Lösungen E+M1 S2 AnUe 1

L=Lektion/Leçon-- Sprechstunde/heures de consultation

W=Work/Arbeit/Travail

A = Anhang/ Annexe / Appendice

Ueb/Ex 1 W

```
Integrate[x^2, {x, 0, 1}] \frac{1}{3}
(* Riemannsche Summe *)
Sum[1/n (k/n)^2, {k,1,n}] \frac{(1+n) (1+2n)}{6 n^2}
%//Apart \frac{1}{3} + \frac{1}{6 n^2} + \frac{1}{2 n}
A = Limit[Sum[1/n (k/n)^2, {k,1,n}], n->Infinity] \frac{1}{3}
```

Ueb/Ex 2 W

а

```
Remove["Global`*"]

f[x_]:=E^(-x^2)

Integrate[f[x], x]

\[ \frac{1}{2} \sqrt{\pi} \text{Erf}[x]

Integrate[f[x],{x,-2,2}]

\[ \sqrt{\pi} \text{Erf}[2]

N[%]

1.76416
```

```
NIntegrate[f[x], \{x,-2,2\}]
1.76416
A = Limit[2 Sum[2/n f[2 k/n], \{k,1,n\}], n->Infinity]
\text{Limit} \Big[ 2 \sum_{k=1}^n \frac{2 e^{-\frac{4 k^2}{n^2}}}{n} \text{, } n \to \infty \Big]
n = 10^2;
2 Sum[N[2/n f[2 k/n]],{k,1,n}]
1.74452
n = 10^3;
2 Sum[N[2/n f[2 k/n]], \{k,1,n\}]
1.7622
n = 10^4;
2 Sum[N[2/n f[2 k/n]], \{k,1,n\}]
1.76397
n = 10^5;
2 Sum[N[2/n f[2 k/n]],{k,1,n}]
1.76414
n = 10^5;
2 Sum[N[2/n f[2 k/n]], \{k,1,n\}]
1.76414
n = 4 10^5;
```

2 $Sum[N[2/n f[2 k/n]], \{k,1,n\}]$

b

```
Series[f[x],{x,0,100}]
```

```
1-x^2+\frac{x^4}{2}-\frac{x^6}{6}+\frac{x^8}{24}-\frac{x^{10}}{120}+\frac{x^{12}}{720}-\frac{x^{14}}{5040}+\frac{x^{16}}{40320}-\frac{x^{18}}{362880}+
            x^{22}
                      x^{24}
                                 x^{26}
                                                x^{28}
           39916800 + 479001600 - 6227020800 + 87178291200 -
 3628800
      x^{30}
                      x^{32}
                                  x^{34}
 1307674368000 + 20922789888000 - 355687428096000 + 6402373705728000 -
        x^{38}
                       x^{40} x^{42}
 121645100408832000 + 2432902008176640000 - 51090942171709440000 +
          x^{44}
                                     x^{46}
 1124000727777607680000 25852016738884976640000 + 620448401733239439360000
                                           x^{52}
 15511210043330985984000000 + 403291461126605635584000000
 10888869450418352160768000000 + 304888344611713860501504000000
               x^{58}
 8841761993739701954543616000000 + 265252859812191058636308480000000
 8222838654177922817725562880000000 + 263130836933693530167218012160000000
 8683317618811886495518194401280000000 + 295232799039604140847618609643520000000
                    x^{70}
 10333147966386144929666651337523200000000 +
                     x^{72}
 371993326789901217467999448150835200000000
                      x^{74}
 13763753091226345046315979581580902400000000 +
                      x^{76}
 523022617466601111760007224100074291200000000
                        x^{78}
 20397882081197443358640281739902897356800000000 +
                        x^{80}
 815915283247897734345611269596115894272000000000
                         x^{82}
 33452526613163807108170062053440751665152000000000 +
                          x^{84}
 1405006117752879898543142606244511569936384000000000
                           x<sup>86</sup>
 60415263063373835637355132068513997507264512000000000 +
 2658271574788448768043625811014615890319638528000000000
                             x^{90}
 119622220865480194561963161495657715064383733760000000000 +
 5502622159812088949850305428800254892961651752960000000000
 258623241511168180642964355153611979969197632389120000000000 +
 12413915592536072670862289047373375038521486354677760000000000
 \frac{1}{30414093201713378043612608166064768844377641568960512000000000000} + O[x]^{101}
```

$Normal[Series[f[x],{x,0,100}]]$

```
\frac{5}{5} + \frac{x^8}{24} - \frac{x^{10}}{120} + \frac{x^{12}}{720} - \frac{x^{14}}{5040} + \frac{x^{16}}{40320} - \frac{x^{18}}{362880} + \frac{x^{10}}{362880} + \frac{x^{10}}{
                                              x^{24}
                                                                       x^{26}
3628800 - 39916800 + 479001600 - 6227020800 + 87178291200
                                   ____ x<sup>32</sup>
        _ x<sup>30</sup>
                                                                         x^{34}
1307674368000 + 20922789888000 - 355687428096000 + 6402373705728000
             x^{38}
                                              x^{40} x^{42}
121645100408832000 + 2432902008176640000 - 51090942171709440000 +
                                                                              x^{46}
                                                                                                                                          x^{48}
                   x^{44}
1124000727777607680000 - 25852016738884976640000 + 620448401733239439360000
                                                                                           x^{52}
                        x^{50}
15511210043330985984000000 + 403291461126605635584000000
                             x^{54}
                                                                                                      x^{56}
10888869450418352160768000000 + 304888344611713860501504000000 -
                               x^{58}
                                                                                                              x^{60}
8841761993739701954543616000000 + 265252859812191058636308480000000
                                  x^{62}
                                                                                                                        x<sup>64</sup>
8222838654177922817725562880000000 + 263130836933693530167218012160000000 -
                                     x^{66}
                                                                                                                                  x^{68}
8683317618811886495518194401280000000 + 295232799039604140847618609643520000000 -
10333147966386144929666651337523200000000 +
                                           x^{72}
371993326789901217467999448150835200000000
                                             x^{74}
13763753091226345046315979581580902400000000 +
                                              x^{76}
523022617466601111760007224100074291200000000
                                                 x^{78}
20397882081197443358640281739902897356800000000 +
815915283247897734345611269596115894272000000000
33452526613163807108170062053440751665152000000000 +
1405006117752879898543142606244511569936384000000000
2658271574788448768043625811014615890319638528000000000
                                                           x^{90}
5502622159812088949850305428800254892961651752960000000000
258623241511168180642964355153611979969197632389120000000000 *
x^{98}
x^{100}
```

Integrate[Normal[Series[$f[x], \{x,0,10\}$]], $\{x,-2.,2.\}$]

```
Integrate[Normal[Series[f[x],{x,0,20}]],{x,-2.,2.}]
1.77809
Integrate[Normal[Series[f[x],{x,0,33}]],{x,-2.,2.}]
1.76417
Integrate[Normal[Series[f[x],{x,0,35}]],{x,-2.,2.}]
1.76416
```

Ueb/Ex 3 W

а

Integrate[
$$\mathbf{x}^{100} - \mathbf{x}^{99}, \mathbf{x}$$
] + c c - $\frac{\mathbf{x}^{100}}{100}$ + $\frac{\mathbf{x}^{101}}{101}$

b

$$\begin{aligned} & \sin[x]^2 + \cos[x]^2 \ / \ \text{Simplify} \\ & 1 \\ & \text{Integrate}[E^* - x - \cos[x] + 1 - 1/x^2, x] + c \\ & c - e^{-x} + \frac{1}{x} + x - \sin[x] \end{aligned}$$

C

d

Integrate[
$$x^{(1/3)}$$
, x] + c

$$c + \frac{3 x^{4/3}}{4}$$

е

Apart[1/(x^2-1)]
$$\frac{1}{2(-1+x)} - \frac{1}{2(1+x)}$$

Integrate[1/(x^2-1), x] + c

$$c + \frac{1}{2} Log[-1+x] - \frac{1}{2} Log[1+x]$$
1/2 (Integrate[1/(x-1),x] + Integrate[1/(x+1),x])+c

$$c + \frac{1}{2} (Log[-1+x] + Log[1+x])$$

Ueb/Ex 4 W

Integrate[$x^2 - x^3$, {x, 0, 1}]

12

N[%]

0.0833333

Ueb/Ex 5 W

а

Integrate[$x Sin[x^2], x] + c$

$$c - \frac{\cos[x^2]}{2}$$

Integrate[x $Sin[x^2], \{x,1,2\}$]

$$\frac{1}{2}$$
 (Cos[1] - Cos[4])

N[%]

0.596973

b

Integrate[Cosh[x],x]+c

c + Sinh[x]

 ${\tt Integrate[Cosh[x],\{x,-1,1\}]}$

2 Sinh[1]

N[%]

C

Integrate[$Cos[\omega x + \phi], \{x,1,2\}]//TrigReduce$

$$\frac{-\sin[\varphi+\omega]+\sin[\varphi+2\,\omega]}{\omega}$$

g

Integrate[$E^{(4x-3)} - 2 \log[4x+3] / (x+3/4), x] + c$

$$c + \frac{1}{4} e^{-3+4x} - Log[3+4x]^2$$

Integrate[$E^{(4x-3)-2} Log[4x+3]/(x+3/4), \{x,1,4\}$]

$$\frac{1}{4} \in (-1 + e^{12}) + \text{Log}[7]^2 - \text{Log}[19]^2$$

N[%]

110598.

h

Integrate $[1/(2x-3)-10x^20+x^40,x]+c$

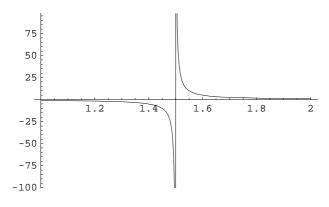
$$c - \frac{10 x^{21}}{21} + \frac{x^{41}}{41} + \frac{1}{2} \text{Log}[-3 + 2 x]$$

Integrate[$1/(2x-3)-10 x^20+x^40, \{x,1,2\}$]

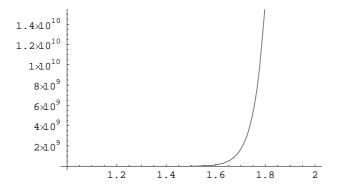
Integrate::idiv: Integral of $-10 \, x^{20} + x^{40} + \frac{1}{-3 + 2 \, x}$ does not converge on $\{1, \, 2\}$. Mehr...

$$\int_{1}^{2} \left(-10 \ x^{20} + x^{40} + \frac{1}{-3 + 2 \ x} \right) \ dx$$

 $Plot[1/(2x-3), \{x,1,2\}];$



$Plot[1/(2x-3)-10 x^20+x^40,{x,1,2}]$



- Graphics -

Integrate[$1/(2x-3)-10 x^20+x^40, \{x,1,3/2-a\}$] +Integrate[$1/(2x-3)-10 x^20+x^40, \{x,3/2+a,2\}$]

$$\begin{split} &\left(\frac{1}{2}-a\right) \\ &\text{If}\left[\text{Re}\left[a\right] \geq 0 \mid \mid \text{Im}\left[a\right] \neq 0\,,\, -\frac{778-861\,\,\mathrm{i}\,\,\pi}{861\,\left(-1+2\,a\right)} + \frac{\frac{21\,\left(3-2\,a\right)^{41}}{1099511627776} + \frac{205\,\left(-3+2\,a\right)^{21}}{524288} + 861\,\text{Log}\left[-2\,a\right]}{861\,\left(1-2\,a\right)} \\ &\text{Integrate}\left[-10\,\left(1+\left(\frac{1}{2}-a\right)\,x\right)^{20} + \left(1+\left(\frac{1}{2}-a\right)\,x\right)^{40} + \frac{1}{-3+2\,\left(1+\left(\frac{1}{2}-a\right)\,x\right)}\,,\, \\ &\left\{x,\,0,\,1\right\}\,,\, \text{Assumptions} \rightarrow !\,\,\left(\text{Re}\left[a\right] \geq 0\mid \mid \text{Im}\left[a\right] \neq 0\right)\,\right] \end{split}$$

$$\begin{split} &\left(\frac{1}{2}-a\right)\,\text{If}\left[\text{Re}\left[\frac{a}{1-2\,a}\right] \geq 0\mid\mid \text{Re}\left[\frac{a}{1-2\,a}\right] \leq -\frac{1}{2}\mid\mid \text{Im}\left[\frac{a}{1-2\,a}\right] \neq 0\,,\\ &-\frac{92357257068544}{861\,\left(-1+2\,a\right)} - \frac{5\,\left(3+2\,a\right)^{21}}{11010048\,\left(-1+2\,a\right)} - \frac{\left(3+2\,a\right)^{41}}{45079976738816\,\left(1-2\,a\right)} - \frac{\text{Log}\left[2\right]}{1-2\,a} - \frac{\text{Log}\left[a\right]}{1-2\,a}\,,\\ &\text{Integrate}\left[-10\,\left(\frac{3}{2}+a+\left(\frac{1}{2}-a\right)\,x\right)^{20} + \left(\frac{3}{2}+a+\left(\frac{1}{2}-a\right)\,x\right)^{40} + \frac{1}{-3+2\,\left(\frac{3}{2}+a+\left(\frac{1}{2}-a\right)\,x\right)}\,,\\ &\left\{x,\,0\,,\,1\right\}\,,\, \text{Assumptions} \rightarrow !\,\left(\text{Re}\left[\frac{a}{1-2\,a}\right] \geq 0\mid\mid \text{Re}\left[\frac{a}{1-2\,a}\right] \leq -\frac{1}{2}\mid\mid \text{Im}\left[\frac{a}{1-2\,a}\right] \neq 0\right)\right]\right] \end{split}$$

N[%]

 5.36337×10^{10}

Ueb/Ex 6 W

$$\begin{aligned} & \phi(\mathbf{x}0_{-}) := \operatorname{ArcTan}\{-1) / f^*(\mathbf{x}0_{-})\}; \ \phi(\mathbf{x}0_{-}) \\ & -\operatorname{ArcTan}\{\frac{1}{2\pi 0}\} \\ & \alpha(\mathbf{x}0_{-}) := \phi(\mathbf{x}0_{-}) - 10 + 2 \operatorname{Pi} / 360; \ \alpha(\mathbf{x}0_{-}) \\ & -\frac{\pi}{18} - \operatorname{ArcTan}[\frac{1}{2\pi 0}] \\ & \operatorname{Remove}(\mathbf{nt}, \mathbf{solv}, \mathbf{ntx0}) \\ & \operatorname{nt}(\mathbf{x}0_{-}, \mathbf{x}_{-}) := f(\mathbf{x}0_{-}) + (\mathbf{x} - \mathbf{x}0_{-}) \operatorname{Tan}(\phi(\mathbf{x}0_{-})); \\ & \operatorname{nt}(\mathbf{x}0_{-}, \mathbf{x}_{-}) := f(\mathbf{x}0_{-}) + (\mathbf{x} - \mathbf{x}0_{-}) \operatorname{Tan}(\phi(\mathbf{x}0_{-})); \\ & \operatorname{nt}(\mathbf{x}0_{-}, \mathbf{x}_{-}) := f(\mathbf{x}0_{-}) + (\mathbf{x} - \mathbf{x}0_{-}) \operatorname{Tan}(\phi(\mathbf{x}0_{-})); \\ & \operatorname{nt}(\mathbf{x}0_{-}, \mathbf{x}_{-}) := f(\mathbf{x}0_{-}) + (\mathbf{x} - \mathbf{x}0_{-}) \operatorname{Tan}(\phi(\mathbf{x}0_{-})); \\ & \operatorname{nt}(\mathbf{x}0_{-}, \mathbf{x}_{-}) := f(\mathbf{x}0_{-}) + (\mathbf{x} - \mathbf{x}0_{-}) \operatorname{Tan}(\phi(\mathbf{x}0_{-})); \\ & \operatorname{nt}(\mathbf{x}0_{-}, \mathbf{x}_{-}) := f(\mathbf{x}0_{-}) + (\mathbf{x} - \mathbf{x}0_{-}) \operatorname{Tan}(\phi(\mathbf{x}0_{-})); \\ & \operatorname{nt}(\mathbf{x}0_{-}, \mathbf{x}_{-}) := f(\mathbf{x}0_{-}) - (\mathbf{x}0_{-}) \operatorname{Tan}(\phi(\mathbf{x}0_{-})); \\ & \operatorname{solv}(\mathbf{x}0_{-}, \mathbf{x}_{-}) := f(\mathbf{x}0_{-}) - (\mathbf{x}0_{-}) \operatorname{Tan}(\phi(\mathbf{x}0_{-})); \\ & \operatorname{solv}(\mathbf{x}0_{-}, \mathbf{x}) := f(\mathbf{x}0_{-}) \operatorname{Tan}(\phi(\mathbf{x}0_{-})); \\ & \operatorname{solv}(\mathbf{x}0_{-}, \mathbf{x}0_{-}) := f(\mathbf{x}0_{-}) \operatorname{Tan}(\phi(\mathbf{x}0_{-})); \\ & \operatorname{solv}(\mathbf{x}0_{-}, \mathbf{x}0_{-}) := f(\mathbf{x}0_{-}) \operatorname{Tan}(\phi(\mathbf{x}0_{-})); \\ & \operatorname{solv}(\mathbf{x}0_{-}, \mathbf{x}0_{-}) := f(\mathbf{x}0_{-}) := f(\mathbf{x}0_{-}) := f(\mathbf{x}0_{-$$

x1 = x0 /. solv[[2]]

-1.32285

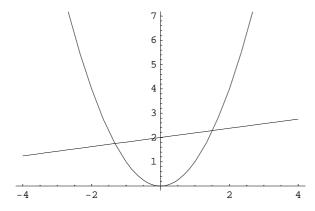
 $ntx0[x_{-}] := nt[x0, x] /. solv[[2]]; ntx0[x]$

General::spell1:

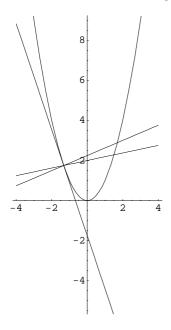
Possible spelling error: new symbol name "ntx0" is similar to existing symbol "nt0". Mehr...

1.74992 + 0.189047 (1.32285 + x)

p1 = Plot[{f[x], ntx0[x]}, {x, -4, 4}];



p2 = Plot[{f[x], ntx0[x], t[x1, x], nt0[x1, x]}, {x, -4, 4}, AspectRatio \rightarrow 16 / 8];



 $solv2 = Solve[{f[x] == ntx0[x]}, {x}] // Flatten$

 $\{x \rightarrow -1.32285, x \rightarrow 1.51189\}$

x2 = x /. solv2[[2]]

1.51189

 $Integrate[ntx0[x]-f[x], \{x, x1, x2\}]$

Ueb/Ex 7 W

а

```
Integrate[x^5, {x, 3, t}]
         -\,\frac{243}{2}\,+\,\frac{t^6}{6}
         Solve[Integrate[x^5, \{x, 3, t\}] = 10, \{t\}]
         \{\,\{\,t\to -789^{1/6}\,\} , \,\{\,t\to 789^{1/6}\,\} , \,\{\,t\to -\,(-1)^{\,1/3}\,\,789^{1/6}\,\} ,
          \{t \to (-1)^{1/3} \ 789^{1/6}\} \text{, } \{t \to -(-1)^{2/3} \ 789^{1/6}\} \text{, } \{t \to (-1)^{2/3} \ 789^{1/6}\} \}
        N[%]
         \{\{t\rightarrow -3.03981\}\,,\ \{t\rightarrow 3.03981\}\,,\ \{t\rightarrow -1.5199-2.63255\ \dot{\mathbb{1}}\}\,,
           \{t \rightarrow 1.5199 + 2.63255 i\}, \{t \rightarrow 1.5199 - 2.63255 i\}, \{t \rightarrow -1.5199 + 2.63255 i\}\}
b
         Integrate[1/(2x+1), \{x, 4, 6\}]
         \frac{1}{2} \operatorname{Log} \left[ \frac{13}{9} \right]
        N[%]
         0.183862
C
         Integrate[x^2Sin[x], {x, 0, Pi}]
         -4 + \pi^2
        N[%]
         5.8696
d
         Integrate[1/(4x^2-1), \{x, 2, 4\}]
         \frac{1}{4} \log \left[ \frac{35}{27} \right]
         Apart[1 / (4 x ^ 2 - 1)]
         \frac{1}{2 \, (-1 + 2 \, x)} \, - \, \frac{1}{2 \, (1 + 2 \, x)}
```

Integrate[Apart[1/ $(4x^2-1)$], $\{x, 2, 4\}$]

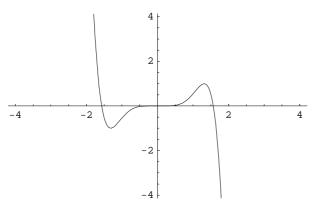
$$\frac{1}{4} \log \left[\frac{35}{27} \right]$$

N[%]

0.0648778

е

 $Plot[x^5Cos[x], \{x, -4, 4\}];$



Integrate $[x^5 Cos[x], \{x, -4, 4\}]$

0

f

Integrate $[x^2 \cos[4x^3 + 5], x]$

$$\frac{1}{12} \sin[5 + 4 x^3]$$

$$\frac{1}{12} \cos[4 \, x^3] \, \sin[5] + \frac{1}{12} \cos[5] \, \sin[4 \, x^3] \, // \, TrigReduce$$

$$\frac{1}{12}\,\text{Sin}[5+4\,x^3]$$

g

Integrate[Cos[x] E^Sin[x], x]

eSin[x]

 $Integrate[Cos[x] \ E^sin[x], \{x, 0, Pi\}]$

0

Plot[Cos[x] E^Sin[x],{x,0,Pi}];

