Lösungen

In[1]:=
```
Remove["Global`*"]
```

```
Remove::rmnsm : There are no symbols matching "Global`*". Mehr…
```

Problem 1

In[2]:=
```
vecLen[r_] := Sqrt[r.r]
```

In[3]:=
```
r01 = {2, 1, 3}; r02 = {-3, -1, 1}; a = {1, -2, 1}; b = {-2, 1, 4};
r = {x, y, z};
```

In[4]:=
```
rΦ[{λ_, μ_}] := r01 + 1 / 2 (r02 - r01) + λ a + μ b
```

In[5]:=
```
n = Cross[a, b]
```

Out[5]=
$$\{-9, -6, -3\}$$

In[6]:=
```
Φ[{x_, y_, z_}] := n.{x, y, z} / vecLen[n] + C1; Φ[r]
```

Out[6]=
$$C1 + \frac{-9\,x - 6\,y - 3\,z}{3\sqrt{14}}$$

In[7]:=
```
en = n / vecLen[n]
```

Out[7]=
$$\left\{-\frac{3}{\sqrt{14}}, -\sqrt{\frac{2}{7}}, -\frac{1}{\sqrt{14}}\right\}$$

In[8]:=
```
N[%]
```

Out[8]=
$$\{-0.801784, -0.534522, -0.267261\}$$

In[9]:=
```
solv1 = Solve[Φ[r01 + 1 / 2 (r02 - r01)] == 0, {C1}] // Flatten
```

Out[9]=
$$\left\{C1 \to \frac{1}{2\sqrt{14}}\right\}$$

In[10]:=
```
Φ[r] /. solv1
```

Out[10]=
$$\frac{1}{2\sqrt{14}} + \frac{-9x - 6y - 3z}{3\sqrt{14}}$$

In[11]:=
```
N[%] // Expand
```

Out[11]=
$$0.133631 - 0.801784\,x - 0.534522\,y - 0.267261\,z$$

Problem 2

In[12]:=
```
a = {5, 2, 1}; b = {1, 4, 8}; c = {-2, y, 4}
```

Out[12]=
$$\{-2, y, 4\}$$

In[13]:=
```
V = Det[{a, b, c}]
```

Out[13]=
$$48 - 39\,y$$

In[14]:=
```
Solve[V == 100, {y}] // Flatten
```

Out[14]=
$$\left\{y \to -\frac{4}{3}\right\}$$

In[15]:=
```
N[%]
```

Out[15]=
$$\{y \to -1.33333\}$$

Problem 3

In[16]:=
```
a = {2, 1, 3}; b = {-3, -1, 1}; c = {5, 2, 1};
```

In[17]:=
```
d1 = Cross[Cross[a, b], c]
```

Out[17]=
$$\{-13, 1, 63\}$$

In[18]:=
```
d2 = Cross[a, Cross[b, c]]
```
Out[18]=
```
{-25, -7, 19}
```

In[19]:=
```
Cross[d1, d2]
```
Out[19]=
```
{460, -1328, 116}
```

Problem 4

In[20]:=
```
d0 = Det[{{4, -2, 3}, {8, -2, 4}, {9, α, β}}]
```
Out[20]=
```
-18 + 8 α + 8 β
```

In[21]:=
```
Solve[{
   4 x - 2 y + 3 z - 5 == 0,
   8 x - 2 y + 4 z - 6 == 0,
   9 x + α y + β z + 4 == 0}, {x, y, z}] // Flatten
```
Out[21]=
$$\left\{x \to -\frac{-4+\alpha-\beta}{-9+4\alpha+4\beta},\ y \to -\frac{7+8\beta}{-9+4\alpha+4\beta},\ z \to -\frac{25-8\alpha}{-9+4\alpha+4\beta}\right\}$$

Repetitionsaufgaben

In[22]:=
```
Remove["Global`*"]
```

Problem 5

In[23]:=
```
solv = Solve[{a == e1 + e2 + e3, b == e1 - e2, c == e1 + 2 e2 + 4 e3},
   {e1, e2, e3}] // Flatten
```
Out[23]=
$$\left\{e1 \to \frac{1}{5}(4a+2b-c),\ e2 \to \frac{1}{5}(4a-3b-c),\ e3 \to \frac{1}{5}(-3a+b+2c)\right\}$$

In[24]:=
```
% // ExpandAll
```
Out[24]=
$$\left\{e1 \to \frac{4a}{5}+\frac{2b}{5}-\frac{c}{5},\ e2 \to \frac{4a}{5}-\frac{3b}{5}-\frac{c}{5},\ e3 \to -\frac{3a}{5}+\frac{b}{5}+\frac{2c}{5}\right\}$$

In[25]:=
```
% // N
```

Out[25]=
$$\{e1 \to 0.8\,a + 0.4\,b - 0.2\,c,\ e2 \to 0.8\,a - 0.6\,b - 0.2\,c,\ e3 \to -0.6\,a + 0.2\,b + 0.4\,c\}$$

Problem 6

In[26]:=
```
Remove["Global`*"]
```

In[27]:=
```
len[v_] := Sqrt[v.v];
```

In[28]:=
```
a = {0, 0}; b = {8, 0}; c = {5, 6};
ac = c - a; bc = c - b; abb = 2 / 3 ac;  cbb = ac - abb;  baa = 2 / 5 bc;
caa = bc - baa;
{λ = len[baa] / len[caa], μ = len[ cbb] / len[abb], ν = 1 / (λ μ)}
```

Out[30]=
$$\left\{\frac{2}{3},\ \frac{1}{2},\ 3\right\}$$

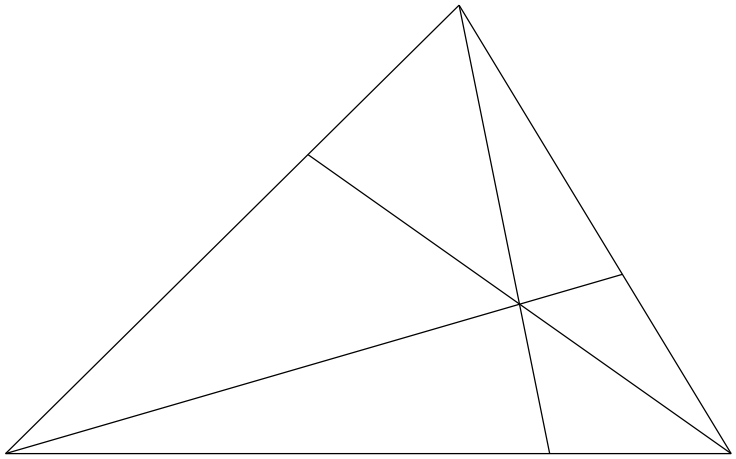
In[31]:=
```
s = Solve[ν == x / (8 - x), {x}] // Flatten
```

Out[31]=
$$\{x \to 6\}$$

In[32]:=
```
cc = {x, 0} /. s
```

Out[32]=
$$\{6,\ 0\}$$

In[33]:=
```
Show[Graphics[{Line[{a, b}], Line[{b, c}], Line[{c, a}],
    Line[{b, abb}], Line[{a, b + baa}], Line[{c, cc}]}]];
```



Problem 7

In[34]:=
```
Remove["Global`*"]
```

In[35]:=
```
a = 67.54; b = 59.18; γ = (98 + 12 / 60 + 14 / (60^2)) 2 Pi / 360
```

Out[35]=
$$\frac{176767\,\pi}{324000}$$

In[36]:=
```
N[%]
```

Out[36]=
```
1.71398
```

In[37]:=
```
(98 + 12 / 60 + 14 / (60^2) ) Degree // N
```

Out[37]=
```
1.71398
```

In[38]:=
```
c = Sqrt[a^2 + b^2 - 2 a b Cos[γ]]
```

Out[38]=
```
95.9408
```

In[39]:=
```
α = ArcSin[a Sin[γ] / c]
```

Out[39]=
```
0.770887
```

In[40]:=
```
α / Degree
```

Out[40]=
```
44.1686
```

Problem 8

In[41]:=
```
Remove["Global`*"]
```

---

a

In[42]:=
```
a = t; b = t; c = q t;
```

In[43]:=
```
oo = 6; aa = 10 - 6; bb = 16 - 10; cc = 25 - 16;
v1 = -aa / bb;
v2 = (aa + bb + cc) / (cc);
```

In[45]:=
```
{dv = v1 / v2, v1, v2}
```

Out[45]=
$$\left\{-\frac{6}{19}, -\frac{2}{3}, \frac{19}{9}\right\}$$

In[46]:=
```
{dv, v3 = -a / b, v4 = (a + b + c) / c, v3 / v4} // Simplify
```

Out[46]=
$$\left\{-\frac{6}{19}, -1, \frac{2+q}{q}, -\frac{q}{2+q}\right\}$$

In[47]:=
```
s = Solve[v3 / v4 == dv, {q}] // Flatten
```

Out[47]=
$$\left\{q \to \frac{12}{13}\right\}$$

In[48]:=
```
c / b /. s
```

Out[48]=
$$\frac{12}{13}$$

b

In[49]:=
```
b1 = a1; c1 = a1;
dv2 = (-a1 / b1) / ((a1 + b1 + c1) / (c1))
```

Out[50]=
$$-\frac{1}{3}$$

In[51]:=
```
dv == dv2
```

Out[51]=
```
False
```

Problem 9

a

In[52]:=
```
Remove["Global`*"]
```

In[53]:=
```
a = {1, 1}; b = {10, 4}; c = {5, 9};
ac = c - a; cb = b - c;
ba = a - b;
```

In[55]:=
```
len[v_] := Sqrt[v.v];
```

In[56]:=
```
s[u_, v_] := u + 1 / 2 v + {-v[[2]], v[[1]]} * 1 / 3 * Sqrt[3] / 2
```

In[57]:=
```
sb = s[a, ac]
```

Out[57]=
$$\left\{3-\frac{4}{\sqrt{3}},\ 5+\frac{2}{\sqrt{3}}\right\}$$

In[58]:=
```
N[%]
```

Out[58]=
$$\{0.690599, 6.1547\}$$

In[59]:=
```
sa = s[c, cb]
```

Out[59]=
$$\left\{\frac{15}{2} + \frac{5}{2\sqrt{3}}, \frac{13}{2} + \frac{5}{2\sqrt{3}}\right\}$$

In[60]:=
```
N[%]
```

Out[60]=
$$\{8.94338, 7.94338\}$$

In[61]:=
```
sc = s[b, ba]
```

Out[61]=
$$\left\{\frac{11}{2} + \frac{\sqrt{3}}{2}, \frac{5}{2} - \frac{3\sqrt{3}}{2}\right\}$$

In[62]:=
```
N[%]
```

Out[62]=
$$\{6.36603, -0.0980762\}$$

b

In[63]:=
```
len[sa - sb] // Simplify
```

Out[63]=
$$\sqrt{\frac{110}{3} + 20\sqrt{3}}$$

In[64]:=
```
N[%]
```

Out[64]=
$$8.44439$$

In[65]:=
```
len[sb - sc] // Simplify
```

Out[65]=
$$\sqrt{\frac{110}{3} + 20\sqrt{3}}$$

In[66]:=
```
N[%]
```

Out[66]=
```
8.44439
```

In[67]:=
```
len[sc - sa] // Simplify
```

Out[67]=
$$\sqrt{\frac{110}{3} + 20\sqrt{3}}$$

In[68]:=
```
N[%]
```

Out[68]=
```
8.44439
```

Arbeiten mit Matlab oder Octave sowie CAS

Lösung Aufgabe 1 mit MatLab

```
>> a1=[0 0 0 -1 1 0 0 0 0 (50+1)]
a1 =

         0         0         0        -1         1         0
0         0         0        51

>> a2=[0 -1 1 -1 0 0 0 0 0 20]
a2 =

         0        -1         1        -1         0         0
0         0         0        20

>> a3=[-1 1 0 0 0 0 0 0 0 (50-10)]
a3 =

        -1         1         0         0         0         0
0         0         0        40

>> a4=[0 0 0 0 1 1 0 0 0  50+56]
a4 =

        0         0         0         0         1         1         0
0         0      106

>> a5=[0 0 1 0 0 1 -1 -1 0 0]
a5 =

         0         0         1         0         0         1
-1        -1         0         0

>> a6=[-1 0 0 0 0 0 1 0 -1 100]
a6 =

        -1         0         0         0         0         0
1         0        -1       100

>> a7=[0 0 0 0 0 0 0 1 1 10+5]
a7 =

        0         0         0         0         0         0         0
1         1       15

>> m=[a1;a2;a3;a4;a5;a6;a7]
m =
```

```
          0          0          0         -1          1          0
0          0          0         51
          0         -1          1         -1          0          0
0          0          0         20
         -1          1          0          0          0          0
0          0          0         40
          0          0          0          0          1          1
0          0          0        106
          0          0          1          0          0          1
-1         -1          0          0
         -1          0          0          0          0          0
1          0         -1        100
          0          0          0          0          0          0
0          1          1         15

>> rref(m)
ans =

          1          0          0          0          0          0
-1         0          1       -100
          0          1          0          0          0          0
-1         0          1        -60
          0          0          1          0          0          1
-1         0          1         15
          0          0          0          1          0          1
-0        -0         -0         55
          0          0          0          0          1          1
0          0          0        106
          0          0          0          0          0          0
0          1          1         15
          0          0          0          0          0          0
0          0          0          0

Transponieren um besser  lesen zu können:

>> rref(m)'
ans =

          1          0          0          0          0          0          0
          0          1          0          0          0          0          0
          0          0          1          0          0          0          0
          0          0          0          1          0          0          0
          0          0          0          0          1          0          0
          0          0          1          1          1          0          0
         -1         -1         -1         -0          0          0          0
          0          0          0         -0          0          1          0
          1          1          1         -0          0          1          0
       -100        -60         15         55        106         15          0
```

Mit einem andern Programm

In[69]:=
```
a1={0, 0, 0, -1, 1, 0, 0, 0, 0};
a2={0, -1, 1, -1, 0, 0, 0, 0, 0};
a3={-1, 1, 0, 0, 0, 0, 0, 0, 0};
a4={0, 0, 0, 0, 1, 1, 0, 0, 0};
a5={0, 0, 1, 0, 0, 1, -1, -1, 0};
a6={-1, 0, 0, 0, 0, 0, 1, 0, -1};
a7={0, 0, 0, 0, 0, 0, 0, 1, 1};
m={a1,a2,a3,a4,a5,a6,a7};
u=RowReduce[m];
Print[u//MatrixForm];
b={51,20,40,106,0,100,15};
Print[RowReduce[Append[Transpose[m],b]]//MatrixForm];
x={x1,x2,x3,x4,x5,x6,x7,x8,x9};
Solve[m.x==b,x] //Flatten
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
Solve::svars :
 Equations may not give solutions for all "solve" variables. Mehr…
```

Out[82]=
$$\{x1 \to -100 + x7 - x9,\ x2 \to -60 + x7 - x9,\ x3 \to 15 - x6 + x7 - x9,\ x4 \to 55 - x6,\ x5 \to 106 - x6,\ x8 \to 15 - x9\}$$

x6, x7, x9 sind hier Parameter!

In[83]:=
```
Prepend[Transpose[m],b];
```

Nochmals

In[84]:=
```
Solve[m.x==b,{x1,x2,x3,x4,x5,x8}] //Flatten
```

Out[84]=
```
{x1 → -100 + x7 - x9, x2 → -60 + x7 - x9,
 x3 → 15 - x6 + x7 - x9, x4 → 55 - x6, x5 → 106 - x6, x8 → 15 - x9}
```

x6, x7, x9 sind hier Parameter!