9}, {17, b, c}}}
```

```
In[112]:= Print[matrix1];
Flatten[matrix1]

{{1, 2, 3, 4, 5}, {6, 7, 8, 9, 10}}
```

```
Out[113]= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
```

```
In[114]:= m = {{1, 2, 3, a}, {4, 5, 6, b}}
```

```
Out[114]= {{1, 2, 3, 17}, {4, 5, 6, b}}
```

```
In[115]:= Transpose[m]
```

```
Out[115]= {{1, 4}, {2, 5}, {3, 6}, {17, b}}
```

```
In[116]:= Print[MatrixForm[m]];
MatrixForm[Transpose[m]]


$$\begin{pmatrix} 1 & 2 & 3 & 17 \\ 4 & 5 & 6 & b \end{pmatrix}$$


Out[117]//MatrixForm=
```

$$\begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \\ 17 & b \end{pmatrix}$$

6.7. Elemente herauspicken

■ Choisir quelques éléments

6.7.1. Befehle

■ Ordres

Oben sind schon "First", "Last" und "Part erwähnt worden.

■ Plus haut on a déjà nommé "First", "Last", et "Post".

6.7.2. Anwendungen

■ Applications

```
In[118]:= liste1 = {a, b, c, d, e, f, g}

Out[118]= {17, b, c, d, e, f, g}

In[119]:= First[liste1]

Out[119]= 17

In[120]:= Last[liste1]

Out[120]= g

In[121]:= liste1[[4]]

Out[121]= d

In[122]:= Print[{liste1[[2]], liste1[[3]], liste1[[4]],
liste1[[5]]}]

{b, c, d, e}
```

```
In[123]:= listel[[ Range[2, 5]]]

Out[123]= {b, c, d, e}

In[124]:= array1 = Array[a, {2, 2}]

Out[124]= {{17[1, 1], 17[1, 2]}, {17[2, 1], 17[2, 2]}}

In[125]:= array1[[2]]

Out[125]= {17[2, 1], 17[2, 2]}

In[126]:= array1[[1, 2]]

Out[126]= 17[1, 2]
```

6.8. Auswahl von Daten

■ Choisir des données

6.8.1. Befehle

■ Ordres

Probiere aus: ■ Essaie:

```
In[127]:= ?Select

Select[list, crit] picks out all elements ei of list for which crit[ei] is True. Select[
list, crit, n] picks out the first n elements for which crit[ei] is True. Mehr...
```

In[128]:= **?*Q**

System`

ArgumentCountQ	MatrixQ
ArrayQ	MemberQ
AtomQ	NameQ
DigitQ	NumberQ
EllipticNomeQ	NumericQ
EvenQ	OddQ
ExactNumberQ	OptionQ
FreeQ	OrderedQ
HypergeometricPFQ	PartitionsQ
InexactNumberQ	PolynomialQ
IntegerQ	PrimeQ
IntervalMemberQ	SameQ
InverseEllipticNomeQ	StringFreeQ
LegendreQ	StringMatchQ
LetterQ	StringQ
LinkConnectedQ	SyntaxQ
LinkReadyQ	TensorQ
ListQ	TrueQ
LowerCaseQ	UnsameQ
MachineNumberQ	UpperCaseQ
MatchLocalNameQ	ValueQ
MatchQ	VectorQ

In[129]:=

?DigitQ

DigitQ[string] yields True if all the characters in the string
are digits in the range 0 through 9, and yields False otherwise. Mehr...

In[130]:=

?IntegerQ

IntegerQ[expr] gives True if expr is an integer, and False otherwise. Mehr...

In[131]:=

?LetterQ

LetterQ[string] yields True if all the characters
in the string are letters, and yields False otherwise. Mehr...

In[132]:=

?LowerCaseQ

LowerCaseQ[string] yields True if all the characters in
the string are lower-case letters, and yields False otherwise. Mehr...

In[133]:=

?MachineNumberQ

MachineNumberQ[expr] returns True if expr is a machine-
precision real or complex number, and returns False otherwise. Mehr...

```
In[134]:= ?MatrixQ
MatrixQ[expr] gives True if expr is a list of lists or a two-dimensional SparseArray object
that can represent a matrix, and gives False otherwise. MatrixQ[expr, test] gives True
only if test yields True when applied to each of the matrix elements in expr. Mehr...
In[135]:= ?NameQ
NameQ["string"] yields True if there are any symbols whose
names match the string pattern given, and yields False otherwise. Mehr...
In[136]:= ?NumberQ
NumberQ[expr] gives True if expr is a number, and False otherwise. Mehr...
In[137]:= ?OddQ
OddQ[expr] gives True if expr is an odd integer, and False otherwise. Mehr...
In[138]:= ?EvenQ
EvenQ[expr] gives True if expr is an even integer, and False otherwise. Mehr...
In[139]:= ?PrimeQ
PrimeQ[expr] yields True if expr is a prime number, and yields False otherwise. Mehr...
In[140]:= ?UpperCaseQ
UpperCaseQ[string] yields True if all the characters in
the string are upper-case letters, and yields False otherwise. Mehr...
In[141]:= ?ValueQ
ValueQ[expr] gives True if a value has been defined for expr, and gives False otherwise.
Mehr...
In[142]:= ?VectorQ
VectorQ[expr] gives True if expr is a list or a one-dimensional SparseArray object, none of
whose elements are themselves lists, and gives False otherwise. VectorQ[expr, test]
gives True only if test yields True when applied to each of the elements in expr. Mehr...
In[143]:= ?Positiv*
Positive[x] gives True if x is a positive number. Mehr...
In[144]:= ?Negat*
Negative[x] gives True if x is a negative number. Mehr...
```

6.8.2. Anwendungen

■ Applications

Probiere aus: ■ Essaie:

```
In[145]:= MemberQ[listel, 3]
Out[145]= False

In[146]:= MemberQ[listel, a]
Out[146]= True

In[147]:= menge = {-4, 3, 5.678, emil, Emma, 4, 0, -2, -4,
Pi, 2^(1/2)}
Out[147]= {-4, 3, 5.678, emil, Emma, 4, 0, -2, -4, \[Pi], \[Sqrt]2}

In[148]:= Select[menge, Positive]
Out[148]= {3, 5.678, 4, \[Pi], \[Sqrt]2}

In[149]:= Select[{a, b, c, d, x, y, z}, Positive]
Out[149]= {17}
```

Bei "Select" steht also hinten das Selektionskriterium. Solche Kriterien kann man selbst definieren. Beispiel:
 ■ Quant à "Select", le critère de sélection se trouve à la fin. On peut définir soi-même de tels critères. Exemple:

```
In[150]:= ?Precision
Precision[x] gives the effective number of digits of precision in the number x. Mehr...
In[151]:= ?===
lhs === rhs yields True if the expression
lhs is identical to rhs, and yields False otherwise. Mehr...
In[152]:= exakt[x_] := Precision[x] === Infinity
In[153]:= Select[{-6, 0, 3, 3.1414333, 1.111111, Pi, E,
N[Sqrt[2], 10], 5.66}, exakt]
Out[153]= {-6, 0, 3, \[Pi], \[Epsilon]}
```

6.9. Rechnen mit Listen

■ Calculer avec des listes

6.9.1. Befehle

■ Ordres

In[154]:=

?Sum

Sum[f, {i, imax}] evaluates the sum of the expressions f as evaluated for each i from 1 to imax. Sum[f, {i, imin, imax}] starts with i = imin. Sum[f, {i, imin, imax, di}] uses steps di. Sum[f, {i, imin, imax}, {j, jmin, jmax}, ...] evaluates a sum over multiple indices. Mehr...

In[155]:=

?Plus

x + y + z represents a sum of terms. Mehr...

In[156]:=

?Apply

Apply[f, expr] or f @@ expr replaces the head of expr by f. Apply[f, expr, levelspec] replaces heads in parts of expr specified by levelspec. Mehr...

6.9.2. Wie man's macht:

■ C'est ainsi que cela se fait:

In[157]:=

Sum[n^2, {n, 7}]

Out[157]=

140

In[158]:=

meineDaten = {3.4, 6.1, 5.3}

Out[158]=

{3.4, 6.1, 5.3}

In[159]:=

Sum[meineDaten[[n]], {n, Length[meineDaten]}]

Out[159]=

14.8

In[160]:=

Plus[1, 1, 1, 1, 1]

Out[160]=

5

```
In[161]:= Plus[1, 2, 3, 4, 5]
Out[161]= 15

In[162]:= Plus[a, b, c, d]
Out[162]= 17 + b + c + d

In[163]:= Plus[{a, b, c, d}]
Out[163]= {17, b, c, d}

In[164]:= ??Plus
x + y + z represents a sum of terms. Mehr...
Attributes[Plus] = {Flat, Listable, NumericFunction, OneIdentity, Orderless, Protected}
Default[Plus] := 0
```

"Plus" hat das Attribut "Listable".

■ "Plus" a l'attribut "Listable".

```
In[165]:= ??Listable
Listable is an attribute that can be assigned to a symbol f to indicate that the function f
should automatically be threaded over lists that appear as its arguments. Mehr...
Attributes[Listable] = {Protected}
```

"Plus" auf eine Liste angewandt addiert nur ein Element, nämlich die Liste selbst. Wie nun "Plus" aber auf die Elemente anwenden? - So:

■ "Plus" appliqu  à une liste additionne seulement un  l ment, c'est- -dire la liste m me. Comment appliquer "Plus" aux  l ments? - Ainsi:

```
In[166]:= Apply[Plus, {a, b, c, d}]
Out[166]= 17 + b + c + d
```

Was geht nun schneller, "Sum" oder "Plus" ?

■ Qu'est-ce qui est plus rapide, "Sum" ou "Plus"?

```
In[167]:= Timing[Apply[Plus, Range[500]]]
Out[167]= {0. Second, 125250}
```

```
In[168]:= Timing[Sum[n, {n, 500}]]

Out[168]= {0. Second, 125250}
```

6.10. Auf Listen anwendbare arithmetische Funktionen

■ Fonctions arithmétiques applicables à des listes

Beispiele ■ Exemples

Wir wenden "Log" auf eine Liste an:

■ Nous appliquons "Log" à une liste:

```
In[169]:= ??Log

Log[z] gives the natural logarithm of z (logarithm to base e). Log[b, z] gives the logarithm to base b. Mehr...
Attributes[Log] = {Listable, NumericFunction, Protected}

In[170]:= Log[{1, 2, 3, 4, 5, 6, 7}]

Out[170]= {0, Log[2], Log[3], Log[4], Log[5], Log[6], Log[7]}

In[171]:= N[%]

Out[171]= {0., 0.693147, 1.09861, 1.38629, 1.60944, 1.79176, 1.94591}
```

Oder mit "Sinus":

■ Ou avec "Sinus":

```
In[172]:= Sin[{1, 2, 3, 4, 5, 6, 7}]

Out[172]= {Sin[1], Sin[2], Sin[3], Sin[4], Sin[5], Sin[6], Sin[7]}

In[173]:= Sin[{1, 2, 3, 4, 5, 6, 7}] // N

Out[173]= {0.841471, 0.909297, 0.14112, -0.756802, -0.958924, -0.279415, 0.656987}
```

Oder "Plus":

■ Ou "Plus":

```
In[174]:= ?Plus
x + y + z represents a sum of terms. Mehr...
```

```
In[175]:= ?*Form*
```

System`

AccountingForm	MathMLForm
BaseForm	MatrixForm
BinaryFormat	NumberForm
BlankForm	NumberFormat
BoxFormatTypes	OutputForm
CForm	OutputFormData
ColonForm	OutputToOutputForm
ColumnForm	OutputToStandardForm
CommonDefaultFormatTypes	PaddedForm
DefaultFormatType	ParentForm
DefaultFormatTypeForStyle	PasteBoxFormInlineCells
DefaultInlineFormatType	PointForm
DefaultInputFormatType	PolynomialForm
DefaultOutputFormatType	PrecedenceForm
DefaultTextFormatType	PrintForm
DefaultTextInlineFormatType	PromptForm
DisplayForm	RealBlockForm
DOSTextFormat	ReturnInputFormPacket
EdgeForm	RuleForm
EngineeringForm	ScientificForm
ExcludedForms	SequenceForm
FaceForm	SetBoxFormNamesPacket
FileFormat	ShowShortBoxForm
FontForm	SpaceForm
Format	StandardForm
FormatRules	StringForm
FormatType	StyleForm
FormatTypeAutoConvert	SyntaxForm
FormatValues	TableForm
FormBox	TeXForm
FormBoxOptions	TextForm
FortranForm	TraditionalForm
FullForm	TreeForm
HoldForm	ValueForm
HorizontalForm	VerticalForm
InputAutoFormat	\$BoxForms
InputForm	\$ExportFormats
InputToBoxFormPacket	\$FormatType
InputToInputForm	\$ImportFormats
InputToStandardForm	\$OutputForms
LineForm	\$PrintForms
LongForm	\$SuppressInputFormHeads

```
In[176]:= FullForm[a + b]
Out[176]//FullForm=
Plus[17, b]

In[177]:= Plus[{1, 2, 3, 4}, {a, b, c, d}]
Out[177]= {18, 2+b, 3+c, 4+d}

In[178]:= {1, 2, 3, 4} + {a, b, c, d}
Out[178]= {18, 2+b, 3+c, 4+d}
```

Attribute einiger Funktionen:

■ Attributs de quelques fonctions:

```
In[179]:= Attributes[{Exp, Log, Sin, Cos, Plus, Times}]
Out[179]= {{Listable, NumericFunction, Protected, ReadProtected},
{Listable, NumericFunction, Protected},
{Listable, NumericFunction, Protected}, {Listable, NumericFunction, Protected},
{Flat, Listable, NumericFunction, OneIdentity, Orderless, Protected},
{Flat, Listable, NumericFunction, OneIdentity, Orderless, Protected}}
```



```
In[180]:= Attributes[Attributes]
Out[180]= {HoldAll, Listable, Protected}
```

"Attributes" hat also auch das Attribut "Listable".

■ "Attributes" a donc aussi l'attribut "Listable".

6.11. Listable und Map

■ Listable et Map

6.11.1. Befehle

■ Ordres

Probiere: ■ Essaie:

```
In[181]:= ??Map
Map[f, expr] or f /@ expr applies f to each element on the first level in expr. Map[
f, expr, levelspec] applies f to parts of expr specified by levelspec. Mehr...
Attributes[Map] = {Protected}
Options[Map] = {Heads → False}

In[182]:= ?Listable
Listable is an attribute that can be assigned to a symbol f to indicate that the function f
should automatically be threaded over lists that appear as its arguments. Mehr...

In[183]:= ?Apply
Apply[f, expr] or f @@ expr replaces the head of expr by f. Apply[f, expr,
levelspec] replaces heads in parts of expr specified by levelspec. Mehr...
```

6.11.2. Anwendungen

■ Applications

Es gibt zwei Wege, eine Funktion auf eine Liste anzuwenden: mit "Listable" als Attribut und mit der Funktion "Map".
Probiere:

■ Il y a deux façons d'appliquer une fonction à une liste: avec l'attribut "List" et avec la fonction "Map". Essaie:

```
In[184]:= f[{1, 2, 3, 4, 5, 6}]
```

```
Out[184]= f[{1, 2, 3, 4, 5, 6}]
```

```
In[185]:= ?f
Global`f
```

f ist nicht "Listable".

■ F n'est pas "Listable"

```
In[186]:= Map[f, {1, 2, 3, 4, 5, 6}]
```

```
Out[186]= {f[1], f[2], f[3], f[4], f[5], f[6]}
```

So geht's! Oder so:

■ Voilà! Ou ainsi:

```
In[187]:= Map[f, a x^2 + b x + c]
```

```
Out[187]= f[c] + f[b x] + f[17 x^2]
```

Map wirkt auf die Teile eines Ausdrucks! Beachte:

- Map a un effet sur les parties d'une expression! Remarque:

```
In[188]:= Map[f, {a, b, c, d, e}]
Out[188]= {f[17], f[b], f[c], f[d], f[e]}

In[189]:= Apply[f, {a, b, c, d, e}]
Out[189]= f[17, b, c, d, e]
```

Mit Apply wird f angewandt auf die Elemente der Menge als Ganzes, während mit Map diejenige Menge gebildet wird, die als neue Elemente die Funktionswerte der einzelnen alten Elemente enthält.

- Avec Apply on applique f aux éléments de l'ensemble comme un tout, tandis qu'avec Map on forme l'ensemble qui contient en tant qu'éléments nouveaux les valeurs des fonctions des vieux éléments.

Statt Apply und Map kann man auch /@ und @@ benutzen. Beispiele:

- On peut employer aussi /@ et @@ au lieu de Apply et Map. Exemple:

```
In[190]:= f/@{a, b, c, d, e}
Out[190]= {f[17], f[b], f[c], f[d], f[e]}

In[191]:= f@@{a, b, c, d, e}
Out[191]= f[17, b, c, d, e]

In[192]:= Sin/@{0, Pi, 2 Pi, 2, 3, 4, 5, 6}
Out[192]= {0, 0, 0, Sin[2], Sin[3], Sin[4], Sin[5], Sin[6]}

In[193]:= f1[x_, y_, z_]:= Sin[x z^2] Cos[y]
In[194]:= f1@@{1, 2, 3}
Out[194]= Cos[2] Sin[9]
```

6.12. Formatierung von Listen

■ Formater des listes

6.12.1. Befehle

■ Ordres

Probiere: ■ Essaie:

```
In[195]:= ?ColumnForm
ColumnForm[{e1, e2, ...}] prints as a column with e1 above e2, etc. ColumnForm[list,
horiz] specifies the horizontal alignment of each element. ColumnForm[list,
horiz, vert] also specifies the vertical alignment of the whole column. Mehr...
```

```
In[196]:= ?MatrixForm
MatrixForm[list] prints with the elements of list arranged in a regular array. Mehr...
```

```
In[197]:= ?TableForm
TableForm[list] prints with the elements of list arranged in an array of rectangular cells.
Mehr...
```

6.12.2. Beispiele

■ Exemples

```
In[198]:= ?Binomial
Binomial[n, m] gives the binomial coefficient. Mehr...
```

```
In[199]:= pascal[n_]:= Table[Binomial[n, i],{i, 0, n}]
```

```
In[200]:= pascal[5]
```

```
Out[200]= {1, 5, 10, 10, 5, 1}
```

```
In[201]:= Table[pascal[n],{n, 0, 5}]
```

```
Out[201]= {{1}, {1, 1}, {1, 2, 1}, {1, 3, 3, 1}, {1, 4, 6, 4, 1}, {1, 5, 10, 10, 5, 1}}
```

```
In[202]:= ColumnForm[Table[pascal[n],{n, 0, 5}]]  
  
Out[202]= {1}  
          {1, 1}  
          {1, 2, 1}  
          {1, 3, 3, 1}  
          {1, 4, 6, 4, 1}  
          {1, 5, 10, 10, 5, 1}  
  
In[203]:= ColumnForm[Table[pascal[n],{n, 0, 5}], Center]  
  
Out[203]= {1}  
          {1, 1}  
          {1, 2, 1}  
          {1, 3, 3, 1}  
          {1, 4, 6, 4, 1}  
          {1, 5, 10, 10, 5, 1}  
  
In[204]:= ColumnForm[Table[pascal[n],{n, 0, 5}], Right]  
  
Out[204]= {1}  
          {1, 1}  
          {1, 2, 1}  
          {1, 3, 3, 1}  
          {1, 4, 6, 4, 1}  
          {1, 5, 10, 10, 5, 1}  
  
In[205]:= ColumnForm[Table[pascal[n],{n, 0, 5}], Left]  
  
Out[205]= {1}  
          {1, 1}  
          {1, 2, 1}  
          {1, 3, 3, 1}  
          {1, 4, 6, 4, 1}  
          {1, 5, 10, 10, 5, 1}  
  
In[206]:= TableForm[Table[pascal[n],{n, 0, 5}]]  
  
Out[206]//TableForm=  
1  
1      1  
1      2      1  
1      3      3      1  
1      4      6      4      1  
1      5      10     10     5      1
```

```
In[207]:= MatrixForm[Table[pascal[n],{n, 0, 5}]]
```

```
Out[207]//MatrixForm=
```

$$\left(\begin{array}{c} \{1\} \\ \{1, 1\} \\ \{1, 2, 1\} \\ \{1, 3, 3, 1\} \\ \{1, 4, 6, 4, 1\} \\ \{1, 5, 10, 10, 5, 1\} \end{array} \right)$$

```
In[208]:= MatrixForm[Table[n - i,{i,5},{n, 5}]]
```

```
Out[208]//MatrixForm=
```

$$\left(\begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \\ -1 & 0 & 1 & 2 & 3 \\ -2 & -1 & 0 & 1 & 2 \\ -3 & -2 & -1 & 0 & 1 \\ -4 & -3 & -2 & -1 & 0 \end{array} \right)$$

"Putzmaschine" einsetzen

■ Employer la "machine de nettoyage"

```
In[209]:= (* Old Form: Remove["Global`*"] *)
```

```
In[210]:= Remove["Global`*"]
```