Lösungen

1

```
LaengeC[f_, x_] := Integrate[Evaluate[Sqrt[D[f,x]^2+1]],x];
Laengeab[f_, x_,a_, b_] := Integrate[Evaluate[Sqrt[D[f,x]^2+1]],{x,a,b}];
```

a

```
f[x_] := x^2;
a = -1;
b = 1;
```

```
LaengeC[f[x], x]
```

$$\frac{1}{2} x \sqrt{1+4 x^2} + \frac{1}{4} \text{ArcSinh}[2 x]$$

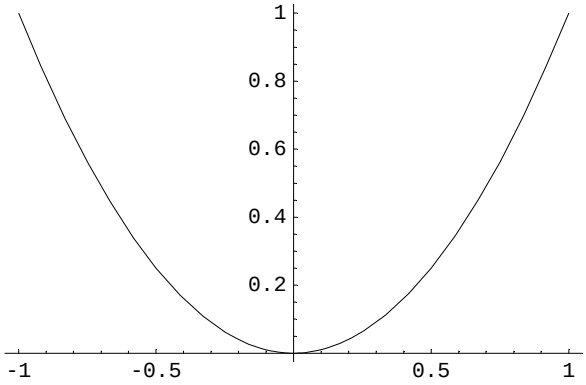
```
Laengeab[f[x], x,a,b]
```

$$\sqrt{5} + \frac{\text{ArcSinh}[2]}{2}$$

```
Laengeab[f[x], x,a,b]//N
```

```
2.95789
```

```
Plot[f[x],{x,a,b}];
```



b

```
f[x_] := E^x;
a = 0;
b = 100;
```

LaengeC[f[x], x]

$\sqrt{1+e^{2x}} - \text{ArcTanh}\left[\sqrt{1+e^{2x}}\right]$

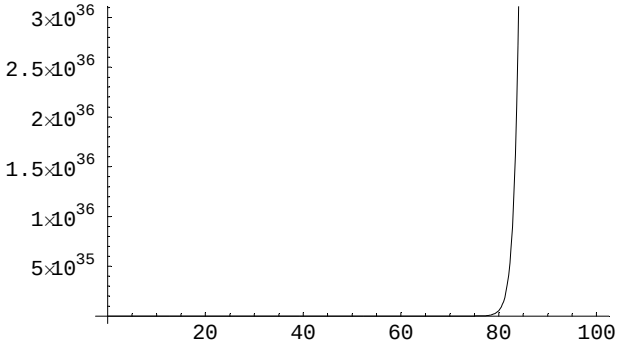
Laengeab[f[x], x,a,b]

$-\sqrt{2} + \sqrt{1+e^{200}} + \text{ArcTanh}\left[\sqrt{2}\right] - \text{ArcTanh}\left[\sqrt{1+e^{200}}\right]$

Laengeab[f[x], x,a,b] // N // Re

2.68812×10^{43}

Plot[f[x],{x,a,b}];



C

```
f[x_] := Sin[x];
a = 0;
b = 2 Pi;
```

LaengeC[f[x], x]

$\sqrt{2}\ \text{EllipticE}\left[x, \frac{1}{2}\right]$

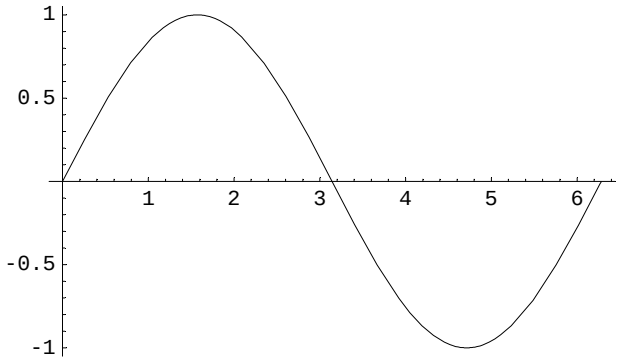
Laengeab[f[x], x,a,b]

$4\sqrt{2}\ \text{EllipticE}\left[\frac{1}{2}\right]$

Laengeab[f[x], x,a,b] // N

7.6404

```
Plot[f[x],{x,a,b}];
```



2

```
FlaecheC[r_, φ_] := 1/2 Integrate[r^2,φ];
Flaecheab[r_, φ_,a_, b_] := 1/2 Integrate[r^2,{φ,a,b}];
```

a

```
r[ϑ_] := E^φ;
a = 0;
b = 2 Pi;

FlaecheC[r[φ], φ]
```

$$\frac{e^{2\varphi}}{4}$$

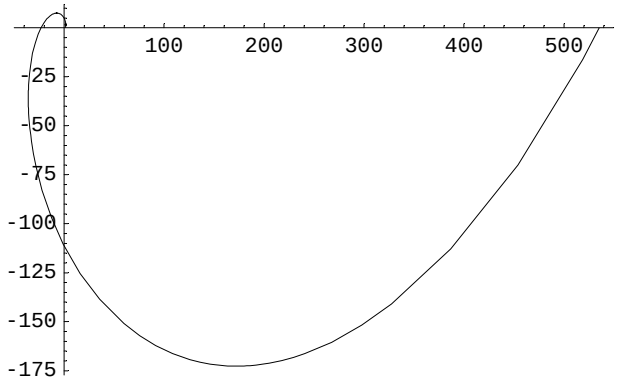
```
Flaecheab[r[φ], φ,a,b]
```

$$\frac{1}{4}\left(-1+e^{4\pi}\right)$$

```
Flaecheab[r[φ], φ,a,b]//N
```

```
71687.6
```

```
ParametricPlot[{r[φ] Cos[φ],r[φ] Sin[φ]},{φ,a,b}];
```



b

```
r[φ_] := Log[φ];
a = 0;
b = 2 Pi;

FlaecheC[r[φ], φ]
```

$$\frac{1}{2}\left(2\varphi - 2\varphi\,\text{Log}[\varphi] + \varphi\,\text{Log}[\varphi]^2\right)$$

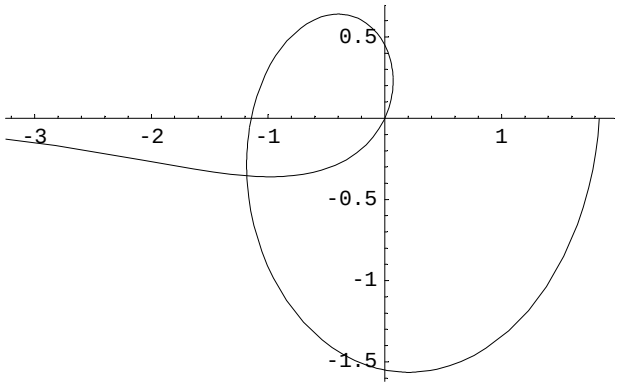
```
Flaecheab[r[φ], φ,a,b]
```

$$\pi\left(2 + \text{Log}[2]^2 + \text{Log}[4]\left(-1 + \text{Log}[\pi]\right) + \left(-2 + \text{Log}[\pi]\right)\text{Log}[\pi]\right)$$

```
Flaecheab[r[φ], φ,a,b]//N
```

```
5.34711
```

```
ParametricPlot[{r[φ] Cos[φ],r[φ] Sin[φ]},{φ,a,b}];
```



c

```
r[φ_] := 1/(1+φ^2);
a = 0;
b = Pi/2;

FlaecheC[r[φ], φ]
```

$$\frac{1}{4}\left(\frac{\varphi}{1+\varphi^2} + \text{ArcTan}[\varphi]\right)$$

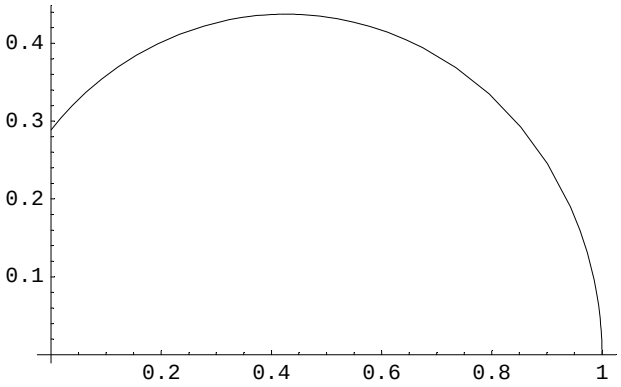
```
Flaecheab[r[φ], φ,a,b]
```

$$\frac{1}{2}\left(\frac{\pi}{4+\pi^2} + \frac{1}{2}\,\text{ArcTan}\left[\frac{\pi}{2}\right]\right)$$

```
Flaecheab[r[φ], φ,a,b]//N
```

```
0.364226
```

```
ParametricPlot[{r[φ] Cos[φ],r[φ] Sin[φ]},{φ,a,b}];
```



3

```
LaengePC[r_, φ_] := Integrate[Evaluate[Sqrt[r^2+D[r,φ]^2]],φ];
LaengePab[r_, φ_,a_, b_] := Integrate[Evaluate[Sqrt[r^2+D[r,φ]^2]],{φ,a,b}];
```

a

```
r[ϑ_] := E^φ;
a = 0;
b = 2 Pi;
```

```
LaengeC[r[φ], φ]
```

$$\sqrt{1+e^{2\varphi}}-\text{ArcTanh}\left[\sqrt{1+e^{2\varphi}}\right]$$

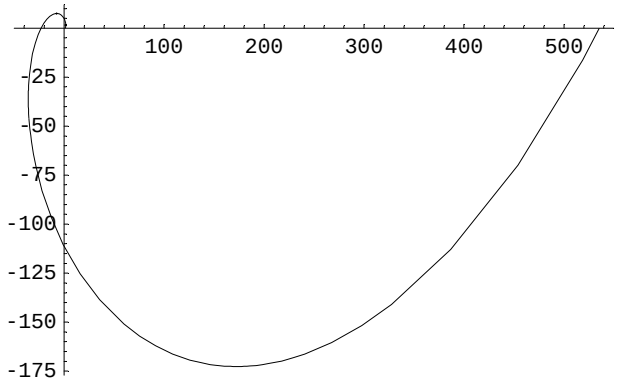
```
Laengeab[r[φ], φ,a,b]
```

$$-\sqrt{2}+\sqrt{1+e^{4\pi}}+\text{ArcTanh}\left[\sqrt{2}\right]-\text{ArcTanh}\left[\sqrt{1+e^{4\pi}}\right]$$

```
Laengeab[r[φ], φ,a,b]// N //Chop
```

```
534.958
```

```
ParametricPlot[{r[φ] Cos[φ],r[φ] Sin[φ]},{φ,a,b}];
```



b

```
r[φ_] := Log[φ];
a = 0;
b = 2 Pi;
```

```
LaengeC[r[φ], φ]
```

$$\frac{\sqrt{1+\frac{1}{\varphi^2}}\ \varphi\ \left(\sqrt{1+\varphi^2}+\text{Log}[\varphi]-\text{Log}\left[1+\sqrt{1+\varphi^2}\right]\right)}{\sqrt{1+\varphi^2}}$$

```
Laengeab[r[φ], φ,a,b]
```

```
Integrate::idiv : Integral of √(1 + 1/φ^2) does not converge on {0, 2 π}. Mehr…
```

$$\int_0^{2\pi}\sqrt{1+\frac{1}{\varphi^2}}\ \mathrm{d}\varphi$$

```
Laengeab[r[φ], φ,a,b]//N
```

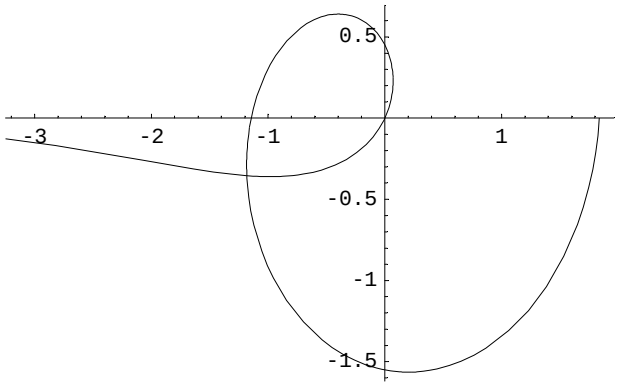
```
Integrate::idiv : Integral of √(1 + 1/φ^2) does not converge on {0, 2 π}. Mehr…

NIntegrate::slwcon :
 Numerical integration converging too slowly; suspect one of the following: singularity, value of
   the integration being 0, oscillatory integrand, or insufficient WorkingPrecision. If your
   integrand is oscillatory try using the option Method->Oscillatory in NIntegrate. Mehr…

NIntegrate::ncvb : NIntegrate failed to converge to prescribed
   accuracy after 7 recursive bisections in φ near φ = 2.7457480509295168`*^-56. Mehr…

23957.1
```

```
ParametricPlot[{r[φ] Cos[φ],r[φ] Sin[φ]},{φ,a,b}];
```



c

```
r[φ_] := 1/(1+φ^2);
a = 0;
b = Pi/2;
```

LaengeC[r[φ], φ]

$$\int \sqrt{1 + \frac{4\,\varphi^2}{(1+\varphi^2)^4}}\; d\varphi$$

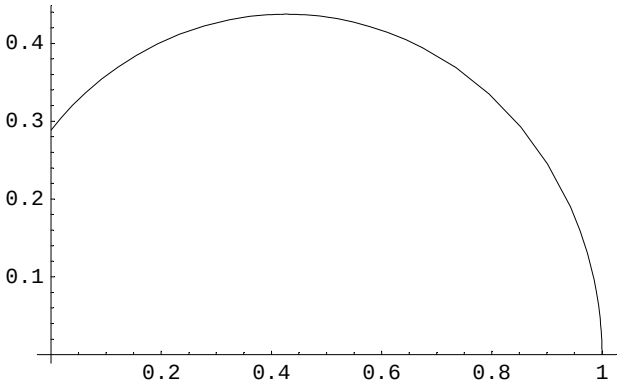
Laengeab[r[φ], φ,a,b]

$$\int_0^{\frac{\pi}{2}} \sqrt{1 + \frac{4\,\varphi^2}{(1+\varphi^2)^4}}\; d\varphi$$

Flaecheab[r[φ], φ,a,b]//N

```
0.364226
```

ParametricPlot[{r[φ] Cos[φ],r[φ] Sin[φ]},{φ,a,b}];



4

VolumenC[f_, x_] := Pi Integrate[f^2,x];
Volumenab[f_, x_,a_, b_] := Pi Integrate[f^2,{x,a,b}];

a

f[x_] := x^2;
a = -1;
b = 1;

VolumenC[f[x], x]

$$\frac{\pi\, x^5}{5}$$

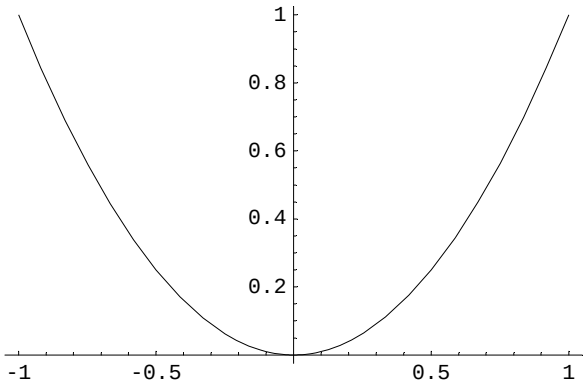
Volumenab[f[x], x,a,b]

$$\frac{2\,\pi}{5}$$

Volumenab[f[x], x,a,b]//N

```
1.25664
```

```
Plot[f[x],{x,a,b}];
```



b

```
f[x_] := E^x;
a = 0;
b = 100;
```

```
LaengeC[f[x], x]
```

$$\sqrt{1+e^{2x}}-\text{ArcTanh}\left[\sqrt{1+e^{2x}}\right]$$

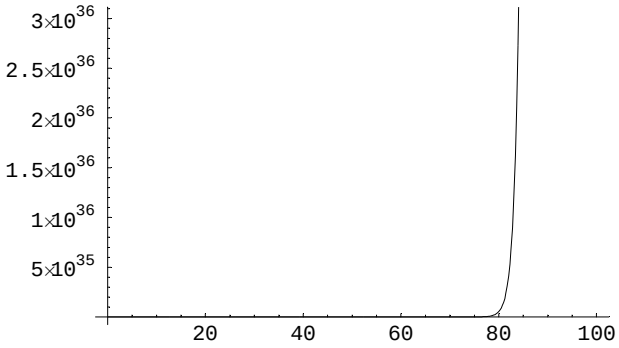
```
Volumenab[f[x], x,a,b]
```

$$\frac{1}{2}\left(-1+e^{200}\right)\pi$$

```
Volumenab[f[x], x,a,b]//N
```

$$1.13505\times 10^{87}$$

```
Plot[f[x],{x,a,b}];
```



c

```
f[x_] := Sin[x];
a = 0;
b = Pi;
```

LaengeC[f[x], x]

$\sqrt{2}\ \text{EllipticE}\left[x, \frac{1}{2}\right]$

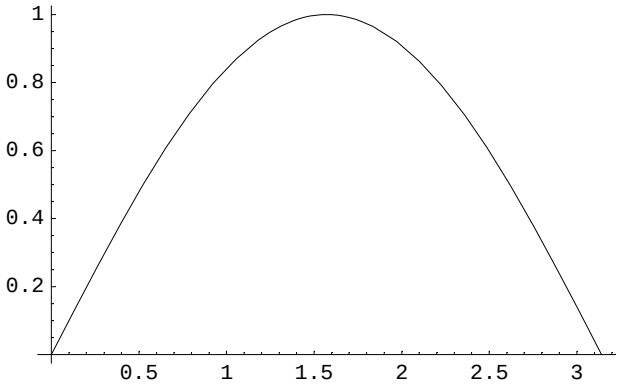
Volumenab[f[x], x,a,b]

$\frac{\pi^2}{2}$

Volumenab[f[x], x,a,b]//N

```
4.9348
```

Plot[f[x],{x,a,b}];



5

```
OberflC[f_, x_] := 2 Pi Integrate[Evaluate[f Sqrt[1+D[f,x]^2]],x];
Oberflab[f_, x_,a_, b_] := 2 Pi Integrate[Evaluate[f Sqrt[1+D[f,x]^2]],{x,a,b}];
```

a

```
f[x_] := x^2;
a = -1;
b = 1;
```

OberflC[f[x], x]

$\frac{1}{32}\pi\left(2\sqrt{1+4x^2}\ (x+8x^3) - \text{ArcSinh}[2x]\right)$

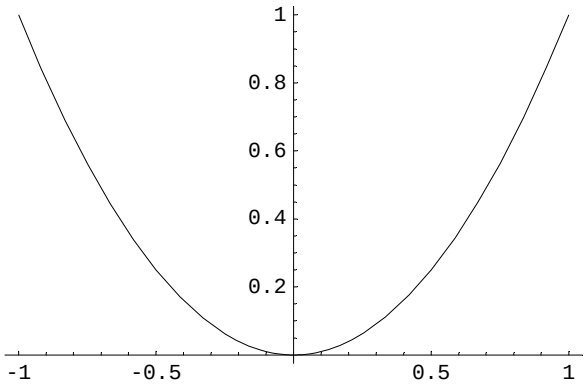
Oberflab[f[x], x,a,b]

$\frac{1}{16}\pi\left(18\sqrt{5} - \text{ArcSinh}[2]\right)$

Oberflab[f[x], x,a,b]//N

```
7.61946
```

```
Plot[f[x],{x,a,b}];
```



b

```
f[x_] := E^x;
a = 0;
b = 100;
```

```
OberflC[f[x], x]
```

$$\pi \left(e^{x} \sqrt{1+e^{2 x}}+\text{ArcSinh}\left[e^{x}\right]\right)$$

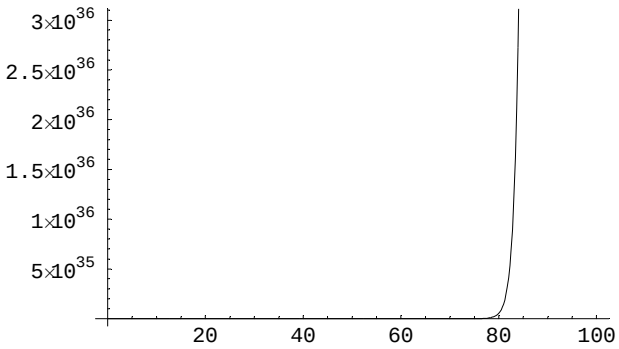
```
Oberflab[f[x], x,a,b]
```

$$\pi \left(-\sqrt{2}+e^{100} \sqrt{1+e^{200}}-\text{ArcSinh}[1]+\text{ArcSinh}\left[e^{100}\right]\right)$$

```
Oberflab[f[x], x,a,b]//N
```

$$2.27011 \times 10^{87}$$

```
Plot[f[x],{x,a,b}];
```



c

```
f[x_] := Sin[x];
a = 0;
b = Pi;
```

OberflC[f[x], x]

$$2\pi\left(-\frac{1}{2}\,\text{ArcSinh}[\text{Cos}[x]]-\frac{1}{2}\,\text{Cos}[x]\,\sqrt{1+\text{Cos}[x]^2}\right)$$

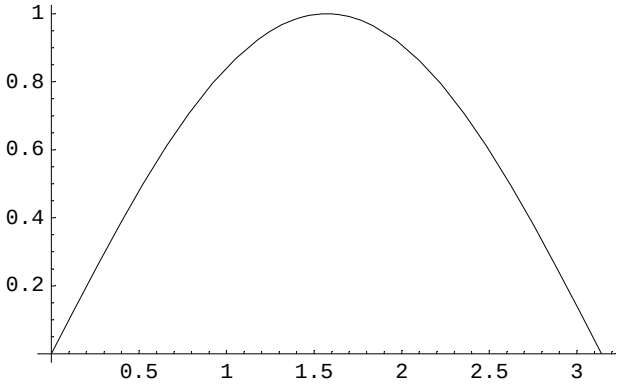
Oberflab[f[x], x,a,b]

$$2\pi\left(\sqrt{2}+\text{ArcSinh}[1]\right)$$

Oberflab[f[x], x,a,b]//N

14.4236

Plot[f[x],{x,a,b}];



6

Remove["Global`*"]

a

```
vec[t_]:= {x[t],y[t]};
x[t_]:= t^2; y[t_]:= t;
f[x_,y_]:= x^2+x y-y^2;
a=-1;
b=1;
LinienintegralC= Integrate[Evaluate[f[x[t],y[t]]*Sqrt[D[x[t],t]^2+D[y[t],t]^2]],t]
```

$$\frac{\sqrt{1+4\,t^2}\;\left(-32-135\,t+64\,t^2-920\,t^3+768\,t^4+640\,t^5\right)}{3840}+\frac{9}{512}\,\text{ArcSinh}[2\,t]$$

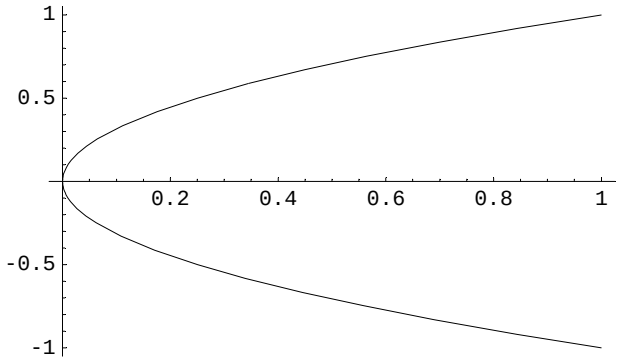
```
Linienintegralab=
Integrate[Evaluate[f[x[t],y[t]]*Sqrt[D[x[t],t]^2+D[y[t],t]^2]],{t,a,b}]
```

$$\frac{1}{768}\left(-166\sqrt{5}+27\,\text{ArcSinh}[2]\right)$$

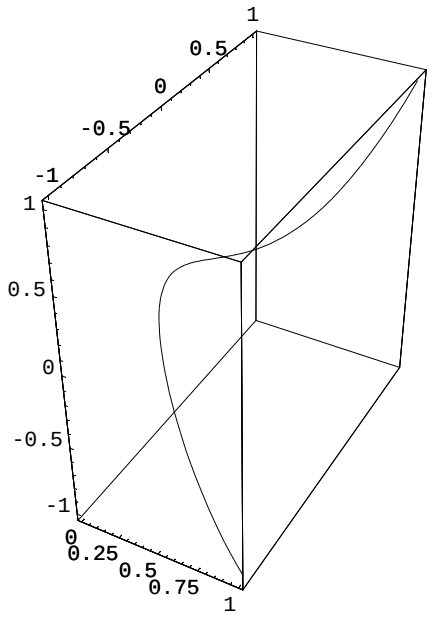
N[%]

-0.432564

```
ParametricPlot[{x[t],y[t]},{t,a,b}];
```




```
ParametricPlot3D[{x[t],y[t],f[x[t],y[t]]},{t,a,b}];
```



b

```
vec[t_]:= {x[t],y[t],z[t]};
x[t_]:= Cos[t]; y[t_]:= Sin[t]; z[t_]:= 1;
f[x_,y_,z_]:= Cos[x^2] + y z;
a= 0;
b= 2 Pi;
LinienintegralC=
Integrate[Evaluate[f[x[t],y[t],z[t]]*Sqrt[D[x[t],t]^2+D[y[t],t]^2+D[z[t],t]^2],t]
```

$$\int (\text{Cos}[\text{Cos}[t]^2] + \text{Sin}[t])\, \sqrt{\text{Cos}[t]^2 + \text{Sin}[t]^2}\, \text{d}t$$

```
Linienintegralab=
Integrate[Evaluate[f[x[t],y[t],z[t]]*Sqrt[D[x[t],t]^2+D[y[t],t]^2+D[z[t],t]^2],{t,
a,b}]
```

$$2\pi\, \text{BesselJ}\left[0, \frac{1}{2}\right] \text{Cos}\left[\frac{1}{2}\right]$$

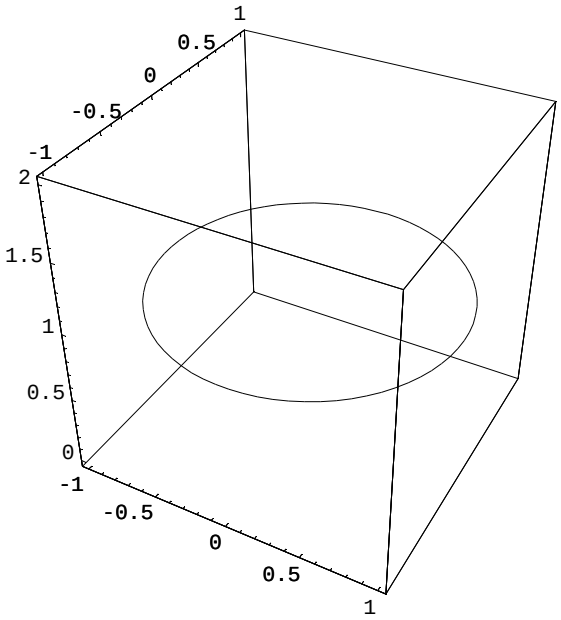
```
N[%]
```

```
5.17474
```

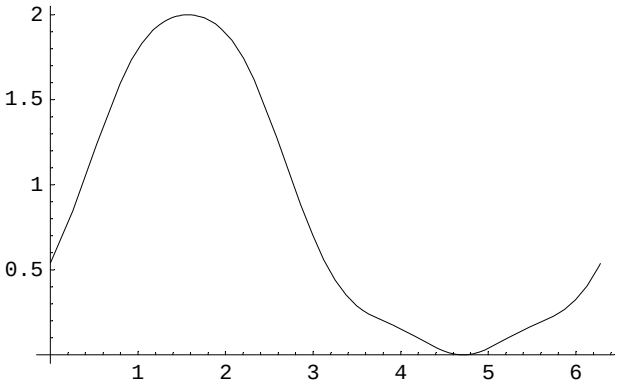
```
NIntegrate[Evaluate[f[x[t],y[t],z[t]]*Sqrt[D[x[t],t]^2+D[y[t],t]^2]+D[z[t],t]^2],{t
,a,b}]
```

```
5.17474
```

```
ParametricPlot3D[{x[t],y[t],z[t]},{t,a,b}];
```




```
Plot[f[x[t],y[t],z[t]],{t,a,b}];
```



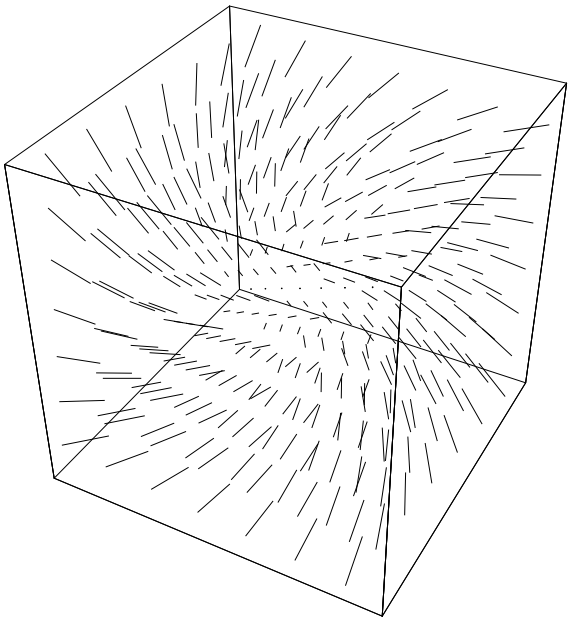
C

```
Remove["Global`*"]
```

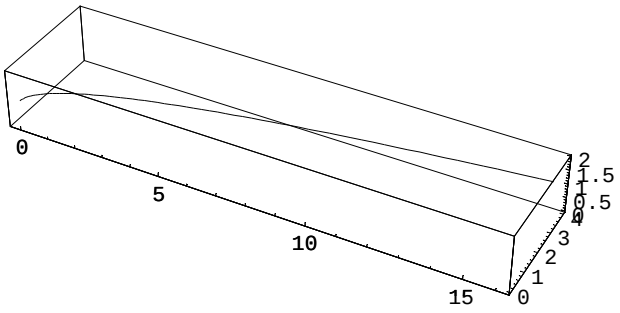
```
vec[t_]:= {x[t],y[t],z[t]};
x[t_]:= t^2; y[t_]:= t; z[t_]:= 1;
f[x_,y_,z_]:= {x+y,y+z,z-x};
a= 0;
b= 4;
```

```
<<Graphics`PlotField3D`
```

```
PlotVectorField3D[f[x,y,z] ,
   {x, -2, 2}, {y, -2, 2}, {z, -2, 2}];
```



```
ParametricPlot3D[{x[t],y[t],z[t]},{t,a,b}];
```




```
LinienintegralC=
Integrate[Evaluate[{f[x[t],y[t],z[t]]}.{D[x[t],t],D[y[t],t],D[z[t],t]}],t]
```

$$\left\{t + \frac{t^2}{2} + \frac{2\,t^3}{3} + \frac{t^4}{2}\right\}$$

```
LinienintegralC=
Integrate[Evaluate[{f[x[t],y[t],z[t]]}.{D[x[t],t],D[y[t],t],D[z[t],t]}],{t,a,b}]
```

$$\left\{\frac{548}{3}\right\}$$

```
N[%]
```

```
{182.667}
```

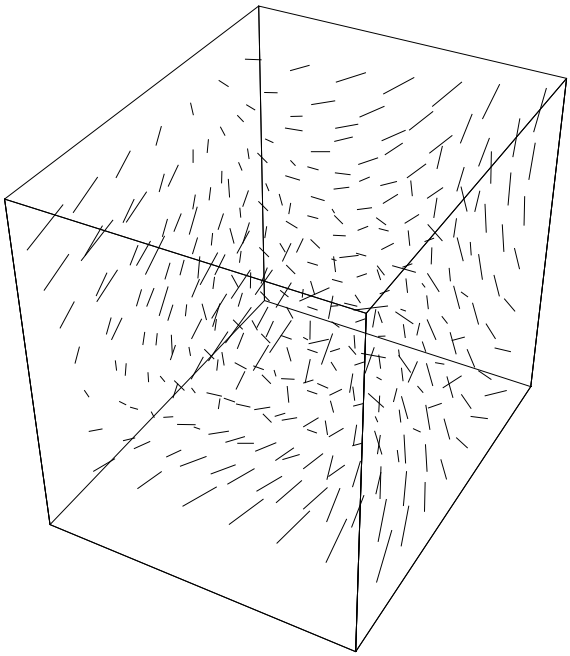
d

```
Remove["Global`*"]
```

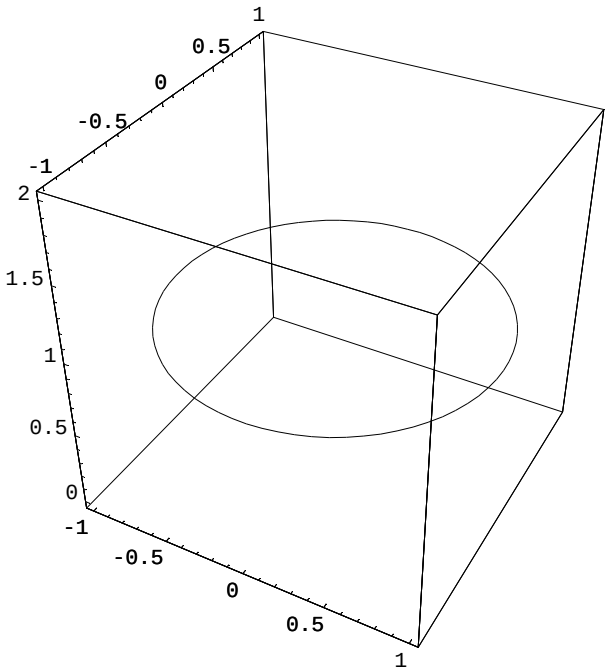
```
vec[t_]:= {x[t],y[t],z[t]};
x[t_]:= Cos[t]; y[t_]:= Sin[t]; z[t_]:= 1;
f[x_,y_,z_]:= {Cos[x^2],y z,x};
a= 0;
b= 2Pi;

<<Graphics`PlotField3D`

PlotVectorField3D[f[x,y,z] ,
   {x, -2, 2}, {y, -2, 2}, {z, -2, 2}];
```



```
ParametricPlot3D[{x[t],y[t],z[t]},{t,a,b}];
```



LinienintegralC=
Integrate[Evaluate[{f[x[t],y[t],z[t]]}.{D[x[t],t],D[y[t],t],D[z[t],t]}],t]

$$\left\{-\frac{1}{2}\,\text{Cos}[t]^2+\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left[\sqrt{\frac{2}{\pi}}\,\text{Cos}[t]\right]\right\}$$

LinienintegralC=
Integrate[Evaluate[{f[x[t],y[t],z[t]]}.{D[x[t],t],D[y[t],t],D[z[t],t]}],{t,a,b}]

{0}

N[%]

{0.}