Lösungen

0. Materialbereitstellung

```
Di1 = {{1,0},{0,-2}};
Dr21 = {{3,5},{0,1}};
Dr22 = {{4,0},{12,6}};
Map[MatrixForm,{Di1,Dr21,Dr22}]
```

$$\left\{\begin{pmatrix}1 & 0\\ 0 & -2\end{pmatrix}, \begin{pmatrix}3 & 5\\ 0 & 1\end{pmatrix}, \begin{pmatrix}4 & 0\\ 12 & 6\end{pmatrix}\right\}$$

```
A2 = {{1,2},{1,-1}};
B2 = {{3,5},{4,7}};
C2 = {{4,1},{5,6}};
Map[MatrixForm,{A2,B2,C2}]
```

$$\left\{\begin{pmatrix}1 & 2\\ 1 & -1\end{pmatrix}, \begin{pmatrix}3 & 5\\ 4 & 7\end{pmatrix}, \begin{pmatrix}4 & 1\\ 5 & 6\end{pmatrix}\right\}$$

```
A3 = {{1,2,3},{1,-1,0},{-2,1,5}};
B3 = {{3,5,1},{4,7,9},{3,2,6}};
C3 = {{4,1,3},{5,6,5},{5,8,8}};
Map[MatrixForm,{A3,B3,C3}]
```

$$\left\{\begin{pmatrix}1 & 2 & 3\\ 1 & -1 & 0\\ -2 & 1 & 5\end{pmatrix}, \begin{pmatrix}3 & 5 & 1\\ 4 & 7 & 9\\ 3 & 2 & 6\end{pmatrix}, \begin{pmatrix}4 & 1 & 3\\ 5 & 6 & 5\\ 5 & 8 & 8\end{pmatrix}\right\}$$

```
A4 = {{1,2,3,4},{1,-1,0,1},{-2,1,5,2},{-2,2,1,5}};
B4 = {{3,5,1,1},{4,7,9,5},{3,2,6,8},{5,6,5,1}};
Map[MatrixForm,{A4,B4}]
```

$$\left\{\begin{pmatrix}1 & 2 & 3 & 4\\ 1 & -1 & 0 & 1\\ -2 & 1 & 5 & 2\\ -2 & 2 & 1 & 5\end{pmatrix}, \begin{pmatrix}3 & 5 & 1 & 1\\ 4 & 7 & 9 & 5\\ 3 & 2 & 6 & 8\\ 5 & 6 & 5 & 1\end{pmatrix}\right\}$$

```
C4 = {{3,5,1,1},{3,5,2,1},{3,4,1,1},{4,7,9,5}};
D3 = {{1,2,2},{1,-1,-1},{-2,1,1}};
Map[MatrixForm,{C4,D3}]
```

$$\left\{\begin{pmatrix}3 & 5 & 1 & 1\\ 3 & 5 & 2 & 1\\ 3 & 4 & 1 & 1\\ 4 & 7 & 9 & 5\end{pmatrix}, \begin{pmatrix}1 & 2 & 2\\ 1 & -1 & -1\\ -2 & 1 & 1\end{pmatrix}\right\}$$

```
VdM4 = {{1,2,3,4},{2,3,4,5},{-3,-2,-1,0},{3,4,5,6}};
NVdM5 = {{1,2,3,4,5},{2,3,4,5,6},{-3,-2,-1,0,1},{3,4,5,6,7},{3,3,3,3,3}};
Map[MatrixForm,{VdM4,NVdM5}]
```

$$\left\{\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ -3 & -2 & -1 & 0 \\ 3 & 4 & 5 & 6 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ -3 & -2 & -1 & 0 & 1 \\ 3 & 4 & 5 & 6 & 7 \\ 3 & 3 & 3 & 3 & 3 \end{pmatrix}\right\}$$

Programme für unten:

```
detOut[M_]:=Print[Det[ToExpression[M]], " = ", "det(", M, ");"];
invOut[M_]:=Print[MatrixForm[Inverse[ToExpression[M]]], " = ", "Inverse(", M,
");"];
invNOut[M_]:=Print[MatrixForm[N[Inverse[ToExpression[M]]]], " = ", "Inverse(", M,
");"];
detInvOut[M_]:=Print[Det[Inverse[ToExpression[M]]] Det[ToExpression[M]], " = ",
"det(", "Inv(", M, "))", ";" ];
detInvKontrOut[M_]:=Print[Det[Inverse[ToExpression[M]]] Det[ToExpression[M]], " =
", "det(", "Inv(", M, "))", "*",  "det(", M, ")", ";" ];
```

---

1. Determinanten von A4, B4, C4

Selbststudium nach Uebungsblatt

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2. Determinanten von A4, B4, C4

```
Map[detOut,{"A4","B4","C4"}];
```

```
-111 = det(A4);
```

```
-430 = det(B4);
```

```
11 = det(C4);
```

3. Inverse von Di2, Dr21, Dr22, A2, B2, C2, A3, B3, C3, A4, B4, C4

```
Map[detOut,{"Di1", "Dr21", "Dr22", "A2", "B2", "C2", "A3", "B3", "C3", "A4", "B4",
"C4"}];
```

```
-2 = det(Di1);

3 = det(Dr21);

24 = det(Dr22);

-3 = det(A2);

1 = det(B2);

19 = det(C2);

-18 = det(A3);

74 = det(B3);

47 = det(C3);

-111 = det(A4);

-430 = det(B4);

11 = det(C4);
```

Map[invOut,{"Di1", "Dr21", "Dr22", "A2", "B2", "C2", "A3", "B3", "C3", "A4", "B4", "C4"}];

$$\begin{pmatrix} 1 & 0 \\ 0 & -\frac{1}{2} \end{pmatrix} = \text{Inverse}(\text{Di1});$$

$$\begin{pmatrix} \frac{1}{3} & -\frac{5}{3} \\ 0 & 1 \end{pmatrix} = \text{Inverse}(\text{Dr21});$$

$$\begin{pmatrix} \frac{1}{4} & 0 \\ -\frac{1}{2} & \frac{1}{6} \end{pmatrix} = \text{Inverse}(\text{Dr22});$$

$$\begin{pmatrix} \frac{1}{3} & \frac{2}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{pmatrix} = \text{Inverse}(\text{A2});$$

$$\begin{pmatrix} 7 & -5 \\ -4 & 3 \end{pmatrix} = \text{Inverse}(\text{B2});$$

$$\begin{pmatrix} \frac{6}{19} & -\frac{1}{19} \\ -\frac{5}{19} & \frac{4}{19} \end{pmatrix} = \text{Inverse}(\text{C2});$$

$$\begin{pmatrix} \frac{5}{18} & \frac{7}{18} & -\frac{1}{6} \\ \frac{5}{18} & -\frac{11}{18} & -\frac{1}{6} \\ \frac{1}{18} & \frac{5}{18} & \frac{1}{6} \end{pmatrix} = \text{Inverse}(\text{A3});$$

$$\begin{pmatrix} \frac{12}{37} & -\frac{14}{37} & \frac{19}{37} \\ \frac{3}{74} & \frac{15}{74} & -\frac{23}{74} \\ -\frac{13}{74} & \frac{9}{74} & \frac{1}{74} \end{pmatrix} = \text{Inverse}(\text{B3});$$

$$\begin{pmatrix} \frac{8}{47} & \frac{16}{47} & -\frac{13}{47} \\ -\frac{15}{47} & \frac{17}{47} & -\frac{5}{47} \\ \frac{10}{47} & -\frac{27}{47} & \frac{19}{47} \end{pmatrix} = \text{Inverse}(\text{C3});$$

$$\begin{pmatrix} \frac{32}{111} & \frac{7}{111} & -\frac{5}{37} & -\frac{7}{37} \\ \frac{31}{111} & -\frac{73}{111} & -\frac{6}{37} & -\frac{1}{37} \\ \frac{7}{111} & \frac{5}{111} & \frac{7}{37} & -\frac{5}{37} \\ -\frac{1}{111} & \frac{31}{111} & -\frac{1}{37} & \frac{6}{37} \end{pmatrix} = \text{Inverse}(\text{A4});$$

$$\begin{pmatrix} -\frac{23}{215} & -\frac{15}{43} & \frac{38}{215} & \frac{94}{215} \\ \frac{61}{215} & \frac{8}{43} & -\frac{26}{215} & -\frac{53}{215} \\ -\frac{23}{86} & \frac{11}{86} & -\frac{5}{86} & \frac{4}{43} \\ \frac{73}{430} & -\frac{1}{86} & \frac{57}{430} & -\frac{37}{215} \end{pmatrix} = \text{Inverse}(\text{B4});$$

$$\begin{pmatrix} -\frac{17}{11} & \frac{4}{11} & \frac{18}{11} & -\frac{1}{11} \\ 1 & 0 & -1 & 0 \\ -1 & 1 & 0 & 0 \\ \frac{18}{11} & -\frac{23}{11} & \frac{1}{11} & \frac{3}{11} \end{pmatrix} = \text{Inverse}(\text{C4});$$

```
Map[invNOut,{"Di1", "Dr21", "Dr22", "A2", "B2", "C2", "A3", "B3", "C3", "A4",
"B4", "C4"}];
```

$$\begin{pmatrix} 1. & 0. \\ 0. & -0.5 \end{pmatrix} = \text{Inverse}(\text{Di1});$$

$$\begin{pmatrix} 0.333333 & -1.66667 \\ 0. & 1. \end{pmatrix} = \text{Inverse}(\text{Dr21});$$

$$\begin{pmatrix} 0.25 & 0. \\ -0.5 & 0.166667 \end{pmatrix} = \text{Inverse}(\text{Dr22});$$

$$\begin{pmatrix} 0.333333 & 0.666667 \\ 0.333333 & -0.333333 \end{pmatrix} = \text{Inverse}(\text{A2});$$

$$\begin{pmatrix} 7. & -5. \\ -4. & 3. \end{pmatrix} = \text{Inverse}(\text{B2});$$

$$\begin{pmatrix} 0.315789 & -0.0526316 \\ -0.263158 & 0.210526 \end{pmatrix} = \text{Inverse}(\text{C2});$$

$$\begin{pmatrix} 0.277778 & 0.388889 & -0.166667 \\ 0.277778 & -0.611111 & -0.166667 \\ 0.0555556 & 0.277778 & 0.166667 \end{pmatrix} = \text{Inverse}(\text{A3});$$

$$\begin{pmatrix} 0.324324 & -0.378378 & 0.513514 \\ 0.0405405 & 0.202703 & -0.310811 \\ -0.175676 & 0.121622 & 0.0135135 \end{pmatrix} = \text{Inverse}(\text{B3});$$

$$\begin{pmatrix} 0.170213 & 0.340426 & -0.276596 \\ -0.319149 & 0.361702 & -0.106383 \\ 0.212766 & -0.574468 & 0.404255 \end{pmatrix} = \text{Inverse}(\text{C3});$$

$$\begin{pmatrix} 0.288288 & 0.0630631 & -0.135135 & -0.189189 \\ 0.279279 & -0.657658 & -0.162162 & -0.027027 \\ 0.0630631 & 0.045045 & 0.189189 & -0.135135 \\ -0.00900901 & 0.279279 & -0.027027 & 0.162162 \end{pmatrix} = \text{Inverse}(\text{A4});$$

$$\begin{pmatrix} -0.106977 & -0.348837 & 0.176744 & 0.437209 \\ 0.283721 & 0.186047 & -0.12093 & -0.246512 \\ -0.267442 & 0.127907 & -0.0581395 & 0.0930233 \\ 0.169767 & -0.0116279 & 0.132558 & -0.172093 \end{pmatrix} = \text{Inverse}(\text{B4});$$

$$\begin{pmatrix} -1.54545 & 0.363636 & 1.63636 & -0.0909091 \\ 1. & 0. & -1. & 0. \\ -1. & 1. & 0. & 0. \\ 1.63636 & -2.09091 & 0.0909091 & 0.272727 \end{pmatrix} = \text{Inverse}(\text{C4});$$

```
Map[detInvKontrOut,{"Di1", "Dr21", "Dr22", "A2", "B2", "C2", "A3", "B3", "C3",
"A4", "B4", "C4"}];
```

```
1 = det(Inv(Di1))*det(Di1);

1 = det(Inv(Dr21))*det(Dr21);

1 = det(Inv(Dr22))*det(Dr22);

1 = det(Inv(A2))*det(A2);

1 = det(Inv(B2))*det(B2);

1 = det(Inv(C2))*det(C2);

1 = det(Inv(A3))*det(A3);

1 = det(Inv(B3))*det(B3);

1 = det(Inv(C3))*det(C3);

1 = det(Inv(A4))*det(A4);

1 = det(Inv(B4))*det(B4);

1 = det(Inv(C4))*det(C4);
```

4. Inverse von D3, VdM4, NVdM5

```
Map[detOut,{"D3", "VdM4", "NVdM5"}];
```

```
0 = det(D3);

0 = det(VdM4);

0 = det(NVdM5);
```

```
Map[invOut,{"D3", "VdM4", "NVdM5"}];
```

```
Inverse::sing : Matrix {{1, 2, 2}, {1, -1, -1}, {-2, 1, 1}} is singular. Mehr…

Inverse[{{1, 2, 2}, {1, -1, -1}, {-2, 1, 1}}] = Inverse(D3);

Inverse::sing : Matrix {{1, 2, 3, 4}, {2, 3, 4, 5}, {-3, -2, -1, 0}, {3, 4, 5, 6}} is singular. Mehr…

Inverse[{{1, 2, 3, 4}, {2, 3, 4, 5}, {-3, -2, -1, 0}, {3, 4, 5, 6}}] = Inverse(VdM4);

Inverse::sing : Matrix
   {{1, 2, 3, 4, 5}, {2, 3, 4, 5, 6}, {-3, -2, -1, 0, 1}, {3, 4, 5, 6, 7}, {3, 3, 3, 3, 3}} is singular. Mehr…

General::stop : Further output of Inverse::sing will be suppressed during this calculation. Mehr…

Inverse[{{1, 2, 3, 4, 5}, {2, 3, 4, 5, 6}, {-3, -2, -1, 0, 1}, {3, 4, 5, 6, 7}, {3, 3, 3, 3, 3}}]
  = Inverse(NVdM5);
```

Keine der Inversen existiert!

5. Matlabanwendung

Arbeit mit MATLAB - führt zu obigen Resultaten.

6. Jacobi

```
A={{1,0.1,0,0,0,0},{0.2,1,0.1,0,0,0},{0,0.2,1,0.1,0,0},{0,0,0.2,1,0.1,0},{0,0,0,0.2
,1,0.1},{0,0,0,0,0.2,1}};
A//MatrixForm
```

$$\begin{pmatrix} 1 & 0.1 & 0 & 0 & 0 & 0 \\ 0.2 & 1 & 0.1 & 0 & 0 & 0 \\ 0 & 0.2 & 1 & 0.1 & 0 & 0 \\ 0 & 0 & 0.2 & 1 & 0.1 & 0 \\ 0 & 0 & 0 & 0.2 & 1 & 0.1 \\ 0 & 0 & 0 & 0 & 0.2 & 1 \end{pmatrix}$$

```
X[1]={{1,0.1,0.1,0.1,0.1,0.1},{0.2,1,0.1,0.1,0.1,0.1},{0.1,0.2,1,0.1,0.1,0.1},{0.1,
0.1,0.2,1,0.1,0.1},{0.1,0.1,0.1,0.2,1,0.1},{0.1,0.1,0.1,0.1,0.2,1}};
X[1]//MatrixForm
```

$$\begin{pmatrix} 1 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.2 & 1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.2 & 1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.2 & 1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.2 & 1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 0.1 & 0.2 & 1 \end{pmatrix}$$

```
X[n_]:=IdentityMatrix[6]-(A-IdentityMatrix[6]).X[n-1]
```

```
X[2]//MatrixForm
```

$$\begin{pmatrix} 0.98 & -0.1 & -0.01 & -0.01 & -0.01 & -0.01 \\ -0.21 & 0.96 & -0.12 & -0.03 & -0.03 & -0.03 \\ -0.05 & -0.21 & 0.96 & -0.12 & -0.03 & -0.03 \\ -0.03 & -0.05 & -0.21 & 0.96 & -0.12 & -0.03 \\ -0.03 & -0.03 & -0.05 & -0.21 & 0.96 & -0.12 \\ -0.02 & -0.02 & -0.02 & -0.04 & -0.2 & 0.98 \end{pmatrix}$$

```
X[3]//MatrixForm
```

$$\begin{pmatrix} 1.021 & -0.096 & 0.012 & 0.003 & 0.003 & 0.003 \\ -0.191 & 1.041 & -0.094 & 0.014 & 0.005 & 0.005 \\ 0.045 & -0.187 & 1.045 & -0.09 & 0.018 & 0.009 \\ 0.013 & 0.045 & -0.187 & 1.045 & -0.09 & 0.018 \\ 0.008 & 0.012 & 0.044 & -0.188 & 1.044 & -0.092 \\ 0.006 & 0.006 & 0.01 & 0.042 & -0.192 & 1.024 \end{pmatrix}$$

```
X[4]//MatrixForm
```

$$\begin{pmatrix} 1.0191 & -0.1041 & 0.0094 & -0.0014 & -0.0005 & -0.0005 \\ -0.2087 & 1.0379 & -0.1069 & 0.0084 & -0.0024 & -0.0015 \\ 0.0369 & -0.2127 & 1.0375 & -0.1073 & 0.008 & -0.0028 \\ -0.0098 & 0.0362 & -0.2134 & 1.0368 & -0.108 & 0.0074 \\ -0.0032 & -0.0096 & 0.0364 & -0.2132 & 1.0372 & -0.106 \\ -0.0016 & -0.0024 & -0.0088 & 0.0376 & -0.2088 & 1.0184 \end{pmatrix}$$

X[5]//MatrixForm

$$\begin{pmatrix} 1.02087 & -0.10379 & 0.01069 & -0.00084 & 0.00024 & 0.00015 \\ -0.20751 & 1.04209 & -0.10563 & 0.01101 & -0.0007 & 0.00038 \\ 0.04272 & -0.2112 & 1.04272 & -0.10536 & 0.01128 & -0.00044 \\ -0.00706 & 0.0435 & -0.21114 & 1.04278 & -0.10532 & 0.01116 \\ 0.00212 & -0.007 & 0.04356 & -0.21112 & 1.04248 & -0.10332 \\ 0.00064 & 0.00192 & -0.00728 & 0.04264 & -0.20744 & 1.0212 \end{pmatrix}$$

X[6]//MatrixForm

$$\begin{pmatrix} 1.02075 & -0.104209 & 0.010563 & -0.001101 & 0.00007 & -0.000038 \\ -0.208446 & 1.04188 & -0.10641 & 0.010704 & -0.001176 & 0.000014 \\ 0.042208 & -0.212768 & 1.04224 & -0.10648 & 0.010672 & -0.001192 \\ -0.008756 & 0.04294 & -0.2129 & 1.04218 & -0.106504 & 0.01042 \\ 0.001348 & -0.008892 & 0.042956 & -0.21282 & 1.04181 & -0.104352 \\ -0.000424 & 0.0014 & -0.008712 & 0.042224 & -0.208496 & 1.02066 \end{pmatrix}$$

X[7]//MatrixForm

$$\begin{pmatrix} 1.02084 & -0.104188 & 0.010641 & -0.0010704 & 0.0001176 & -1.4 \times 10^{-6} \\ -0.208371 & 1.04212 & -0.106337 & 0.0108682 & -0.0010812 & 0.0001268 \\ 0.0425648 & -0.21267 & 1.04257 & -0.106359 & 0.0108856 & -0.0010448 \\ -0.0085764 & 0.0434428 & -0.212744 & 1.04258 & -0.106315 & 0.0106736 \\ 0.0017936 & -0.008728 & 0.0434512 & -0.212659 & 1.04215 & -0.10415 \\ -0.0002696 & 0.0017784 & -0.0085912 & 0.042564 & -0.208362 & 1.02087 \end{pmatrix}$$

Inverse[A]//MatrixForm

$$\begin{pmatrix} 1.02084 & -0.104212 & 0.0106384 & -0.001086 & 0.000110817 & -0.0000110817 \\ -0.208424 & 1.04212 & -0.106384 & 0.01086 & -0.00110817 & 0.000110817 \\ 0.0425536 & -0.212768 & 1.04256 & -0.106428 & 0.01086 & -0.001086 \\ -0.00868802 & 0.0434401 & -0.212856 & 1.04256 & -0.106384 & 0.0106384 \\ 0.00177307 & -0.00886533 & 0.0434401 & -0.212768 & 1.04212 & -0.104212 \\ -0.000354613 & 0.00177307 & -0.00868802 & 0.0425536 & -0.208424 & 1.02084 \end{pmatrix}$$

Fehler nach 7 Schritten (Beachte die nachstehende Differenz):

Round[(Inverse[A]-X[7]) 100000] / 100000.//MatrixForm

$$\begin{pmatrix} 0 & -0.00002 & 0 & -0.00002 & -0.00001 & -0.00001 \\ -0.00005 & 0 & -0.00005 & -0.00001 & -0.00003 & -0.00002 \\ -0.00001 & -0.0001 & -0.00001 & -0.00007 & -0.00003 & -0.00004 \\ -0.00011 & 0 & -0.00011 & -0.00002 & -0.00007 & -0.00004 \\ -0.00002 & -0.00014 & -0.00001 & -0.00011 & -0.00003 & -0.00006 \\ -0.00009 & -0.00001 & -0.0001 & -0.00001 & -0.00006 & -0.00003 \end{pmatrix}$$