# Lösungen

```
Remove["Global`*"]
```

## Uebung 1

```
rg1[t_] := {-2, 4} + t {7, 2};
rg2[t_] := {8, 3} + t {-4, 5};
```

```
s1a = Solve[rg1[λ] == rg2[μ], {λ, μ}] // Flatten
```

$$\left\{\lambda \to \frac{46}{43}, \mu \to \frac{27}{43}\right\}$$

```
s1a // N
```

$$\{\lambda \to 1.06977, \mu \to 0.627907\}$$

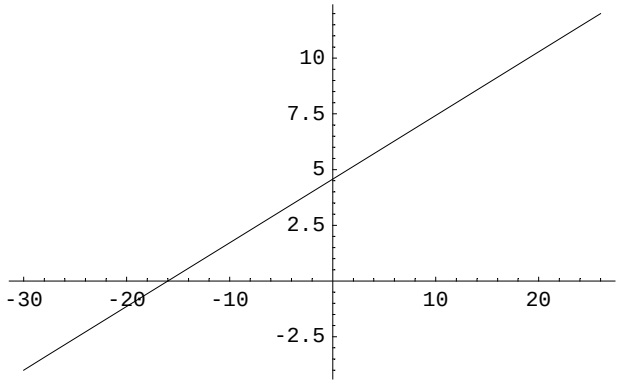
```
res1a = {rg1[λ], rg2[μ]} /. s1a
```

$$\left\{\left\{\frac{236}{43}, \frac{264}{43}\right\}, \left\{\frac{236}{43}, \frac{264}{43}\right\}\right\}$$

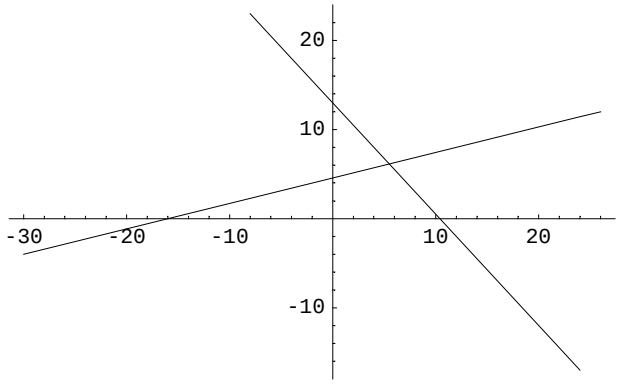
```
res1a // N
```

$$\{\{5.48837, 6.13953\}, \{5.48837, 6.13953\}\}$$

```
ParametricPlot[Evaluate[rg1[t]], {t, -4, 4}];
```



```
ParametricPlot[Evaluate[{rg1[t], rg2[t]}], {t, -4, 4}];
```

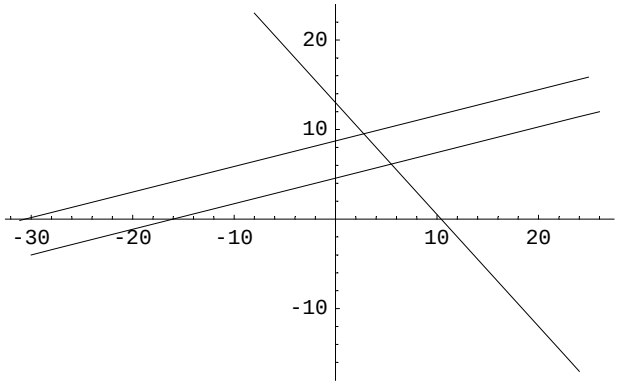



```
h[t_] := rg1[t] + {-2, 7} / Sqrt[(-2)^2 + 7^2] 4
```

```
h[t] // N
```

```
{-3.09888 + 7. t, 7.8461 + 2. t}
```

```
ParametricPlot[Evaluate[{rg1[t], rg2[t], h[t]}], {t, -4, 4}];
```




```
s1b = Solve[h[λ] == rg2[μ], {λ, μ}] // Flatten
```

$$\left\{\lambda \to -\frac{2\left(-1219+36\sqrt{53}\right)}{2279},\ \mu \to \frac{1}{43}\left(27+4\sqrt{53}\right)\right\}$$

```
s1b // N
```

```
{λ → 0.839768, μ → 1.30513}
```

```
{h[λ], rg2[μ]} /. s1b // N
```

```
{{2.77949, 9.52563}, {2.77949, 9.52563}}
```

# Uebung 2

## ■ a

```
g[t_] := {-2, 4, 1} + t {7, 2, -1}
```

```
Φ[λ_, μ_] := {1, 0, -2} + λ {4, 1, 4} + μ {-3, -2, 1}
```

```
s2a = Solve[g[t] == Φ[λ, μ], {t, λ, μ}] // Flatten
```

$$\left\{t \to \frac{53}{18},\ \lambda \to \frac{10}{9},\ \mu \to -\frac{79}{18}\right\}$$

```
c
```

c

```
res2a // N
```

res2a

## ■ b

```
Ψ[λ_, μ_] := {0, 1, 1} + λ {-1, 2, 3} + μ {1, -1, 1}
```

```
s2b = Solve[Ψ[λ, μ] == Φ[ξ, η], {μ, ξ, η}] // Flatten
```

$$\left\{\mu \to \frac{2}{5}\ (5 + 7\,\lambda),\ \xi \to \frac{1}{5}\ (5 + 6\,\lambda),\ \eta \to 1 + \lambda\right\}$$

```
res2b = {Ψ[λ, μ], Φ[ξ, η]} /. s2b
```

$$\left\{\left\{-\lambda + \frac{2}{5}\ (5 + 7\,\lambda),\ 1 + 2\,\lambda - \frac{2}{5}\ (5 + 7\,\lambda),\ 1 + 3\,\lambda + \frac{2}{5}\ (5 + 7\,\lambda)\right\},\right.$$
$$\left.\left\{1 - 3\ (1 + \lambda) + \frac{4}{5}\ (5 + 6\,\lambda),\ -2\ (1 + \lambda) + \frac{1}{5}\ (5 + 6\,\lambda),\ -1 + \lambda + \frac{4}{5}\ (5 + 6\,\lambda)\right\}\right\}$$

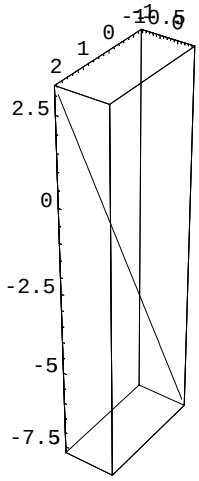
```
res2b // N
```

$$\{\{-1.\,\lambda + 0.4\ (5. + 7.\,\lambda),\ 1. + 2.\,\lambda - 0.4\ (5. + 7.\,\lambda),\ 1. + 3.\,\lambda + 0.4\ (5. + 7.\,\lambda)\},$$
$$\{1. - 3.\ (1. + \lambda) + 0.8\ (5. + 6.\,\lambda),\ -2.\ (1. + \lambda) + 0.2\ (5. + 6.\,\lambda),\ -1. + \lambda + 0.8\ (5. + 6.\,\lambda)\}\}$$

```
gs[λ_] := res2b[[1]]; gs[λ]
```

$$\left\{-\lambda + \frac{2}{5}\ (5 + 7\,\lambda),\ 1 + 2\,\lambda - \frac{2}{5}\ (5 + 7\,\lambda),\ 1 + 3\,\lambda + \frac{2}{5}\ (5 + 7\,\lambda)\right\}$$

```
ParametricPlot3D[Evaluate[gs[λ]], {λ, -1.8, 0}, ViewPoint -> {2.265, 1.593, 1.945}];
```



```
Solve[gs[λ] == {x, y, 0}, {x, y, λ}] // Flatten
```

$$\left\{x \to \frac{31}{29},\ y \to -\frac{17}{29},\ \lambda \to -\frac{15}{29}\right\}$$

```
% // N
```

$$\{x \to 1.06897,\ y \to -0.586207,\ \lambda \to -0.517241\}$$

```
Solve[gs[λ] == {x, 0, z}, {x, z, λ}] // Flatten
```

$$\left\{x \to -\frac{1}{4},\ z \to -\frac{17}{4},\ \lambda \to -\frac{5}{4}\right\}$$

```
% // N
```

$$\{x \to -0.25,\ z \to -4.25,\ \lambda \to -1.25\}$$

```
Solve[gs[λ] == {0, y, z}, {y, z, λ}] // Flatten
```

$$\left\{y \to -\frac{1}{9},\ z \to -\frac{31}{9},\ \lambda \to -\frac{10}{9}\right\}$$

```
% // N
```

$$\{y \to -0.111111,\ z \to -3.44444,\ \lambda \to -1.11111\}$$

---

# Uebung 3

## ■ a

```
r = Sqrt[1^2 + 2^2 + 3^2]
```

$$\sqrt{14}$$

```
α = ArcCos[1 / r]
```

$$\text{ArcCos}\left[\frac{1}{\sqrt{14}}\right]$$

```
N[α]
```

```
1.30025
```

```
β = ArcCos[2 / r]
```

$$\text{ArcCos}\left[\sqrt{\frac{2}{7}}\right]$$

```
N[β]
```

```
1.00685
```

```
ɣ = ArcCos[3 / r]
```

$$\text{ArcCos}\left[\frac{3}{\sqrt{14}}\right]$$

```
N[γ]
```

```
0.640522
```

### ■ b

```
x = 1; y = 2; z = 3;
```

```
aStrich = {x Cos[φ] - y Sin[φ], x Sin[φ] + y Cos[φ], z}
```

$$\{\text{Cos}[\varphi] - 2\,\text{Sin}[\varphi],\ 2\,\text{Cos}[\varphi] + \text{Sin}[\varphi],\ 3\}$$

```
φ = π / 7; aStrich
```

$$\left\{\text{Cos}\left[\frac{\pi}{7}\right] - 2\,\text{Sin}\left[\frac{\pi}{7}\right],\ 2\,\text{Cos}\left[\frac{\pi}{7}\right] + \text{Sin}\left[\frac{\pi}{7}\right],\ 3\right\}$$

```
% // N
```

```
{0.0332014, 2.23582, 3.}
```

## Uebung 4

### ■ a

```
Remove[x, y, z]
```

```
Ψ[λ_, μ_] := {0, 1, 1} + λ {-1, 2, 3} + μ {1, -1, 1}
```

```
s4 = Solve[Ψ[λ, μ] == {x, y, z}, {λ, μ, x}] // Flatten
```

$$\left\{\lambda \to \frac{1}{5}\,(-2 + y + z),\ \mu \to \frac{1}{5}\,(1 - 3\,y + 2\,z),\ x \to \frac{1}{5}\,(3 - 4\,y + z)\right\}$$

```
x1 = x /. s4
```

$$\frac{1}{5}\,(3 - 4\,y + z)$$

```
equat = (x - x1 == 0 // Expand)
```

$$x + \frac{1}{5}\,(-3 + 4\,y - z) == 0$$

```
(equat[[1]] * 5) // Expand
```

$$-3 + 5\,x + 4\,y - z$$

```
Remove[h]
```

```
h[{x_, y_, z_}] := (-3 + 5 x + 4 y - z) * 1 / Sqrt[(5)^2 + (4)^2 + (-1)^2] // Simplify
```

```
?h
```

```
Global`h
```

$$h[\{x\_,\ y\_,\ z\_\}] := \text{Simplify}\left[\frac{-3+5\,x+4\,y-z}{\sqrt{5^2+4^2+(-1)^2}}\right]$$

```
h[{x, y, z}]
```

$$\frac{-3+5x+4y-z}{\sqrt{42}}$$

```
h[{0, 1, 1}]
```

```
0
```

```
h[Ψ[λ, μ]] // Simplify
```

```
0
```

```
d = h[{15, 20, 18}]
```

$$67\sqrt{\frac{2}{21}}$$

```
d // N
```

```
20.6766
```

### ■ b

```
gH[t_] := {15, 20, 18} + t {5, 4, -1}
```

```
s4b = Solve[gH[t] == Ψ[λ, μ], {t, λ, μ}] // Flatten
```

$$\left\{t \to -\frac{67}{21},\ \lambda \to \frac{37}{7},\ \mu \to \frac{13}{3}\right\}$$

```
res4b = {gH[t], Ψ[λ, μ]} /. s4b
```

$$\left\{\left\{-\frac{20}{21},\ \frac{152}{21},\ \frac{445}{21}\right\},\ \left\{-\frac{20}{21},\ \frac{152}{21},\ \frac{445}{21}\right\}\right\}$$

```
res4b // N
```

```
{{-0.952381, 7.2381, 21.1905}, {-0.952381, 7.2381, 21.1905}}
```

```
Remove["Global`*"]
```

---

## Uebung 5

```
p = (7 - 4) (7 + 4)
```

```
33
```

```
solv1 = Solve[t^2 == 33, {t}] // Flatten
```

$$\left\{t \to -\sqrt{33},\ t \to \sqrt{33}\right\}$$

```
t1 = t /. solv1[[2]]
```

$$\sqrt{33}$$

```
N[t1]
```

5.74456

```
y = Sqrt[4^2 - 2^2]
```

$2\sqrt{3}$

```
a = Sqrt[y^2 + (2 + 7)^2]
```

$\sqrt{93}$

```
solv2 = Solve[p == a b, {b}] // Flatten
```

$\left\{b \to 11\sqrt{\frac{3}{31}}\right\}$

```
s = a - b /. solv2
```

$-11\sqrt{\frac{3}{31}} + \sqrt{93}$

```
N[%]
```

6.22171

## Uebung 6

```
Remove[x, y]
```

```
Solve[{3 x + 4 y == 7, 2 x - 3 y == 5}, {x, y}]
```

$\left\{\left\{x \to \frac{41}{17}, y \to -\frac{1}{17}\right\}\right\}$

```
D0 = Det[{{3, 4}, {2, -3}}]
```

$-17$

```
D1 = Det[{{4, -7}, {-3, -5}}]
```

$-41$

```
x = D1 / D0
```

$\frac{41}{17}$

```
D1 = Det[{{-7, 3}, {-5, 2}}]
```

1

```
y = D1 / D0
```

$-\frac{1}{17}$

## Uebung 7

```
a = {a1, a2, a3}; b = {b1, b2, b3}; Cross[a, b]
```

```
{-a3 b2 + a2 b3, a3 b1 - a1 b3, -a2 b1 + a1 b2}
```

```
a = {1, 2, 3}; b = {4, -1, 5}; n = Cross[a, b]
```

```
{13, 7, -9}
```

```
Len[n_] := Sqrt[n.n]
```

```
A = Len[n]
```

$$\sqrt{299}$$

```
A // N
```

```
17.2916
```

```
c = {-4, -3, -2};
```

```
h = c.n / Len[n]
```

$$-\frac{55}{\sqrt{299}}$$

```
v = A h
```

```
-55
```

```
Det[{a, b, c}]
```

```
-55
```

→ V = Det[.....] !!!!!!

## Uebung 8

```
Remove[x, y, z, a, b, n, xD, h]
```

```
a = {1, 2, 3}; b = {4, -1, 5}; n = Cross[a, b] / Norm[Cross[a, b]]
```

$$\left\{\sqrt{\frac{13}{23}}, \frac{7}{\sqrt{299}}, -\frac{9}{\sqrt{299}}\right\}$$

```
%//N
```

```
{0.751809, 0.40482, -0.520483}
```

```
n.{x, y, z} + dD == 0
```

$$dD + \sqrt{\frac{13}{23}}\, x + \frac{7\, y}{\sqrt{299}} - \frac{9\, z}{\sqrt{299}} == 0$$

```
solv = Solve[{n.{x, y, z} + dD == 0, x == -4, y == -3, z == -2}, {dD}] // Flatten
```

$$\left\{dD \to \frac{55}{\sqrt{299}}\right\}$$

```
%//N
```

$$\{dD \to 3.18073\}$$

```
dOrigo = dD /. solv
```

$$\frac{55}{\sqrt{299}}$$

```
h[{x_, y_, z_}] := n.{x, y, z} + dOrigo

h[{17, 18, 19}]
```

$$17\sqrt{\frac{13}{23}} + \frac{10}{\sqrt{299}}$$

```
%//N
```

```
13.3591
```

## Uebung 9

```
Remove[g, t, Θ, λ, μ]

g[t_] := {-2, 4, 1} + t {7, 2, 2};
Θ[λ_, μ_] := {8, 3, -1} + λ {-4, 5, 1} + μ {7, 2, 2};

Solve[{g[t] == Θ[λ, μ]}, {λ, μ}]
```

```
{}
```

V = a * h → h = V / A, V = Det[....], A = |Cross[....]|

```
d = Det[{{-4, 5, 1}, {7, 2, 2}, {8, 3, -1} - {2, 4, 1}}] / Len[Cross[{-4, 5, 1}, {7, 2, 2}]]
```

$$\frac{119}{\sqrt{2138}}$$

```
N[%]
```

```
2.57361
```