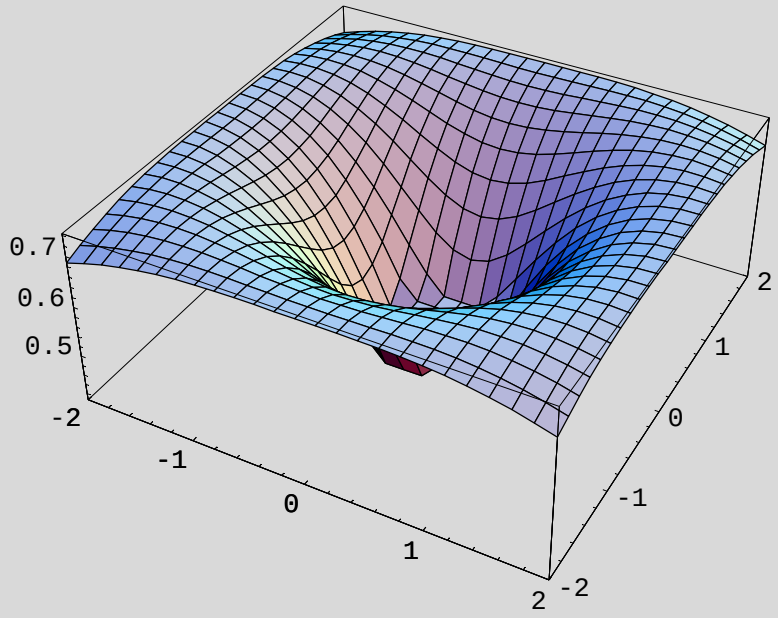
Lösungen

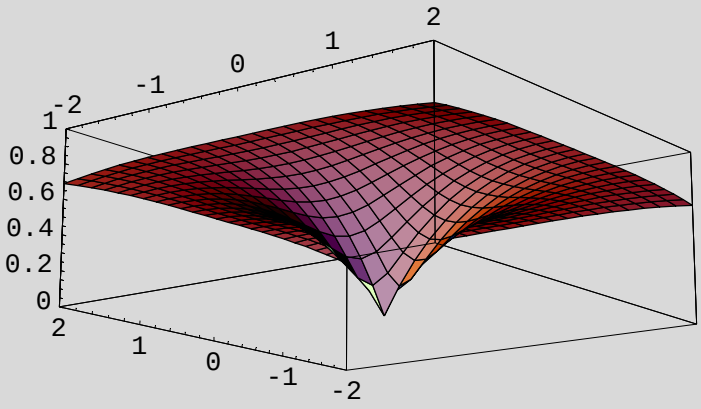
Lösung zu Aufgabe 1

■ **a**

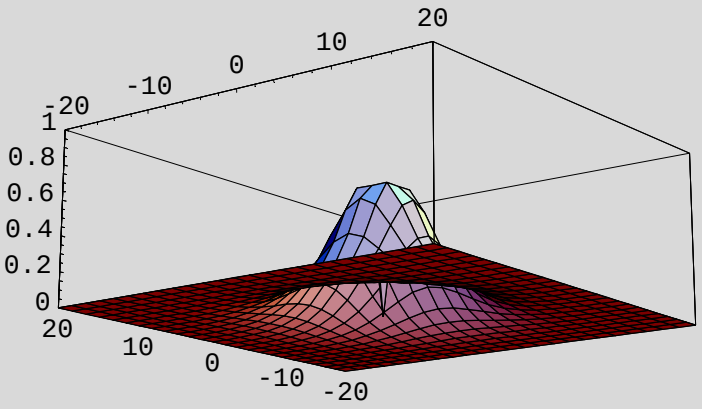
```
f[x_,y_]:= Sqrt[x^2+y^2] E^(-Sqrt[x^2+y^2]/2);
Plot3D[f[x,y],{x,-2,2},{y,-2,2}];
```



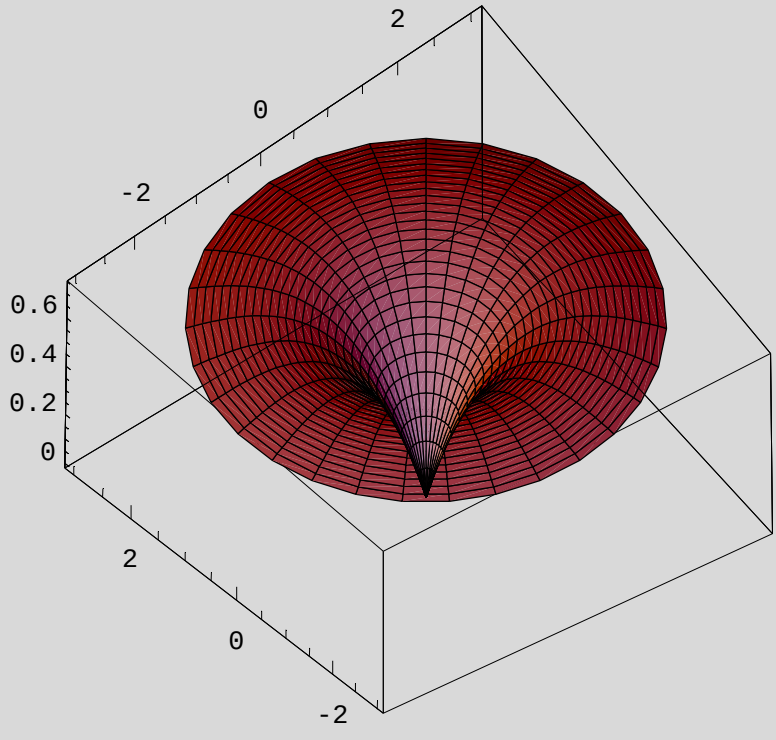

```
Plot3D[f[x,y],{x,-2,2},{y,-2,2},ViewPoint->{3.667, 2.785,
-1.120},PlotRange->{0,1}];
```



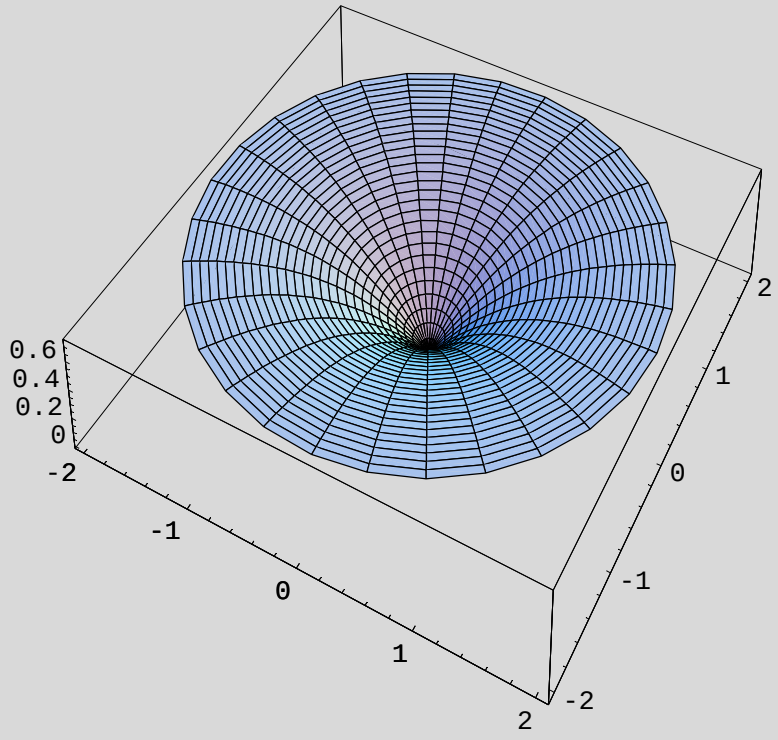
```
Plot3D[f[x,y],{x,-20,20},{y,-20,20},ViewPoint->{3.667,
2.785, -1.120},PlotRange->{0,1}];
```




```
h[r_]:= r E^(-r/2);
ParametricPlot3D[{r Cos[t],r Sin[t], h[r]}, {r,0,3},
{t,0,2Pi}, ViewPoint->{3.667, 2.785, -1.120},
AspectRatio->1];
```



```
ParametricPlot3D[{r Cos[t],r Sin[t], h[r]}, {r,0,2},
{t,0,2Pi}, AspectRatio->1];
```

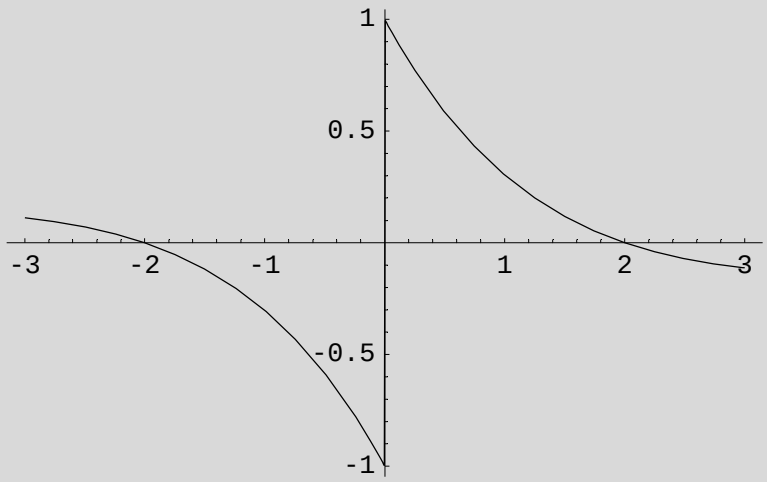


■ b

```
D[f[x,y],x]/.x->u
```

$$-\frac{1}{2}\, e^{-\frac{1}{2}\sqrt{u^2+y^2}}\, u + \frac{e^{-\frac{1}{2}\sqrt{u^2+y^2}}\, u}{\sqrt{u^2+y^2}}$$

```
Plot[Evaluate[D[f[x,0],x]/.x->u],{u,-3,3}];
```




```
Limit[Evaluate[D[f[x,0],x]/.x->u],u->0, Direction → 1]
```

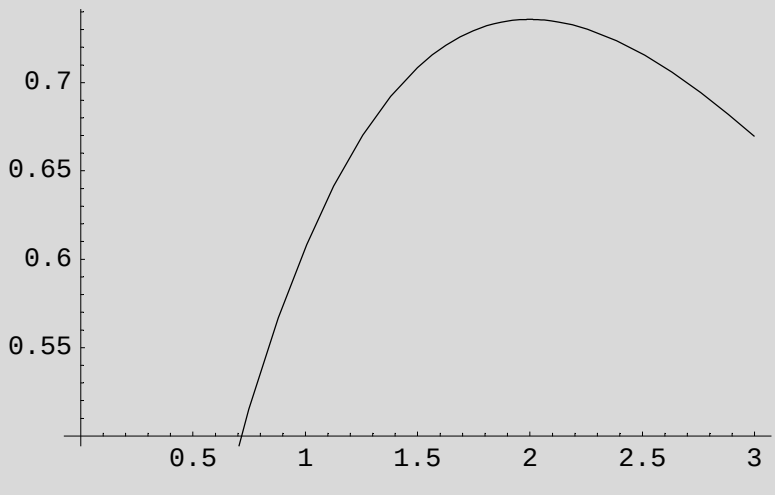
```
-1
```

```
Limit[Evaluate[D[f[x,0],x]/.x->u],u->0, Direction → -1]
```

```
1
```

■ c

```
Plot[h[r],{r,0,3}];
```



```
D[h[r],r]/.r->2
```

```
0
```

```
hMax=h[2]
```

```
General::spell1 : Possible spelling error: new symbol
   name "hMax" is similar to existing symbol "Max". Mehr…
```

$$\frac{2}{e}$$

```
hMax=h[2]//N
```

```
0.735759
```

■ d

```
<<Calculus`VectorAnalysis`
```

```
Drop[Grad[f[x,y], Cartesian[x, y, z]],{3}]
```

$$\left\{-\frac{1}{2}\, e^{-\frac{1}{2}\sqrt{x^2+y^2}}\, x + \frac{e^{-\frac{1}{2}\sqrt{x^2+y^2}}\, x}{\sqrt{x^2+y^2}},\ -\frac{1}{2}\, e^{-\frac{1}{2}\sqrt{x^2+y^2}}\, y + \frac{e^{-\frac{1}{2}\sqrt{x^2+y^2}}\, y}{\sqrt{x^2+y^2}}\right\}$$

```
Drop[Grad[f[x,y], Cartesian[x, y, z]],{3}] /. {x->1,y->1}
```

$$\left\{-\frac{1}{2}\, e^{-\frac{1}{\sqrt{2}}} + \frac{e^{-\frac{1}{\sqrt{2}}}}{\sqrt{2}},\ -\frac{1}{2}\, e^{-\frac{1}{\sqrt{2}}} + \frac{e^{-\frac{1}{\sqrt{2}}}}{\sqrt{2}}\right\}$$

```
N[%]
```

```
{0.102118, 0.102118}
```

■ **e**

```
Oberfl=
Integrate[Evaluate[Sqrt[D[f[x,y],x]^2+D[f[x,y],x]^2+1]/.{x-
>x1,y->y1}],{x1,1,2},{y1,1,2}]
```

$$\int_1^2 \int_1^2 \sqrt{1 + \frac{e^{-\sqrt{x1^2+y1^2}}\, x1^2 \left(-2+\sqrt{x1^2+y1^2}\right)^2}{2\,(x1^2+y1^2)}}\; dy1\, dx1$$

```
NIntegrate[Evaluate[Sqrt[D[f[x,y],x]^2+D[f[x,y],x]^2+1]/.{x
->x1,y->y1}],{x1,1,2},{y1,1,2}]
```

```
1.00133
```

```
NIntegrate[Evaluate[Sqrt[D[f[x,y],x]^2+D[f[x,y],x]^2+1]/.{x
->x1,y->y1}],{x1,0,2},{y1,1,2}]
```

```
2.00319
```

```
Remove["Global`*"]
```

Lösung zu Aufgabe 2

■ **a**

```
DSolve[{y''[x] + 2 y[x] == Cos[x]}, y[x], x ] // Simplify
```

$$\{\{y[x] \to \text{Cos}[x] + C[1]\,\text{Cos}\left[\sqrt{2}\,x\right] + C[2]\,\text{Sin}\left[\sqrt{2}\,x\right]\}\}$$

```
N[%]
```

$$\{\{y[x] \to \text{Cos}[x] + C[1]\,\text{Cos}[1.41421\,x] + C[2]\,\text{Sin}[1.41421\,x]\}\}$$

■ b

```
DSolve[{y''[x]-y'[x] + 2 y[x] ==Cos[x]},y[x],x ]//Simplify
```

$$\left\{\left\{y[x] \to \frac{1}{2}\left(\text{Cos}[x] + 2\, e^{x/2}\, C[2]\, \text{Cos}\left[\frac{\sqrt{7}\, x}{2}\right] - \text{Sin}[x] + 2\, e^{x/2}\, C[1]\, \text{Sin}\left[\frac{\sqrt{7}\, x}{2}\right]\right)\right\}\right\}$$

```
N[%]
```

$$\{\{y[x] \to 0.5\,(\text{Cos}[x] + 2.\, 2.71828^{0.5\,x}\, C[2]\, \text{Cos}[1.32288\, x] - 1.\, \text{Sin}[x] + 2.\, 2.71828^{0.5\,x}\, C[1]\, \text{Sin}[1.32288\, x])\}\}$$

■ c

```
DSolve[{y''[x] - y'[x] + 2 y[x] == Cos[x], y[0] == 0, y'[0] == 0},
  y[x], x ] // Simplify
```

$$\left\{\left\{y[x] \to \frac{1}{14}\left(7\,\text{Cos}[x] - 7\, e^{x/2}\, \text{Cos}\left[\frac{\sqrt{7}\, x}{2}\right] - 7\,\text{Sin}[x] + 3\sqrt{7}\, e^{x/2}\, \text{Sin}\left[\frac{\sqrt{7}\, x}{2}\right]\right)\right\}\right\}$$

```
N[%]
```

$$\{\{y[x] \to 0.0714286\,(7.\, \text{Cos}[x] - 7.\, 2.71828^{0.5\,x}\, \text{Cos}[1.32288\, x] - 7.\, \text{Sin}[x] + 7.93725\, 2.71828^{0.5\,x}\, \text{Sin}[1.32288\, x])\}\}$$

```
solv=DSolve[{y''[x]-y'[x]+2y[x]==Cos[x],y[0]==0,y'[0]==0},y,x
]//Simplify//Flatten
```

$$\left\{y \to \text{Function}\left[\{x\}, \frac{1}{14}\left(-7\, e^{x/2}\cos\left[\frac{\sqrt{7}\,x}{2}\right] + 7\cos[x]\cos\left[\frac{\sqrt{7}\,x}{2}\right]^2 - 7\cos\left[\frac{\sqrt{7}\,x}{2}\right]^2\sin[x] + 3\sqrt{7}\, e^{x/2}\sin\left[\frac{\sqrt{7}\,x}{2}\right] + 7\cos[x]\sin\left[\frac{\sqrt{7}\,x}{2}\right]^2 - 7\sin[x]\sin\left[\frac{\sqrt{7}\,x}{2}\right]^2\right)\right]\right\}$$

```
N[%]
```

$$\{y \to \text{Function}[\{x\}, 0.0714286\,(-7.\,2.71828^{0.5\,x}\cos[0.5\,2.64575\,x] + 7.\cos[x]\cos[0.5\,2.64575\,x]^2 - 7.\cos[0.5\,2.64575\,x]^2\sin[x] + 3.\,2.64575\,2.71828^{0.5\,x}\sin[0.5\,2.64575\,x] + 7.\cos[x]\sin[0.5\,2.64575\,x]^2 - 7.\sin[x]\sin[0.5\,2.64575\,x]^2)]\}$$

```
y=y/.solv;
y[z]//Simplify
```

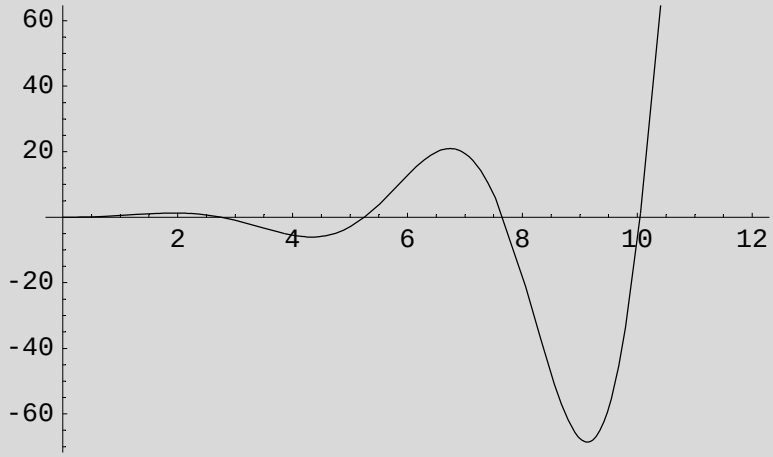
$$\frac{1}{14}\left(7\cos[z] - 7\, e^{z/2}\cos\left[\frac{\sqrt{7}\,z}{2}\right] - 7\sin[z] + 3\sqrt{7}\, e^{z/2}\sin\left[\frac{\sqrt{7}\,z}{2}\right]\right)$$

```
N[%]
```

$$0.0714286\,(7.\cos[z] - 7.\,2.71828^{0.5\,z}\cos[1.32288\,z] - 7.\sin[z] + 7.93725\,2.71828^{0.5\,z}\sin[1.32288\,z])$$

■ d

```
pl=Plot[y[z],{z,0,12}];
```



■ e

```
FindRoot[y[x],{x,3}]
```

$$\{x \to 2.75558\}$$

```
y[2.75558]
```

$$1.56584 \times 10^{-6}$$

```
Remove["Global`*"]
```

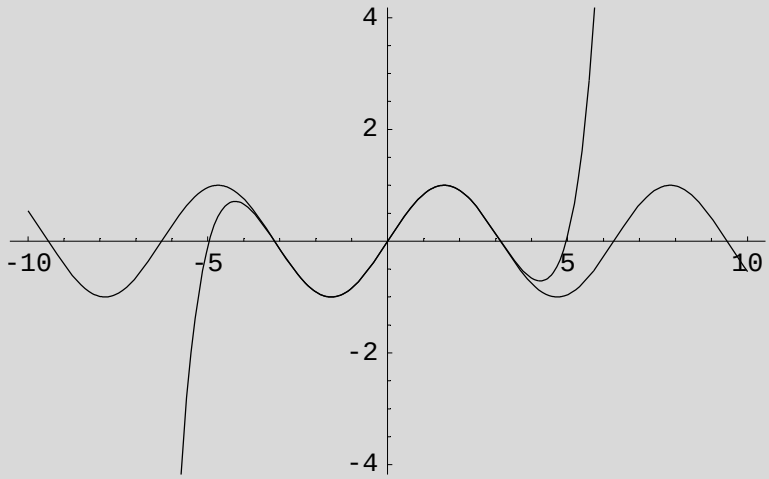
Lösung zu Aufgabe 3

■ a

```
s1=Normal[Series[Sin[x],{x,0,10}]]
```

$$x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \frac{x^9}{362880}$$

```
Plot[{s1,Sin[x]},{x,-10,10}];
```

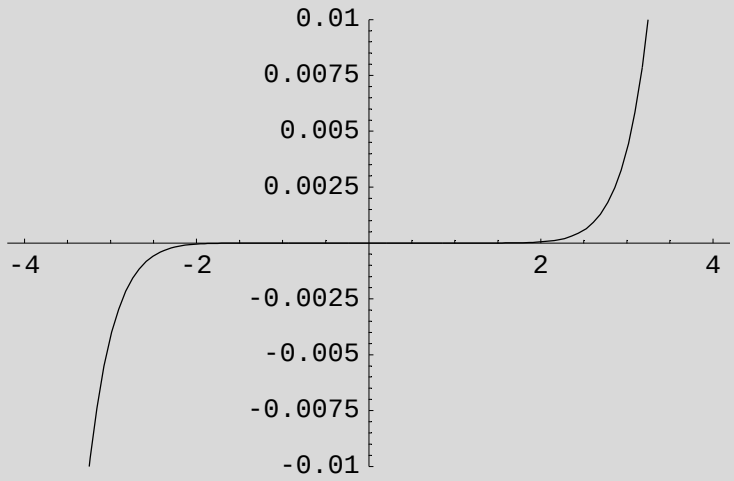



```
s2=Normal[Series[Cos[x],{x,0,10}]]
```

$$1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \frac{x^8}{40320} - \frac{x^{10}}{3628800}$$

■ b

```
Plot[s1-Sin[x],{x,-4,4},PlotRange->{-0.01,0.01}]
```



- Graphics -

■ c

```
D[s1,x]
```

$$1-\frac{x^2}{2}+\frac{x^4}{24}-\frac{x^6}{720}+\frac{x^8}{40320}$$

■ d

```
s1+s2
```

$$1+x-\frac{x^2}{2}-\frac{x^3}{6}+\frac{x^4}{24}+\frac{x^5}{120}-\frac{x^6}{720}-\frac{x^7}{5040}+\frac{x^8}{40320}+\frac{x^9}{362880}-\frac{x^{10}}{3628800}$$

```
Sqrt[2] Sin[x+Pi/4]//TrigExpand
```

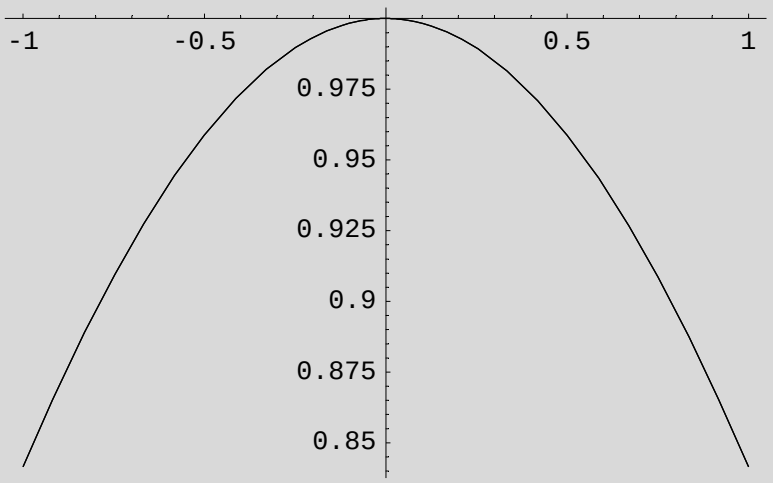
$$\text{Cos}[x] + \text{Sin}[x]$$

```
s3=Normal[Series[Sqrt[2] Sin[x+Pi/4],{x,0,10}]]
```

$$1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} - \frac{x^6}{720} - \frac{x^7}{5040} + \frac{x^8}{40320} + \frac{x^9}{362880} - \frac{x^{10}}{3628800}$$

■ e

```
Plot[{Sin[x]/x,s1/x},{x,-1,1}]
```



- Graphics -

■ f

```
Integrate[s1/x,x]
```

$$x - \frac{x^3}{18} + \frac{x^5}{600} - \frac{x^7}{35280} + \frac{x^9}{3265920}$$

```
NIntegrate[Simplify[s1/x],{x,-1,1}]
```

```
1.89217
```

```
NIntegrate[Sin[x]/x,{x,-1,1}]
```

```
1.89217
```

Lösung zu Aufgabe 4

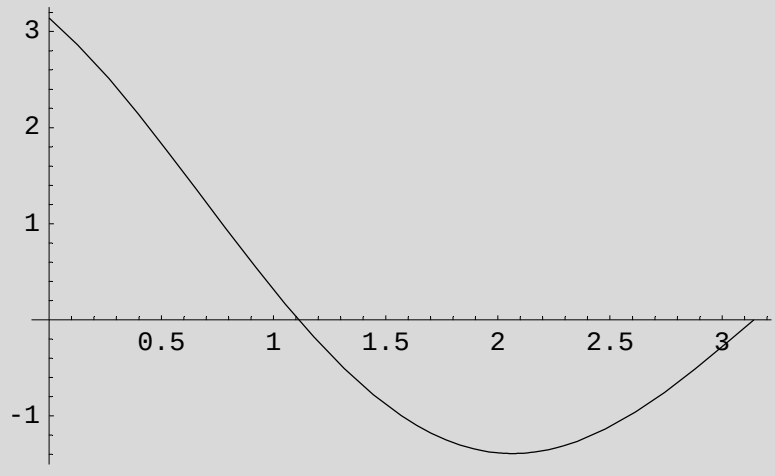
```
Remove["Global`*"]
```

■ **a**

```
tr[x_]:= (Pi+ (Pi-2 x))/2 Sin[x]; tr[x]//Simplify
```

```
(π - x) Sin[x]
```

```
Plot[Evaluate[D[tr[x],x]/.x->u],{u,0,Pi}];
```




```
FindRoot[(Evaluate[D[tr[x],x]]==0) /.x->u,{u,1}]
```

```
{u → 1.11283}
```

```
(Evaluate[D[tr[x],x]]) /.x->1.11283
```

```
0.0000130208
```

```
tr[x] /.x->1.11283
```

```
1.81971
```

```
Integrate[Sin[x],{x,0,Pi}]
```

```
2
```

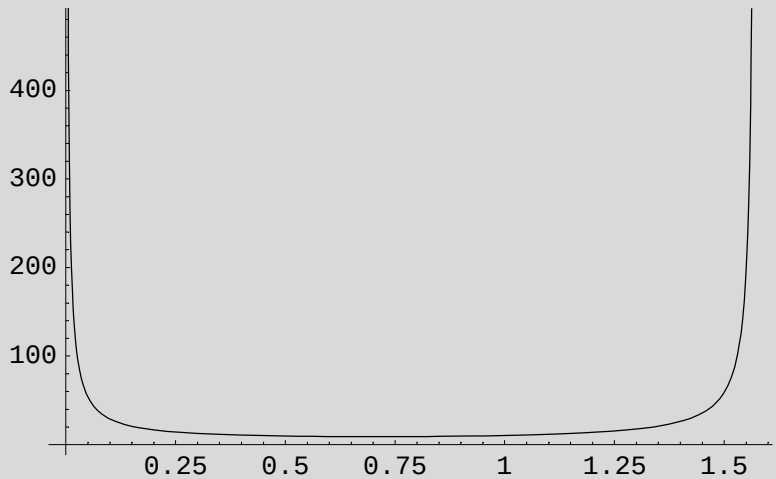
```
Integrate[Sin[x],{x,0,Pi}] - (tr[x] /.x->1.11283)
```

```
0.180294
```

■ b

```
h[α_] := 2.5 + 4 Tan[α];
g[α_] := 1/Tan[α] h[α];
laenge[α_] := Sqrt[h[α]^2+g[α]^2]
```

```
Plot[laenge[α],{α,0,Pi/2}];
```



```
D[laenge[α],α]
```

$$\left(8\,\text{Csc}[\alpha]^2\,(2.5+4\,\text{Tan}[\alpha])+8\,\text{Sec}[\alpha]^2\,(2.5+4\,\text{Tan}[\alpha])-2\,\text{Cot}[\alpha]\,\text{Csc}[\alpha]^2\,(2.5+4\,\text{Tan}[\alpha])^2\right)\Big/\left(2\sqrt{(2.5+4\,\text{Tan}[\alpha])^2+\text{Cot}[\alpha]^2\,(2.5+4\,\text{Tan}[\alpha])^2}\right)$$

```
fr=FindRoot[(Evaluate[D[laenge[α],α]==0] /.α->u),{u,0.75}]
```

$$\{u \to 0.707383\}$$

```
laenge[u] /. fr
```

```
9.10977
```

```
Remove["Global`*"]
```

Lösung zu Aufgabe 5, differenzieren und integrieren

■ a Resultat so kurz wie möglich halten

```
f[x_]:=x^2+ 0.5 Sin[x];
f[x]
```

$$x^2+0.5\,\text{Sin}[x]$$

```
f'[x]
```

$$2\,x+0.5\,\text{Cos}[x]$$

```
f'[x]/.x->1
```

```
2.27015
```

```
ArcTan[f'[x]/.x->1]
```

```
1.15587
```

```
ArcTan[f'[x]/.x->1]/Degree
```

```
66.2265
```

```
f''[x]/.x->1
```

```
1.57926
```

■ b Resultat so kurz wie möglich halten

```
f[x_]:=(((x^(3/4)-3)^3)^(1/4))^5
```

```
f[x]
```

$$\left(\left(-3+x^{3/4}\right)^3\right)^{5/4}$$

```
D[f[x],x]//Simplify
```

$$\frac{45\left(-3+x^{3/4}\right)^2\left(\left(-3+x^{3/4}\right)^3\right)^{1/4}}{16\,x^{1/4}}$$

```
D[f[x],x]//Expand
```

$$\frac{405\left(\left(-3+x^{3/4}\right)^3\right)^{1/4}}{16\,x^{1/4}}-\frac{135}{8}\left(\left(-3+x^{3/4}\right)^3\right)^{1/4}\sqrt{x}+\frac{45}{16}\left(\left(-3+x^{3/4}\right)^3\right)^{1/4}x^{5/4}$$

■ c Resultat so kurz wie möglich halten

```
f[x_]:= 4 x^2 Sin[3x^2-2]
```

```
f[x]
```

$$-4\,x^2\,\text{Sin}[2-3\,x^2]$$

```
D[f[x],x]//Simplify
```

$$24\,x^3\,\text{Cos}[2-3\,x^2]-8\,x\,\text{Sin}[2-3\,x^2]$$

```
D[f[x],x]//Expand
```

$$24\,x^3\,\text{Cos}[2-3\,x^2]-8\,x\,\text{Sin}[2-3\,x^2]$$

■ d Resultat so kurz wie möglich halten

```
f[x_]:= Log[x/(1-x)] (x/(1-x))
```

```
f[x]
```

$$\frac{x\,\text{Log}\left[\frac{x}{1-x}\right]}{1-x}$$

```
D[f[x],x]//Simplify
```

$$\frac{1+\text{Log}\left[\frac{x}{1-x}\right]}{(-1+x)^2}$$

```
D[f[x],x]//Expand
```

$$\frac{1}{1-x}+\frac{x}{(1-x)^2}+\frac{\text{Log}\left[\frac{x}{1-x}\right]}{1-x}+\frac{x\,\text{Log}\left[\frac{x}{1-x}\right]}{(1-x)^2}$$

■ e Resultat so kurz wie möglich halten

```
f[x_]:= 4 E^(-x) Cos[2-E^x]
```

```
f[x]
```

$$4\, e^{-x} \cos[2 - e^{x}]$$

```
D[f[x],x]//Simplify
```

$$-4\, e^{-x} \cos[2 - e^{x}] + 4 \sin[2 - e^{x}]$$

```
D[f[x],x]//Expand
```

$$-4\, e^{-x} \cos[2 - e^{x}] + 4 \sin[2 - e^{x}]$$

■ f Resultat so kurz wie möglich halten

```
Remove[a]
```

```
f[x_]:= x^2+a Sin[x]+2/x
```

```
Integrate[f[x],{x,1,E^2}]
```

$$\frac{1}{3}\left(11 + e^{6} + 3\, a \cos[1] - 3\, a \cos[e^{2}]\right)$$

```
Integrate[f[x],{x,1,E^2}]//Expand
```

$$\frac{11}{3} + \frac{e^{6}}{3} + a \cos[1] - a \cos[e^{2}]$$

```
N[%]//Expand
```

$$138.143 + 0.0919461\, a$$

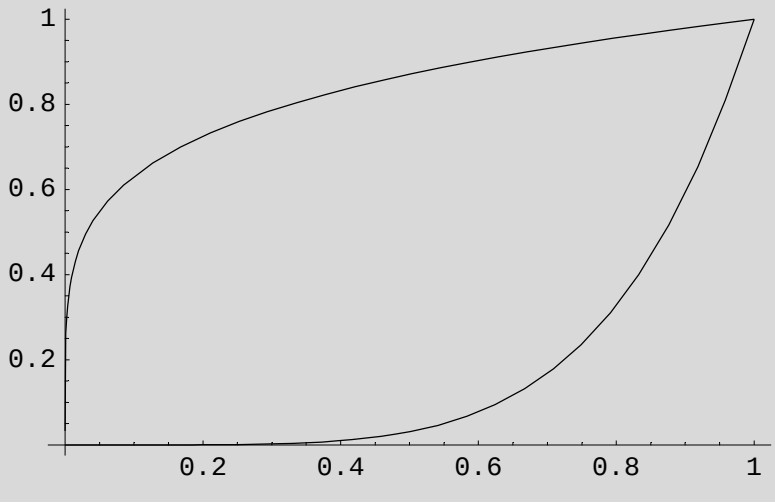
```
Remove["Global`*"]
```

Lösung zu Aufgabe 6, Kurven und Flächen

Gegeben sei die Kurvenschar y=x^a auf I=[0,1]

■ a Plot

```
Plot[{x^5,x^(1/5)},{x,0,1}];
```



■ b y=x^(a) soll die eingeschlossenen Fläche halbieren. Berechne a

```
r1=Evaluate[Integrate[x^5-x^(a),{x,0,1}]==Integrate[x^(a)-x
^(1/5),{x,0,1}]][[1]][[2]];
r2=Evaluate[Integrate[x^5-x^(a),{x,0,1}]==Integrate[x^(a)-x
^(1/5),{x,0,1}]][[2]][[2]];
h=(r1==r2)
```

$$\frac{1}{6} - \frac{1}{1+a} == -\frac{5}{6} + \frac{1}{1+a}$$

```
Solve[h,{a}]
```

$$\{\{a \to 1\}\}$$

```
r1=Evaluate[Integrate[x^5-x^(a),{x,0,1}]==Integrate[x^(a)-x
^(1/5),{x,0,1}]][[1]][[2]];
r2=Evaluate[
Integrate[x^5-x^(a),{x,0,1}]==Integrate[x^(a)-x^(1/5),{x,0,
1}]][[2]][[2]];
h=(r1==3 r2)
```

$$\frac{1}{6} - \frac{1}{1+a} == 3\left(-\frac{5}{6} + \frac{1}{1+a}\right)$$

```
Solve[h,{a}]
```

$$\left\{\left\{a \to \frac{1}{2}\right\}\right\}$$

```
r1=Evaluate[Integrate[x^5-x^(a),{x,0,1}]==Integrate[x^(a)-x
^(1/5),{x,0,1}]][[1]][[2]];
r2=Evaluate[
Integrate[x^5-x^(a),{x,0,1}]==Integrate[x^(a)-x^(1/5),{x,0,
1}]][[2]][[2]];
h=(r1==(Sqrt[5]+1)/2 r2)
```

$$\frac{1}{6} - \frac{1}{1+a} == \frac{1}{2}\left(1+\sqrt{5}\right)\left(-\frac{5}{6} + \frac{1}{1+a}\right)$$

```
solv1=Solve[h,{a}]//Flatten
```

$$\left\{a \to \frac{11+\sqrt{5}}{7+5\sqrt{5}}\right\}$$

```
N[%]
```

$$\{a \to 0.728043\}$$

```
r3=Evaluate[Integrate[x^5-x^(a),{x,0,1}]==Integrate[x^(a)-x
^(1/5),{x,0,1}]][[1]][[2]];
r4=Evaluate[
Integrate[x^5-x^(a),{x,0,1}]==Integrate[x^(a)-x^(1/5),{x,0,
1}]][[2]][[2]];
h=(r3==(Sqrt[5]-1)/2 r2)
```

$$\frac{1}{6}-\frac{1}{1+a}==\frac{1}{2}\left(-1+\sqrt{5}\right)\left(-\frac{5}{6}+\frac{1}{1+a}\right)$$

```
N[%]
```

$$0.166667-\frac{1.}{1.+a}==0.618034\left(-0.833333+\frac{1}{1.+a}\right)$$

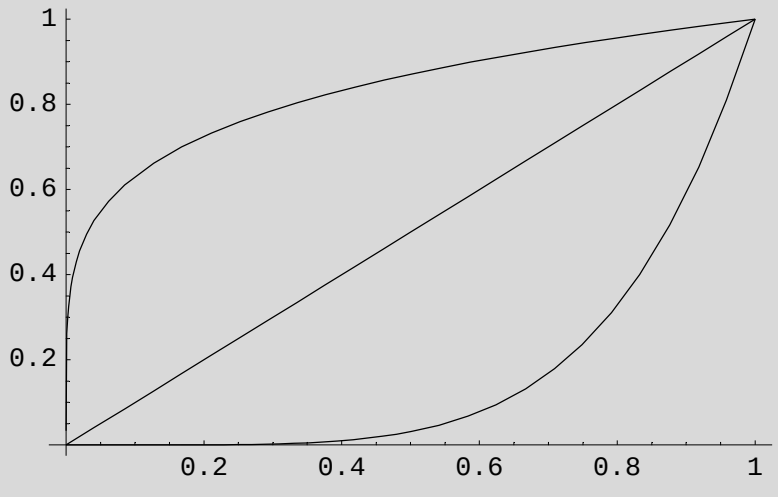
```
solv2=Solve[h,{a}]//Flatten
```

$$\left\{a\to\frac{9+\sqrt{5}}{-3+5\sqrt{5}}\right\}$$

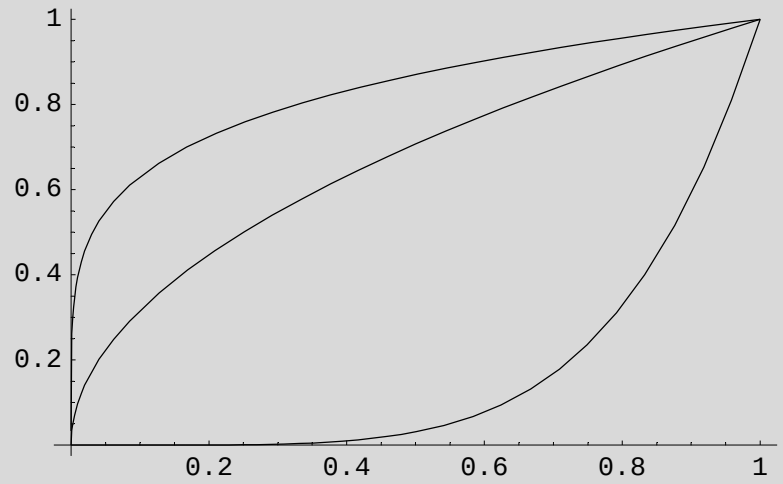
```
N[%]
```

$$\{a\to 1.37355\}$$

```
Plot[{x^5,x^(1/5),x^1},{x,0,1}];
```

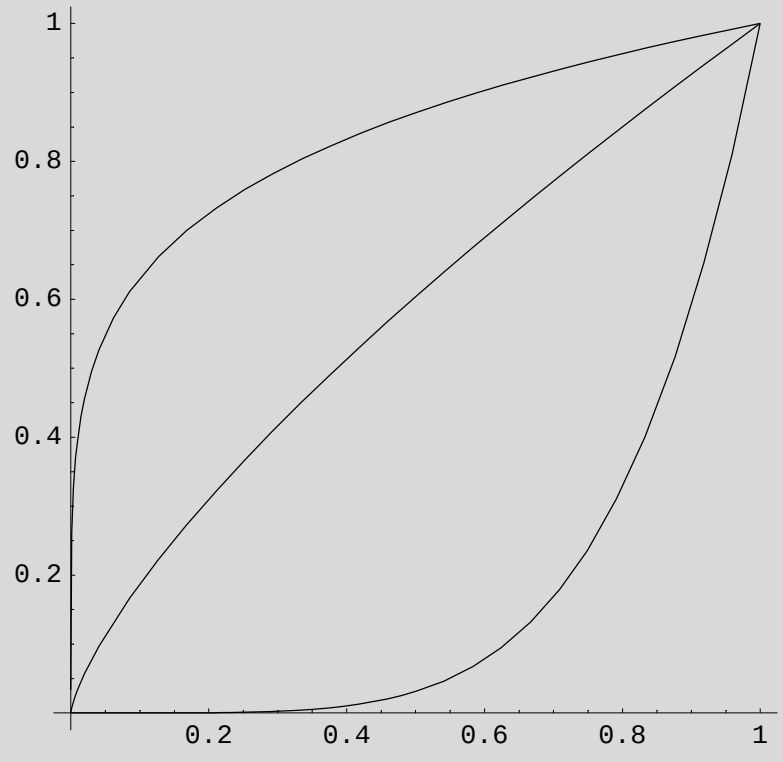


```
Plot[{x^5,x^(1/5),x^(1/2)},{x,0,1}];
```

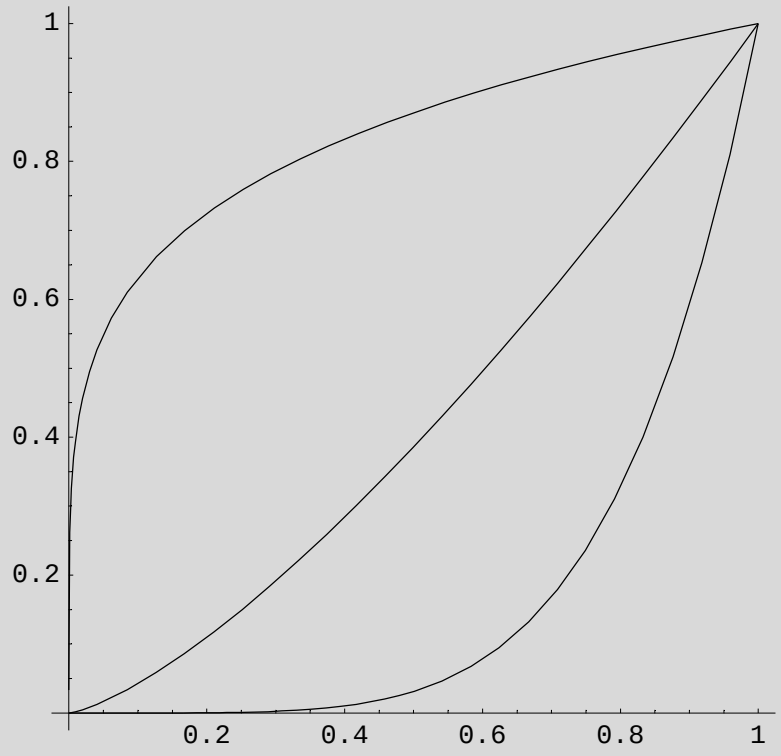


■ Plot Goldener Schnitt

```
Plot[Evaluate[{x^5,x^(1/5),x^(a)/.solv1}],{x,0,1},AspectRat
io->Automatic];
```



```
Plot[Evaluate[{x^5,x^(1/5),x^(a)/.solv2}],{x,0,1},AspectRat
io->Automatic];
```




```
Remove["Global`*"]
```

Lösung zu Aufgabe 7, Rotationskörper, Parabel

Funktion durch Parabel angenähert. Gleiche Ableitung bei -1 und Werte bei -1 und 1, Winkelabweichung bei 1?

```
f[x_]:=(x^2-1) E^(-x);
p[x_,a_,b_,c_]:=a x^2+b x + c
```

```
f'[x]/.x->-1
```

$-2\,e$

```
solv=Solve[Evaluate[{p[-1,a,b,c]==0, p[1,a,b,c]==0,
(D[p[x,a,b,c],x]==f'[x]/.x->-1)}]]//Flatten
```

$$\{a \to e,\ b \to 0,\ c \to -e\}$$

```
%//N
```

$$\{a \to 2.71828,\ b \to 0.,\ c \to -2.71828\}$$

```
p[x_]:=a x^2+b x + c/.solv; p[x]
```

$$-e + e\,x^2$$

```
p'[x]/.x->1
```

$$2\,e$$

```
f'[x]/.x->1
```

$$\frac{2}{e}$$

```
ArcTan[2 E]-ArcTan[2/ E] //N
```

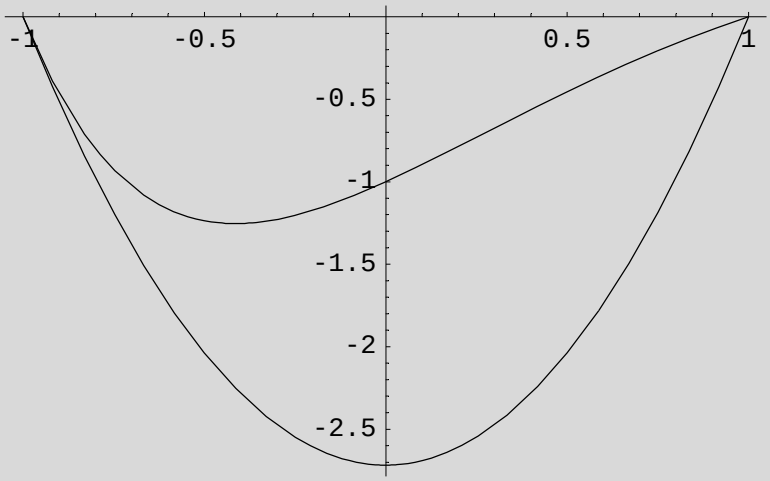
```
0.754566
```

```
(ArcTan[2 E]-ArcTan[2/ E]) /Degree//N
```

```
43.2334
```

■ a Plot

```
Plot[{f[x],p[x]},{x,-1,1}];
```



■ Flächenverhältnis

```
Integrate[(x^2-1) E^(-x),x]
```

$$-e^{-x}\,(1+x)^2$$

```
Integrate[(x^2-1) E^(-x),{x,-1,1}]
```

$$-\frac{4}{e}$$

```
%//N
```

$$-1.47152$$

```
Integrate[p[x],x]
```

$$-e\,x+\frac{e\,x^3}{3}$$

```
Integrate[p[x],{x,-1,1}]
```

$$-\frac{4\,e}{3}$$

```
%//N
```

$$-3.62438$$

```
int=Integrate[(f[x]-p[x]),{x,-1,1}]/Integrate[(p[x]),{x,-1,
1}]
```

$$-\frac{3\left(-\frac{4}{e}+\frac{4\,e}{3}\right)}{4\,e}$$

```
int//N
```

$$-0.593994$$

```
Integrate[(f[x]),{x,-1,1}]/Integrate[(p[x]),{x,-1,1}]
```

$$\frac{3}{e^2}$$

```
%//N
```

$$0.406006$$

```
1/%//N
```

$$2.46302$$

■ Wendepunkt

```
Evaluate[f''[x]]/.x->0
```

```
1
```

```
Solve[Evaluate[f''[x]==0],{x}]
```

```
Solve::ifun :
 Inverse functions are being used by Solve, so some solutions may not
   be found; use Reduce for complete solution information. Mehr…
```

$$\{\{x \to 2 - \sqrt{3}\}, \{x \to 2 + \sqrt{3}\}\}$$

```
Solve::ifun :
 Inverse functions are being used by Solve, so some solutions may not
   be found; use Reduce for complete solution information. Mehr…
```

$$\{\{x \to 2 - \sqrt{3}\}, \{x \to 2 + \sqrt{3}\}\}$$

```
Solve[Evaluate[f''[x]==0],{x}]//N
```

```
Solve::ifun :
 Inverse functions are being used by Solve, so some solutions may not
   be found; use Reduce for complete solution information. Mehr…
```

$$\{\{x \to 0.267949\}, \{x \to 3.73205\}\}$$

■ Rotationsvolunen bei Parabe, Vergleich mit Zylinder gleicher Länge, r = ? (Hat Volumen in Zylinder mit r=2 Platz?)

```
Pi Integrate[p[x]^2,{x,-1,1}]
```

$$\frac{16\, e^2\, \pi}{15}$$

```
%//N
```

```
24.761
```

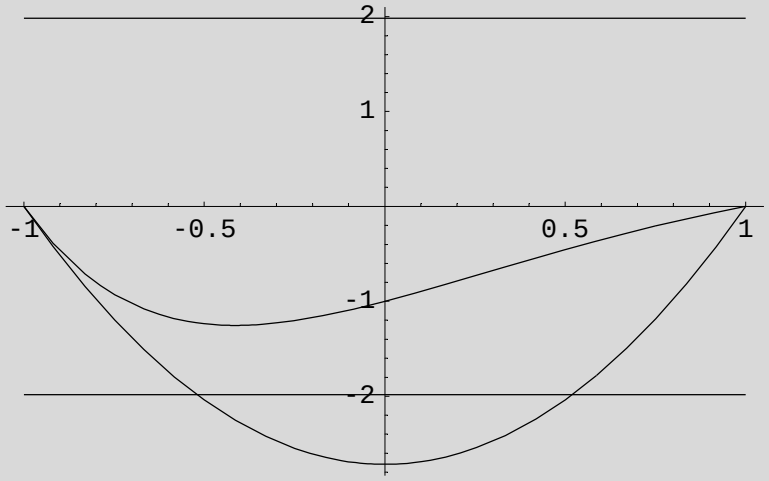
```
Sqrt[Integrate[p[x]^2,{x,-1,1}]/2]
```

$$2\sqrt{\frac{2}{15}}\,e$$

```
%//N
```

1.98515

```
Plot[{f[x],p[x],-1.9851,1.9851},{x,-1,1}];
```



■ Länge der Bögen:

```
{f[x],p[x]}
```

$$\{e^{-x}\,(-1+x^2),\ -e+e\,x^2\}$$

```
l1=NIntegrate[Evaluate[Sqrt[1+D[p[x],x]^2]],{x,-1,1}]
```

5.96824

```
l2=NIntegrate[Evaluate[Sqrt[1+D[f[x],x]^2]],{x,-1,1}]
```

3.37052

```
l3=2
```

```
2
```

```
l1/l2
```

```
1.77072
```

```
l1/l3
```

```
2.98412
```

```
l2/l3
```

```
1.68526
```

```
Remove["Global`*"]
```

Lösung zu Aufgabe 7, Variante: Rotationskörper, Parabel mit Punkteverwechslung

Funktion durch Parabel angenähert. Gleiche Ableitung bei -1 und Werte bei -1 und 1, Winkelabweichung bei 1?

```
f[x_]:=(x^2-1) E^(-x);
p[x_,a_,b_,c_]:=a x^2+b x + c
```

```
f'[x]/.x->1
```

$$\frac{2}{e}$$

```
solv=Solve[Evaluate[{p[-1,a,b,c]==0, p[1,a,b,c]==0,
(D[p[x,a,b,c],x]==f'[x]/.x->1)}]]//Flatten
```

$$\left\{c \to -\frac{1}{e},\ a \to \frac{1}{e},\ b \to 0\right\}$$

```
%//N
```

$$\{c \to -0.367879,\ a \to 0.367879,\ b \to 0.\}$$

```
p[x_]:=a x^2+b x + c/.solv; p[x]
```

$$-\frac{1}{e} + \frac{x^2}{e}$$

```
p'[x]/.x->1
```

$$\frac{2}{e}$$

```
f'[x]/.x->1
```

$$\frac{2}{e}$$

```
ArcTan[2 E]-ArcTan[2/ E] //N
```

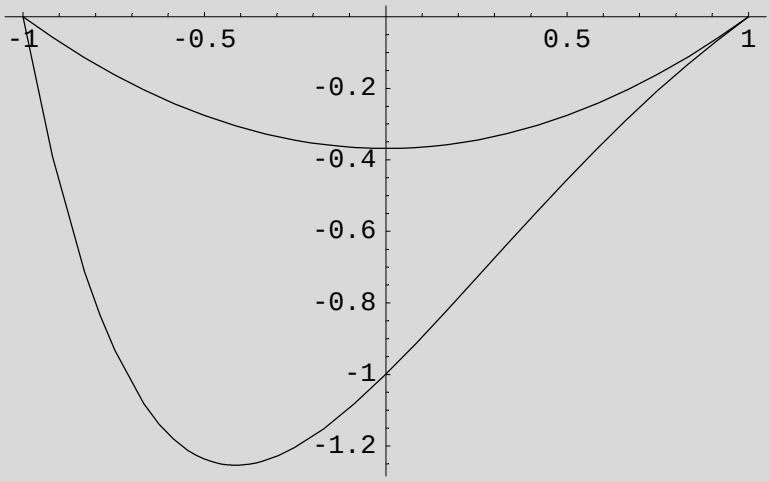
```
0.754566
```

```
(ArcTan[2 E]-ArcTan[2/ E]) /Degree//N
```

```
43.2334
```

■ a Plot

```
Plot[{f[x],p[x]},{x,-1,1}];
```



■ Flächenverhältnis

```
Integrate[(x^2-1) E^(-x),x]
```

$$-e^{-x}\,(1+x)^2$$

```
Integrate[(x^2-1) E^(-x),{x,-1,1}]
```

$$-\frac{4}{e}$$

```
%//N
```

$$-1.47152$$

```
Integrate[p[x],t]
```

$$\frac{t\,(-1+x^2)}{e}$$

```
Integrate[p[x],{x,-1,1}]
```

$$-\frac{4}{3\,e}$$

```
%//N
```

$$-0.490506$$

```
int=Integrate[(f[x]-p[x]),{x,-1,1}]/Integrate[(p[x]),{x,-1,
1}]
```

$$2$$

```
int//N
```

$$2.$$

```
Integrate[(f[x]),{x,-1,1}]/Integrate[(p[x]),{x,-1,1}]
```

$$3$$

```
%//N
```

$$3.$$

```
1/%//N
```

$$0.333333$$

■ Wendepunkt

```
Evaluate[f''[x]]/.x->0
```

$$1$$

```
Solve[Evaluate[f''[x]==0],{x}]
```

```
Solve::ifun :
 Inverse functions are being used by Solve, so some solutions may not
   be found; use Reduce for complete solution information. Mehr…
```

$$\{\{x \to 2 - \sqrt{3}\}, \{x \to 2 + \sqrt{3}\}\}$$

```
Solve[Evaluate[f''[x]==0],{x}]//N
```

```
Solve::ifun :
 Inverse functions are being used by Solve, so some solutions may not
   be found; use Reduce for complete solution information. Mehr…
```

$$\{\{x \to 0.267949\}, \{x \to 3.73205\}\}$$

■ Rotationsvolunen bei Parabe, Vergleich mit Zylinder gleicher Länge, r = ? (Hat Volumen in Zylinder mit r=2 Platz?)

```
Pi Integrate[p[x]^2,{x,-1,1}]
```

$$\frac{16\,\pi}{15\,e^2}$$

```
%//N
```

```
0.453513
```

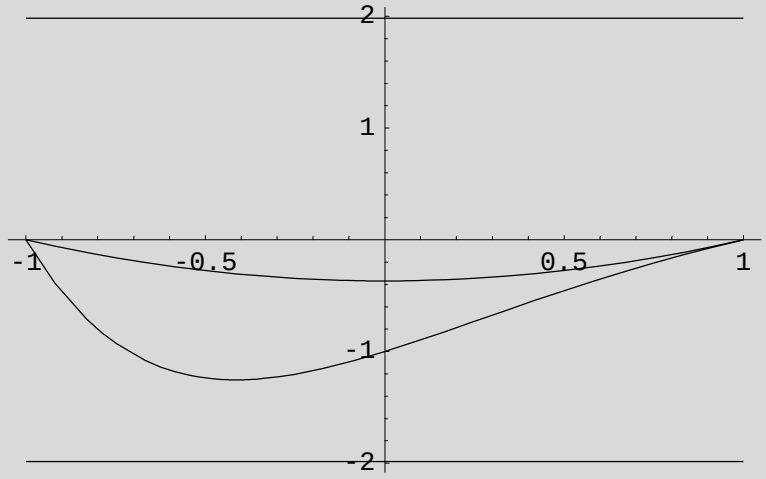
```
Sqrt[Integrate[p[x]^2,{x,-1,1}]/2]
```

$$\frac{2\sqrt{\frac{2}{15}}}{e}$$

```
%//N
```

```
0.268661
```

```
Plot[{f[x],p[x],-1.9851,1.9851},{x,-1,1}];
```



■ Länge der Bögen:

```
{f[x],p[x]}
```

$$\left\{e^{-x}\,(-1+x^2),\ -\frac{1}{e}+\frac{x^2}{e}\right\}$$

```
l1=NIntegrate[Evaluate[Sqrt[1+D[p[x],x]^2]],{x,-1,1}]
```

```
2.16805
```

```
l2=NIntegrate[Evaluate[Sqrt[1+D[f[x],x]^2]],{x,-1,1}]
```

```
3.37052
```

```
l3=2
```

```
2
```

```
l1/l2
```

```
0.643241
```

```
l1/l3
```

```
1.08403
```

```
l2/l3
```

```
1.68526
```

```
Remove["Global`*"]
```

Lösung zu Aufgabe 8, Annäherung Hügel durch Potenzreihe

Funktion durch Parabel angenähert. Gleiche Ableitung bei -1 und Werte bei -1 und 1, Winkelabweichung bei 1?

■ Test

```
Remove["Global`*"]
```

– *Remove::rmnsm : There are no symbols matching "Global`*". Mehr…*

```
q[x_,a_,b_,c_,d_,e_]:=(a x^2+b x + c)/(x^2+d x+e)
```

```
q[x,a,b,c,d,e]//TeXForm
```

```
\frac{a x^2+b x+c}{x^2+d x+e}
```

```
sys={
q[-2,a,b,c,d,e]==0,
q[2,a,b,c,d,e]==0,
q[0,a,b,c,d,e]==2,
(D[q[x,a,b,c,d,e],x]/.x->1)==-1,
(D[q[x,a,b,c,d,e],x]/.x->-1)==1}
```

$$\left\{\frac{4\,a-2\,b+c}{4-2\,d+e}==0,\ \frac{4\,a+2\,b+c}{4+2\,d+e}==0,\ \frac{c}{e}==2,\ -\frac{(a+b+c)\,(2+d)}{(1+d+e)^2}+\frac{2\,a+b}{1+d+e}==-1,\ -\frac{(a-b+c)\,(-2+d)}{(1-d+e)^2}+\frac{-2\,a+b}{1-d+e}==1\right\}$$

```
solv=Solve[Evaluate[sys,{a,b,c,d,e}]]
```

$$\left\{\left\{b\to 0,\ c\to 1,\ a\to -\frac{1}{4},\ d\to 0,\ e\to \frac{1}{2}\right\},\ \{b\to 0,\ c\to 8,\ a\to -2,\ d\to -\sqrt{7},\ e\to 4\},\ \{b\to 0,\ c\to 8,\ a\to -2,\ d\to \sqrt{7},\ e\to 4\}\right\}$$

```
q[x_]:=(a x^2+b x + c)/(x^2+d x+e)/.solv ; q[x]
```

$$\left\{\frac{1-\frac{x^2}{4}}{\frac{1}{2}+x^2},\ \frac{8-2\,x^2}{4-\sqrt{7}\,x+x^2},\ \frac{8-2\,x^2}{4+\sqrt{7}\,x+x^2}\right\}$$

■ Symmetrie

```
Remove["Global`*"]
```

```
q[x_,a_,c_,e_]:=(a x^2+ c)/(x^2+e)
```

```
sys={
q[2,a,c,e]==0,
q[0,a,c,e]==2,
(D[q[x,a,c,e],x]/.x->1)==-1}
```

$$\left\{\frac{4\,a+c}{4+e}==0,\ \frac{c}{e}==2,\ -\frac{2\,(a+c)}{(1+e)^2}+\frac{2\,a}{1+e}==-1\right\}$$

```
solv=Solve[Evaluate[sys,{a,c,e}]]//Flatten
```

$$\left\{c \to 1,\ a \to -\frac{1}{4},\ e \to \frac{1}{2}\right\}$$

```
q[x_]:=(a x^2+ c)/(x^2+e)/.solv ; q[x]
```

$$\frac{1-\frac{x^2}{4}}{\frac{1}{2}+x^2}$$

```
q[x]//Simplify
```

$$\frac{4-x^2}{2+4\,x^2}$$

■ Konvergenzbereich: Partialbruchzerlegung, Vergleich mit geometrischer Reihe! ==> r = 1

```
q1[x_]:= Evaluate[q[x]//Apart]; q1[x]
```

$$-\frac{1}{4}+\frac{9}{4\,(1+2\,x^2)}$$

```
Table[D[q1[x],{x,k}],{k,0,10}]//Together
```

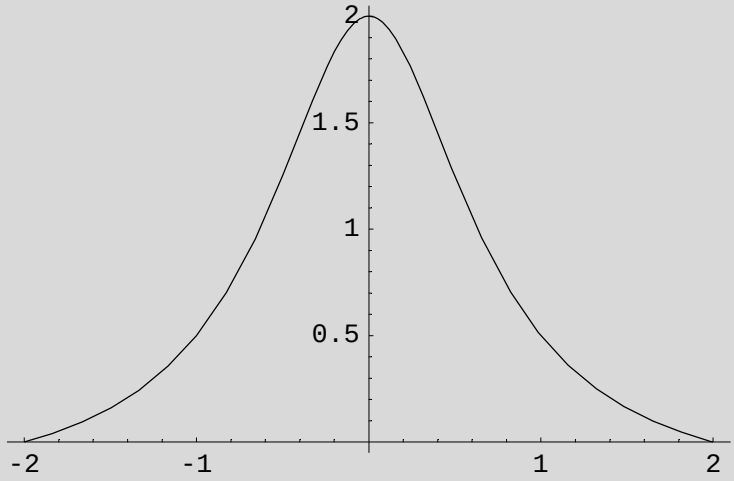
$$\left\{\frac{4-x^2}{2\,(1+2\,x^2)},\ -\frac{9\,x}{(1+2\,x^2)^2},\ \frac{9\,(-1+6\,x^2)}{(1+2\,x^2)^3},\right.$$
$$-\frac{216\,(-x+2\,x^3)}{(1+2\,x^2)^4},\ \frac{216\,(1-20\,x^2+20\,x^4)}{(1+2\,x^2)^5},$$
$$-\frac{4320\,(3\,x-20\,x^3+12\,x^5)}{(1+2\,x^2)^6},\ \frac{12960\,(-1+42\,x^2-140\,x^4+56\,x^6)}{(1+2\,x^2)^7},$$
$$-\frac{1451520\,(-x+14\,x^3-28\,x^5+8\,x^7)}{(1+2\,x^2)^8},$$
$$\frac{1451520\,(1-72\,x^2+504\,x^4-672\,x^6+144\,x^8)}{(1+2\,x^2)^9},$$
$$\left.-\frac{52254720\,(5\,x-120\,x^3+504\,x^5-480\,x^7+80\,x^9)}{(1+2\,x^2)^{10}},\ \frac{1}{(1+2\,x^2)^{11}}\right.$$
$$\left.(261273600\,(-1+110\,x^2-1320\,x^4+3696\,x^6-2640\,x^8+352\,x^{10}))\right\}$$

```
Table[D[q1[x],{x,k}],{k,0,10}]/.x->1.
```

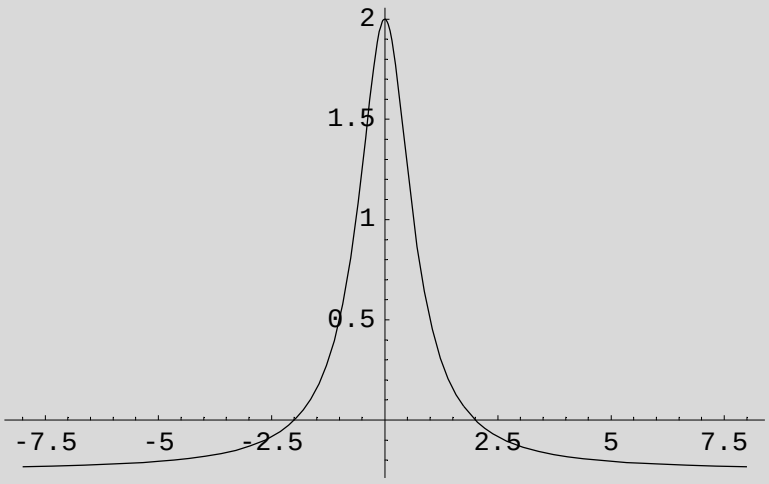
```
{0.5, -1., 1.66667, -2.66667, 0.888889, 29.6296,
 -254.815, 1548.64, -7005.76, 9734.32, 290555.}
```

```
q1[x_]:=Evaluate[q[x]]
```

```
Plot[q1[x],{x,-2,2}];
```




```
Plot[q1[x],{x,-8,8}];
```



■ Potenzreihe, Abweichung

```
q2[x_]:=Normal[Series[q1[z],{z,0,10}]]/.z->x; q2[x]
```

$$2 - \frac{9\,x^2}{2} + 9\,x^4 - 18\,x^6 + 36\,x^8 - 72\,x^{10}$$

```
q2[x_]:=Evaluate[Normal[Series[q1[x],{x,0,10}]]]; q2[x]
```

$$2 - \frac{9\,x^2}{2} + 9\,x^4 - 18\,x^6 + 36\,x^8 - 72\,x^{10}$$

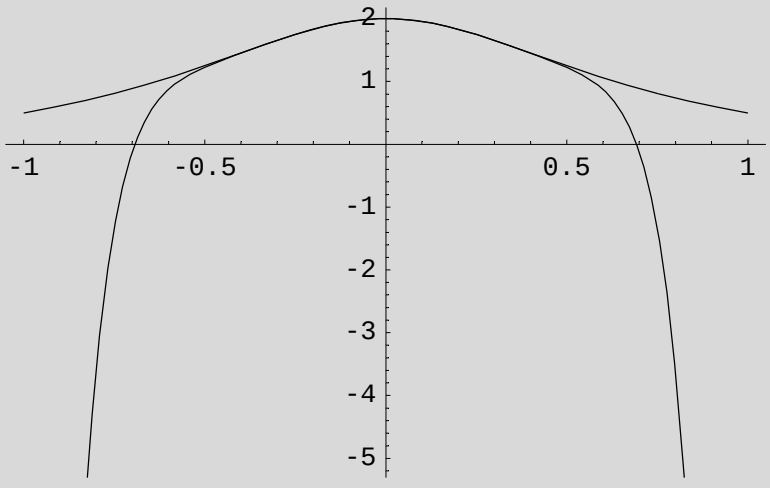
```
{q1[x],q2[x]}
```

$$\left\{\frac{1-\frac{x^2}{4}}{\frac{1}{2}+x^2},\ 2 - \frac{9\,x^2}{2} + 9\,x^4 - 18\,x^6 + 36\,x^8 - 72\,x^{10}\right\}$$

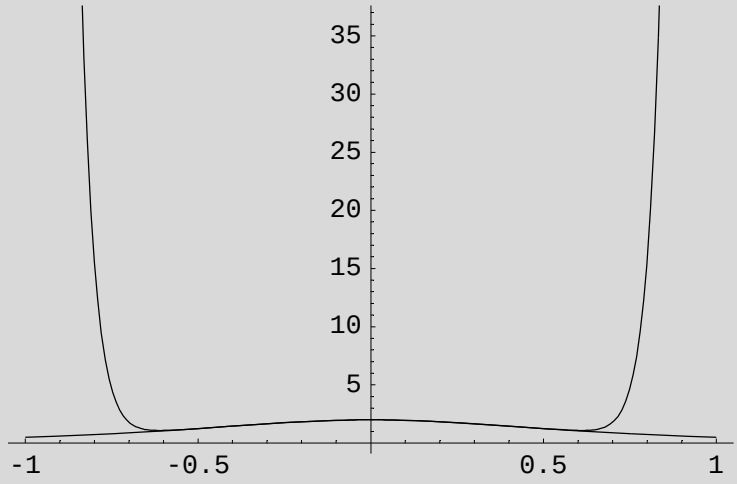
```
{q1[5],q2[5]}//N
```

$$\{-0.205882,\ -6.89338\times 10^8\}$$

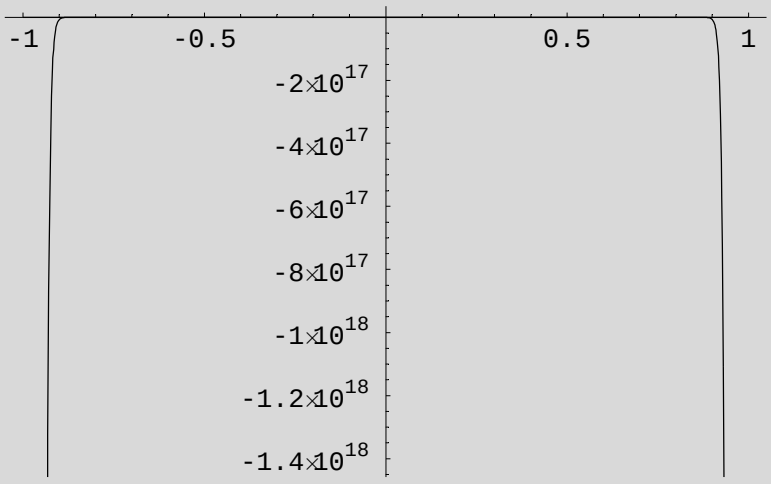
```
Plot[{q1[x],q2[x]},{x,-1,1}];
```



```
q2[x_]:=Evaluate[Normal[Series[q1[x],{x,0,20}]]];
Plot[{q1[x],q2[x]},{x,-1,1}];
```




```
q2[x_]:=Evaluate[Normal[Series[q1[x],{x,0,150}]]];
Plot[{q1[x],q2[x]},{x,-1,1}];
```




```
q2[x_]:=Evaluate[Normal[Series[q1[x],{x,0,10}]]];
d1=q2[x]/.x->0.5
```

```
1.22656
```

```
d2=q[x]/.x->0.5
```

```
1.25
```

```
d2-d1
```

```
0.0234375
```

```
q[x]-q2[x]/.x->0.9
```

```
15.5228
```

```
Remove["Global`*"]
```

Lösung zu Aufgabe 9, Reihe, Maximum Geometrie

■ a

```
Normal[Series[ArcTan[x],{x,0,10}]]
```

$$x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \frac{x^9}{9}$$

```
4(Normal[Series[ArcTan[x],{x,0,10}]]/.x->1)//N
```

```
3.33968
```

```
4(Normal[Series[ArcTan[x],{x,0,2000}]]/.x->1)//N
```

```
3.14059
```

■ b

10m : 2m

Breite 10 m

Höhe: x : 2 = (x+10) : h, l = f[x]

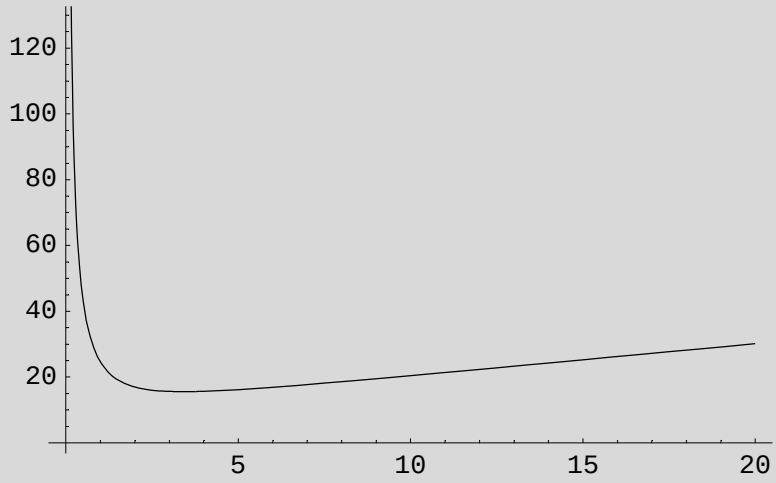
```
Solve[ x / 2 == (x+10) / h,{h}]
```

$$\left\{\left\{h \to \frac{2\,(10+x)}{x}\right\}\right\}$$

```
f[x_] := Sqrt[(2 (10 + x)/x)^2 + (x + 10)^2]; f[x]
```

$$\sqrt{(10+x)^2 + \frac{4\,(10+x)^2}{x^2}}$$

```
Plot[f[x],{x,0,20}];
```




```
f'[x]
```

$$\frac{2\,(10+x) + \frac{8\,(10+x)}{x^2} - \frac{8\,(10+x)^2}{x^3}}{2\sqrt{(10+x)^2 + \frac{4\,(10+x)^2}{x^2}}}$$

```
Solve[Evaluate[f'[x]==0],{x}]
```

$$\left\{\{x \to -2\,(-5)^{1/3}\},\ \{x \to 2\,5^{1/3}\},\ \{x \to 2\,(-1)^{2/3}\,5^{1/3}\}\right\}$$

```
Solve[Evaluate[f'[x]==0],{x}]//N
```

$$\{\{x \to -1.70998 - 2.96177\,i\},\ \{x \to 3.41995\},\ \{x \to -1.70998 + 2.96177\,i\}\}$$

```
(-5)^(1/3) // N
```

```
0.854988 + 1.48088 i
```

```
(-1)^(2/3) // N
```

```
-0.5 + 0.866025 i
```

```
((-1)^(2/3))^2 // N
```

```
-0.5 - 0.866025 i
```

```
((-1)^(2/3))^3 // N
```

```
1.
```

■ c

```
tan1=49/10
```

$$\frac{49}{10}$$

```
weite=2/tan1 -0.7
```

```
-0.291837
```

```
Remove["Global`*"]
```

Lösung zu Aufgabe 10

■ a) a

```
f[x_]:= -2(6x^3 + 5x^2 - 6x^1 + x^0)/(-x^2+2x+1)
```

```
f'[x]//Expand
```

$$\frac{4}{(1+2x-x^2)^2}-\frac{28x}{(1+2x-x^2)^2}+\frac{44x^2}{(1+2x-x^2)^2}+\frac{4x^3}{(1+2x-x^2)^2}-\frac{24x^4}{(1+2x-x^2)^2}+\frac{12}{1+2x-x^2}-\frac{20x}{1+2x-x^2}-\frac{36x^2}{1+2x-x^2}$$

```
f'[x]//Simplify
```

$$\frac{4(4-6x-11x^2-12x^3+3x^4)}{(-1-2x+x^2)^2}$$

```
f'[x]/.x->0
```

```
16
```

```
ArcTan[(f'[x]/.x->0)]
```

```
ArcTan[16]
```

```
ArcTan[16] // N
```

```
1.50838
```

```
ArcTan[(f'[x]/.x->0)]//N
```

```
1.50838
```

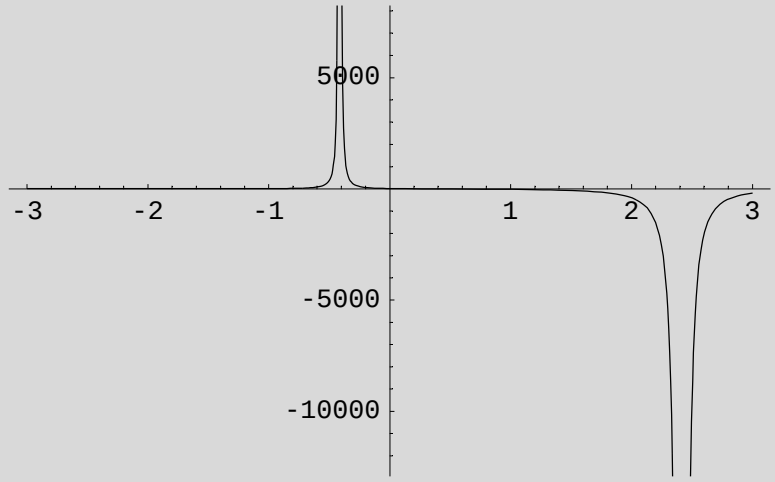
```
Pi/ Degree//N
```

```
180.
```

```
N[ArcTan[(f'[x]/.x->0)]]/(1 Degree)
```

```
86.4237
```

```
Plot[Evaluate[f'[x]],{x,-3,3}];
```




```
Evaluate[f'[x]==0]
```

$$-\frac{2\,(-6+10\,x+18\,x^2)}{1+2\,x-x^2}+\frac{2\,(2-2\,x)\,(1-6\,x+5\,x^2+6\,x^3)}{(1+2\,x-x^2)^2}==0$$

```
Solve[Evaluate[f'[x]==0],{x}]//Simplify
```

$$\left\{\left\{x \to 1-\frac{1}{6}\sqrt{\left(58+7^{2/3}\left(331-234\sqrt{2}\right)^{1/3}+7^{2/3}\left(331+234\sqrt{2}\right)^{1/3}\right)}-\frac{1}{6}\,i\sqrt{\left(-116+7^{2/3}\left(331-234\sqrt{2}\right)^{1/3}+7^{2/3}\left(331+234\sqrt{2}\right)^{1/3}+936\Big/\left(\sqrt{\left(58+7^{2/3}\left(331-234\sqrt{2}\right)^{1/3}+7^{2/3}\left(331+234\sqrt{2}\right)^{1/3}\right)}\right)\right)}\right\},\right.$$
$$\left\{x \to 1-\frac{1}{6}\sqrt{\left(58+7^{2/3}\left(331-234\sqrt{2}\right)^{1/3}+7^{2/3}\left(331+234\sqrt{2}\right)^{1/3}\right)}+\frac{1}{6}\,i\sqrt{\left(-116+7^{2/3}\left(331-234\sqrt{2}\right)^{1/3}+7^{2/3}\left(331+234\sqrt{2}\right)^{1/3}+936\Big/\left(\sqrt{\left(58+7^{2/3}\left(331-234\sqrt{2}\right)^{1/3}+7^{2/3}\left(331+234\sqrt{2}\right)^{1/3}\right)}\right)\right)}\right\},$$
$$\left\{x \to \frac{1}{6}\left(6+\sqrt{\left(58+7^{2/3}\left(331-234\sqrt{2}\right)^{1/3}+7^{2/3}\left(331+234\sqrt{2}\right)^{1/3}\right)}-\sqrt{\left(116-7^{2/3}\left(331-234\sqrt{2}\right)^{1/3}-7^{2/3}\left(331+234\sqrt{2}\right)^{1/3}+936\Big/\left(\sqrt{\left(58+7^{2/3}\left(331-234\sqrt{2}\right)^{1/3}+7^{2/3}\left(331+234\sqrt{2}\right)^{1/3}\right)}\right)\right)}\right)\right\},$$
$$\left.\left\{x \to \frac{1}{6}\left(6+\sqrt{\left(58+7^{2/3}\left(331-234\sqrt{2}\right)^{1/3}+7^{2/3}\left(331+234\sqrt{2}\right)^{1/3}\right)}+\sqrt{\left(116-7^{2/3}\left(331-234\sqrt{2}\right)^{1/3}-7^{2/3}\left(331+234\sqrt{2}\right)^{1/3}+936\Big/\left(\sqrt{\left(58+7^{2/3}\left(331-234\sqrt{2}\right)^{1/3}+7^{2/3}\left(331+234\sqrt{2}\right)^{1/3}\right)}\right)\right)}\right)\right\}\right\}$$

```
Solve[f'[x]==0,{x}]//N//Chop
```

```
{{x → -0.593629 - 0.652289 i},
 {x → -0.593629 + 0.652289 i}, {x → 0.354691}, {x → 4.83257}}
```

■ a) c

```
f[x_]:= -2(6x^3 + 5x^2 - 6x^1 + x^0)/(x^2-2x+1)
```

```
f'[x]//Expand
```

$$-\frac{4}{(1-2x+x^2)^2}+\frac{28x}{(1-2x+x^2)^2}-\frac{44x^2}{(1-2x+x^2)^2}-\frac{4x^3}{(1-2x+x^2)^2}+\frac{24x^4}{(1-2x+x^2)^2}+\frac{12}{1-2x+x^2}-\frac{20x}{1-2x+x^2}-\frac{36x^2}{1-2x+x^2}$$

```
f'[x]//Simplify
```

$$-\frac{4(2-2x-9x^2+3x^3)}{(-1+x)^3}$$

```
f'[x]/.x->0
```

```
8
```

```
ArcTan[(f'[x]/.x->0)]
```

```
ArcTan[8]
```

```
ArcTan[8] // N
```

```
1.44644
```

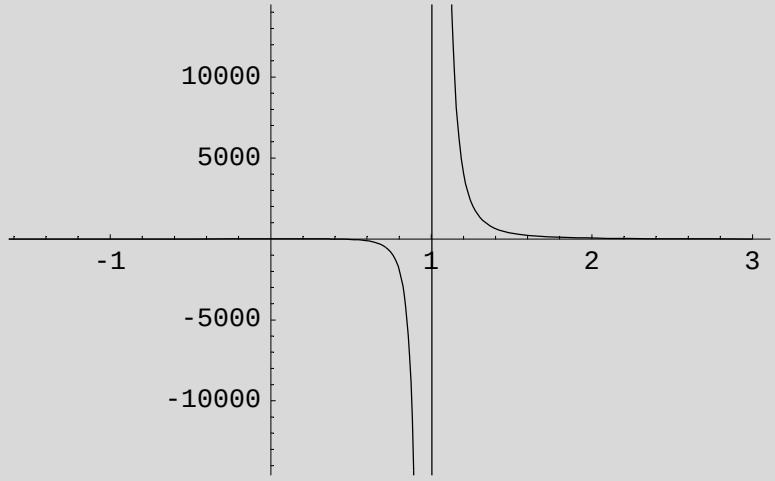
```
Pi/ Degree//N
```

```
180.
```

```
1.44644/ Degree
```

```
82.8749
```

```
Plot[Evaluate[f'[x]],{x,-3,3}];
```




```
Evaluate[f'[x]==0]
```

$$-\frac{2\,(-6+10\,x+18\,x^2)}{1-2\,x+x^2}+\frac{2\,(-2+2\,x)\,(1-6\,x+5\,x^2+6\,x^3)}{(1-2\,x+x^2)^2}==0$$

```
Solve[Evaluate[f'[x]==0],{x}]
```

$$\left\{\left\{x\to 1+\frac{11}{3\left(27+i\sqrt{602}\right)^{1/3}}+\frac{1}{3}\left(27+i\sqrt{602}\right)^{1/3}\right\},\right.$$

$$\left\{x\to 1-\frac{11\left(1+i\sqrt{3}\right)}{6\left(27+i\sqrt{602}\right)^{1/3}}-\frac{1}{6}\left(1-i\sqrt{3}\right)\left(27+i\sqrt{602}\right)^{1/3}\right\},$$

$$\left.\left\{x\to 1-\frac{11\left(1-i\sqrt{3}\right)}{6\left(27+i\sqrt{602}\right)^{1/3}}-\frac{1}{6}\left(1+i\sqrt{3}\right)\left(27+i\sqrt{602}\right)^{1/3}\right\}\right\}$$

```
Solve[f'[x]==0,{x}]//N//Chop
```

```
{{x → 3.14459}, {x → -0.538373}, {x → 0.393787}}
```

■ a) x

```
f[x_]:= -2(6x^3 + 5x^2 - 8x^1 + x^0)/(x^2-2x+1)+x
```

```
f'[x]//Expand
```

$$1-\frac{4}{(1-2x+x^2)^2}+\frac{36x}{(1-2x+x^2)^2}-\frac{52x^2}{(1-2x+x^2)^2}-\frac{4x^3}{(1-2x+x^2)^2}+\frac{24x^4}{(1-2x+x^2)^2}+\frac{16}{1-2x+x^2}-\frac{20x}{1-2x+x^2}-\frac{36x^2}{1-2x+x^2}$$

```
f'[x]//Simplify
```

$$\frac{-13+7x+33x^2-11x^3}{(-1+x)^3}$$

```
f'[x]/.x->0
```

```
13
```

```
ArcTan[(f'[x]/.x->0)]
```

```
ArcTan[13]
```

```
ArcTan[8] // N
```

```
1.44644
```

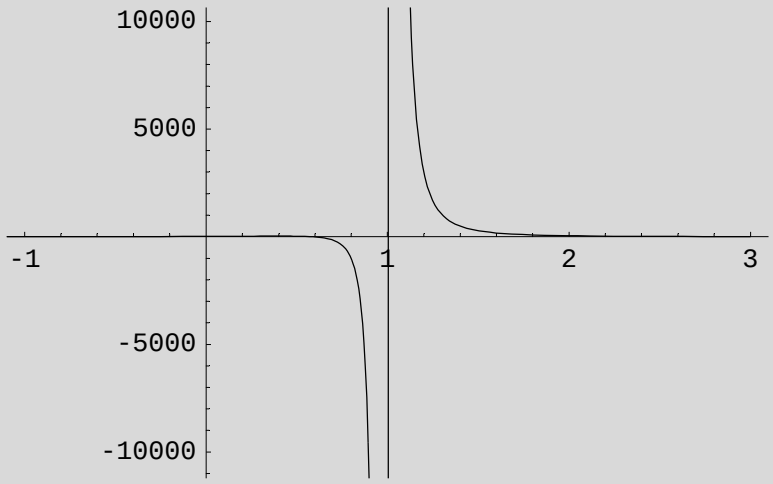
```
Pi/ Degree//N
```

```
180.
```

```
1.44644/ Degree
```

```
82.8749
```

```
Plot[Evaluate[f'[x]],{x,-3,3}];
```




```
Evaluate[f'[x]==0]
```

$$1-\frac{2\,(-8+10\,x+18\,x^2)}{1-2\,x+x^2}+\frac{2\,(-2+2\,x)\,(1-8\,x+5\,x^2+6\,x^3)}{(1-2\,x+x^2)^2}==0$$

```
Solve[Evaluate[f'[x]==0],{x}]//Simpify
```

General::spell1 : Possible spelling error: new symbol name "Simpify" is similar to existing symbol "Simplify". Mehr…

$$\text{Simpify}\Big[\Big\{\Big\{x\to 1+\frac{2\left(99+i\sqrt{23199}\right)^{1/3}}{33^{2/3}}+\frac{20}{\left(33\left(99+i\sqrt{23199}\right)\right)^{1/3}}\Big\},\ \Big\{x\to 1-\frac{\left(1+i\sqrt{3}\right)\left(99+i\sqrt{23199}\right)^{1/3}}{33^{2/3}}-\frac{10\left(1-i\sqrt{3}\right)}{\left(33\left(99+i\sqrt{23199}\right)\right)^{1/3}}\Big\},\ \Big\{x\to 1-\frac{\left(1-i\sqrt{3}\right)\left(99+i\sqrt{23199}\right)^{1/3}}{33^{2/3}}-\frac{10\left(1+i\sqrt{3}\right)}{\left(33\left(99+i\sqrt{23199}\right)\right)^{1/3}}\Big\}\Big\}\Big]$$

```
Solve[f'[x]==0,{x}]//N//Chop
```

$$\{\{x\to 3.08206\},\ \{x\to 0.579562\},\ \{x\to -0.661622\}\}$$

■ b) a, c

```
f[x_]:= Sin[x] E^(x)-Cos[E^(-x)]-(x^3)/(x+1)
```

```
f'[x]//Expand
```

$$\frac{x^3}{(1+x)^2}-\frac{3x^2}{1+x}+e^x\,\text{Cos}[x]-e^{-x}\,\text{Sin}[e^{-x}]+e^x\,\text{Sin}[x]$$

```
((f'[x]//Expand)[[1]]+(f'[x]//Expand)[[2]])//Together
```

$$\frac{-3x^2-2x^3}{(1+x)^2}$$

```
(f'[x]/.x->1.0)//N
```

```
2.37375
```

```
f''[x]
```

$$-\frac{2x^3}{(1+x)^3}+\frac{6x^2}{(1+x)^2}-\frac{6x}{1+x}+e^{-2x}\,\text{Cos}[e^{-x}]+2\,e^x\,\text{Cos}[x]+e^{-x}\,\text{Sin}[e^{-x}]$$

```
f''[x]//Simplify
```

$$-\frac{2x^3}{(1+x)^3}+\frac{6x^2}{(1+x)^2}-\frac{6x}{1+x}+e^{-2x}\,\text{Cos}[e^{-x}]+2\,e^x\,\text{Cos}[x]+e^{-x}\,\text{Sin}[e^{-x}]$$

```
((f''[x]//Expand)[[1]]+(f''[x]//Expand)[[2]]+(f''[x]//Expan
d)[[3]])//Together
```

$$-\frac{2\,(3x+3x^2+x^3)}{(1+x)^3}$$

```
Limit[Evaluate[f''[x]],x->Infinity]
```

```
Interval[{-∞, ∞}]
```

■ b) x

```
f[x_]:= Sin[x] E^( x)-Cos[Log[x]]-(x^3)/(x+1)
```

```
f'[x]//Expand
```

$$\frac{x^3}{(1+x)^2} - \frac{3x^2}{1+x} + e^x \operatorname{Cos}[x] + e^x \operatorname{Sin}[x] + \frac{\operatorname{Sin}[\operatorname{Log}[x]]}{x}$$

```
((f'[x]//Expand)[[1]]+(f'[x]//Expand)[[2]])//Together
```

$$\frac{-3x^2 - 2x^3}{(1+x)^2}$$

```
(f'[x]/.x->1.0)//N
```

```
2.50605
```

```
f''[x]
```

$$-\frac{2x^3}{(1+x)^3} + \frac{6x^2}{(1+x)^2} - \frac{6x}{1+x} + 2e^x \operatorname{Cos}[x] + \frac{\operatorname{Cos}[\operatorname{Log}[x]]}{x^2} - \frac{\operatorname{Sin}[\operatorname{Log}[x]]}{x^2}$$

```
f''[x]//Simplify
```

$$2e^x \operatorname{Cos}[x] + \frac{1}{x^2 (1+x)^3} \left(-2x^3 (3 + 3x + x^2) + (1+x)^3 \operatorname{Cos}[\operatorname{Log}[x]] - (1+x)^3 \operatorname{Sin}[\operatorname{Log}[x]]\right)$$

```
((f''[x]//Expand)[[1]]+(f''[x]//Expand)[[2]]+(f''[x]//Expan
d)[[3]])//Together
```

$$-\frac{2\,(3\,x+3\,x^2+x^3)}{(1+x)^3}$$

```
Limit[Evaluate[f''[x]],x->Infinity]
```

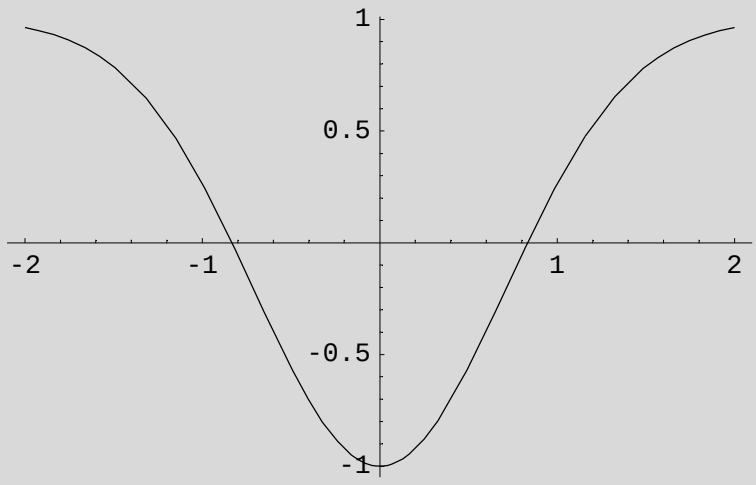
```
Interval[{-∞, ∞}]
```

```
Remove["Global`*"]
```

Lösung zu Aufgabe 11

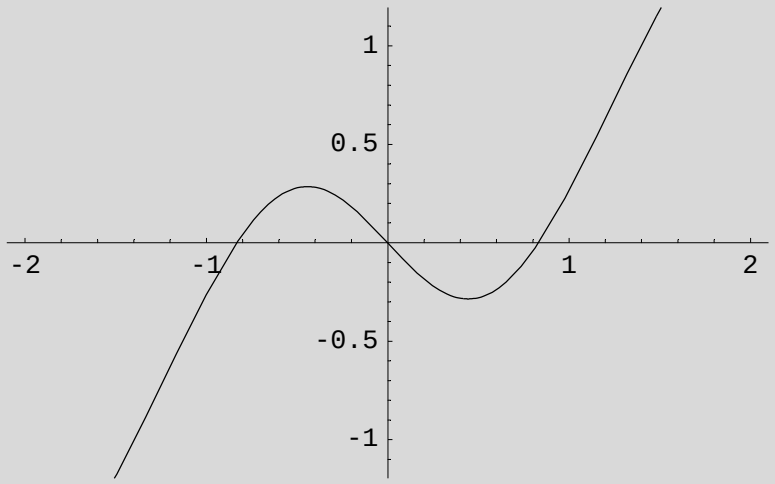
■ **a**

```
f[x_]:=-2 E^(-x^2)+1;
Plot[f[x],{x,-2,2}];
```




```
A[x_]:=f[x] 2x/2
```

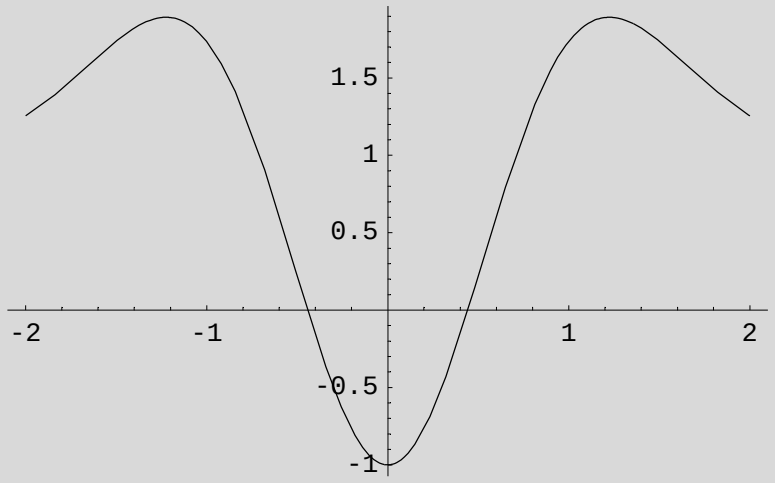
```
Plot[A[x],{x,-2,2}];
```




```
A'[x]
```

$$1 - 2\, e^{-x^2} + 4\, e^{-x^2} x^2$$

```
Plot[A'[x],{x,-2,2}];
```



```
Solve[Evaluate[A'[x]==0],{x}]
```

– *InverseFunction::ifun : Inverse functions are being used. Values may be lost for multivalued inverses.* Mehr…

– *Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.* Mehr…

$$\left\{\left\{x \to -\sqrt{\frac{1}{2}\left(1 - 2\,\text{ProductLog}\left[\frac{\sqrt{e}}{4}\right]\right)}\right\},\ \left\{x \to \sqrt{\frac{1}{2}\left(1 - 2\,\text{ProductLog}\left[\frac{\sqrt{e}}{4}\right]\right)}\right\}\right\}$$

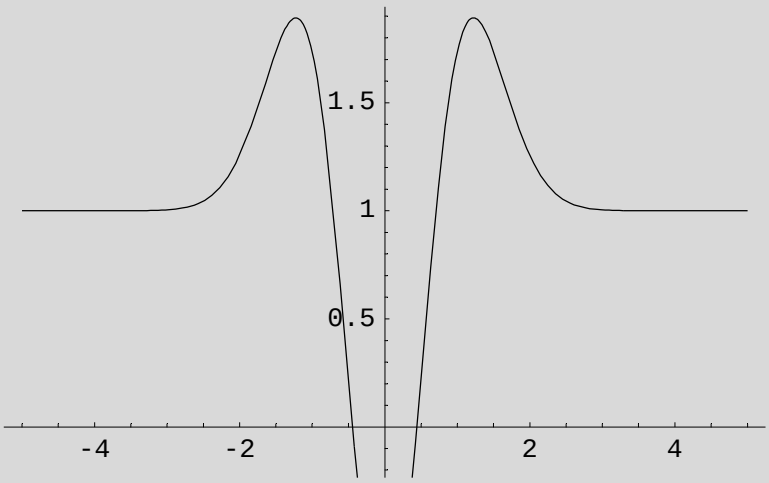
```
Solve[Evaluate[A'[x]==0],{x}]//N
```

– *InverseFunction::ifun : Inverse functions are being used. Values may be lost for multivalued inverses.* Mehr…

– *Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.* Mehr…

```
{{x → -0.442605}, {x → 0.442605}}
```

```
Plot[Evaluate[A'[x]],{x,-5,5}];
```




```
FindRoot[Evaluate[A'[x]==0],{x,0.5}]
```

```
{x → 0.442605}
```

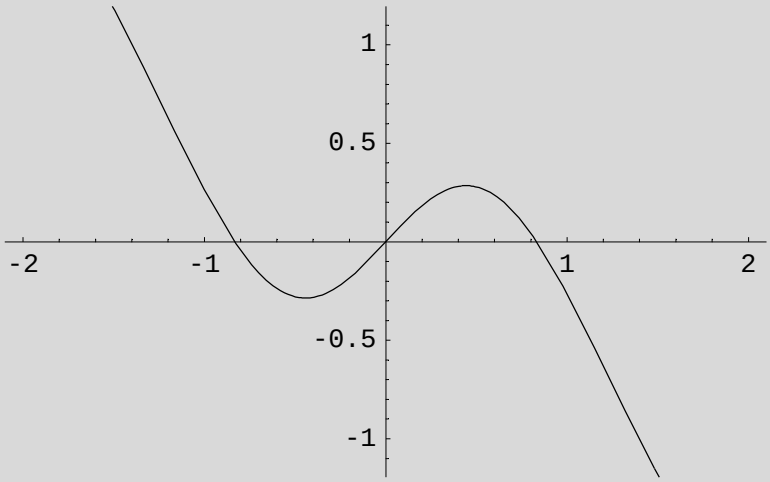
```
u[1]=0.5;
u[n_]:=u[n-1]-A'[u[n-1]]/A''[u[n-1]];
Table[u[n],{n,1,5}]//MatrixForm
```

$$\begin{pmatrix} 0.5 \\ 0.443195 \\ 0.442605 \\ 0.442605 \\ 0.442605 \end{pmatrix}$$

■ c

```
f[x_]:=2 E^(-x^2)-1;
A[x_]:=f[x] 2x/2
```

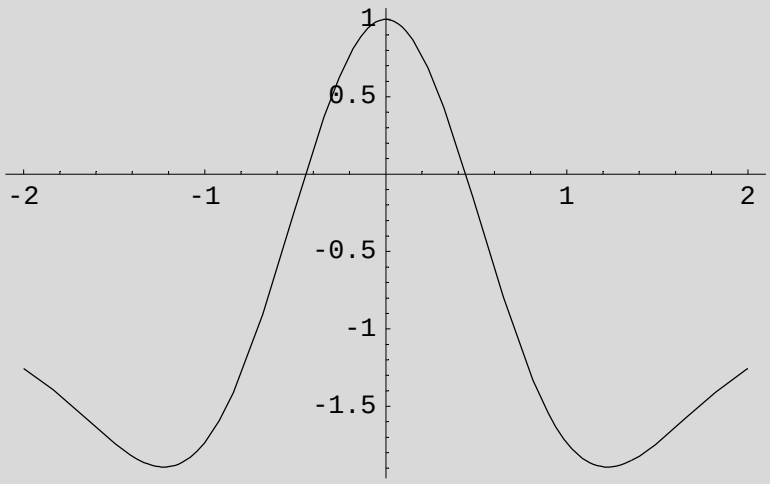
```
Plot[A[x],{x,-2,2}];
```




```
A'[x]
```

$$-1 + 2\, e^{-x^2} - 4\, e^{-x^2}\, x^2$$

```
Plot[A'[x],{x,-2,2}];
```




```
Solve[Evaluate[A'[x]==0],{x}]
```

– *InverseFunction::ifun : Inverse functions are being used. Values may be lost for multivalued inverses.* Mehr…

– *Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.* Mehr…

$$\left\{\left\{x \to -\sqrt{\frac{1}{2}\left(1-2\,\text{ProductLog}\left[\frac{\sqrt{e}}{4}\right]\right)}\right\},\ \left\{x \to \sqrt{\frac{1}{2}\left(1-2\,\text{ProductLog}\left[\frac{\sqrt{e}}{4}\right]\right)}\right\}\right\}$$

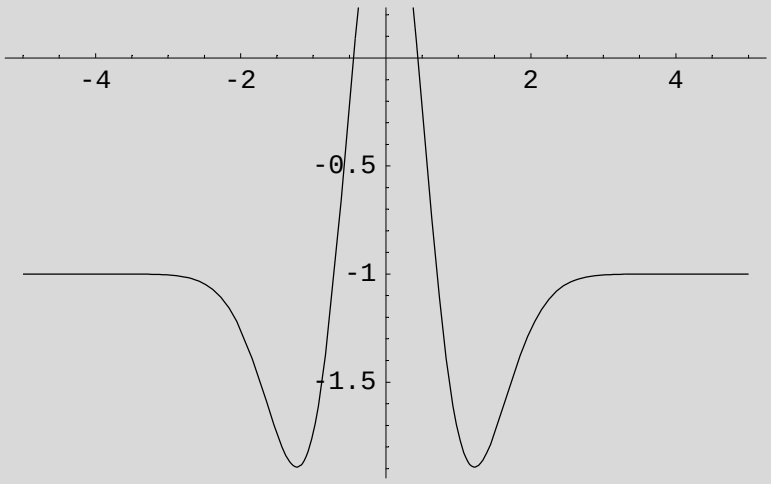
```
Solve[Evaluate[A'[x]==0],{x}]//N
```

– *InverseFunction::ifun : Inverse functions are being used. Values may be lost for multivalued inverses.* Mehr…

– *Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.* Mehr…

$$\{\{x \to -0.442605\},\ \{x \to 0.442605\}\}$$

```
Plot[Evaluate[A'[x]],{x,-5,5}];
```




```
FindRoot[Evaluate[A'[x]==0],{x,0.5}]
```

$\{x \to 0.442605\}$

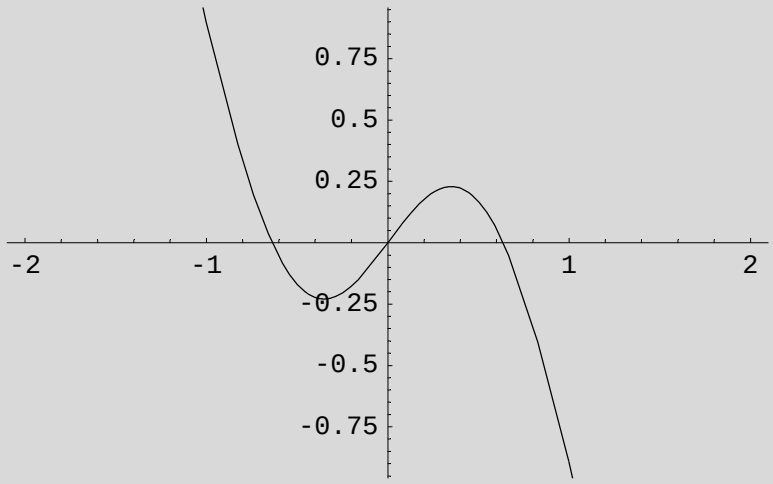
```
u[1]=0.5;
u[n_]:=u[n-1]-A'[u[n-1]]/A''[u[n-1]];
Table[u[n],{n,1,5}]//MatrixForm
```

$$\begin{pmatrix} 0.5 \\ 0.443195 \\ 0.442605 \\ 0.442605 \\ 0.442605 \end{pmatrix}$$

■ x

```
f[x_]:=3 E^(-x^2)-2;
A[x_]:=f[x] 2x/2
```

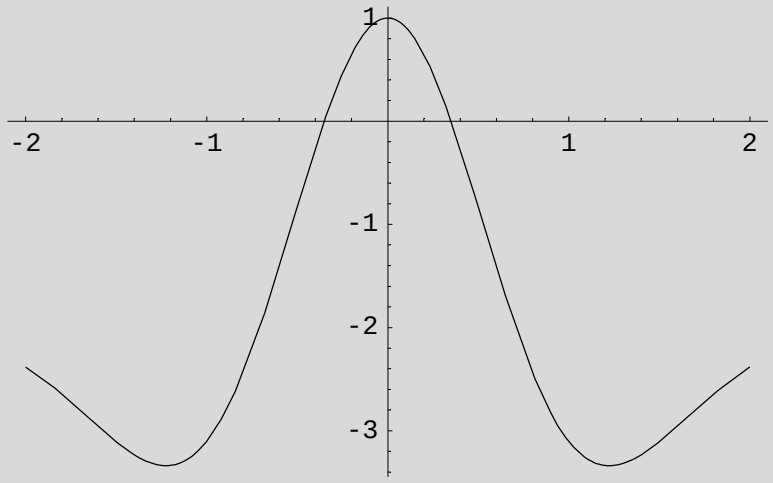
```
Plot[A[x],{x,-2,2}];
```




```
A'[x]
```

$$-2+3\,e^{-x^2}-6\,e^{-x^2}\,x^2$$

```
Plot[A'[x],{x,-2,2}];
```



```
Solve[Evaluate[A'[x]==0],{x}]
```

– *InverseFunction::ifun : Inverse functions are being used. Values may be lost for multivalued inverses.* Mehr…

– *Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.* Mehr…

$$\left\{\left\{x \to -\sqrt{\frac{1}{2}\left(1-2\,\text{ProductLog}\left[\frac{\sqrt{e}}{3}\right]\right)}\right\},\ \left\{x \to \sqrt{\frac{1}{2}\left(1-2\,\text{ProductLog}\left[\frac{\sqrt{e}}{3}\right]\right)}\right\}\right\}$$

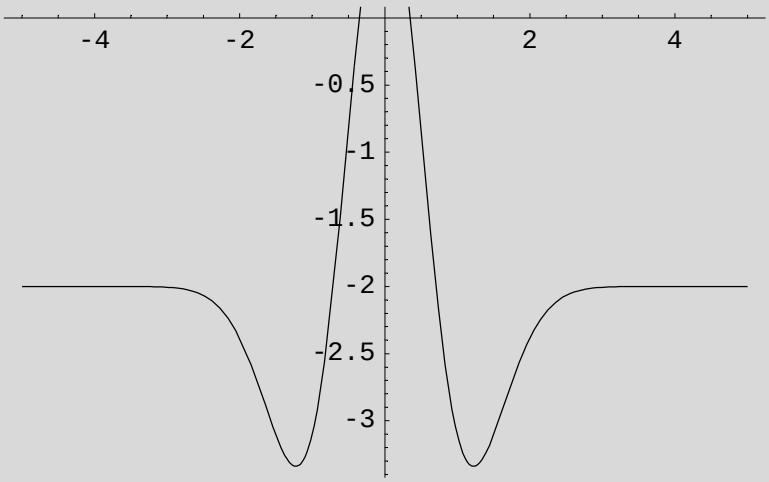
```
Solve[Evaluate[A'[x]==0],{x}]//N
```

– *InverseFunction::ifun : Inverse functions are being used. Values may be lost for multivalued inverses.* Mehr…

– *Solve::ifun : Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.* Mehr…

```
{{x → -0.350753}, {x → 0.350753}}
```

```
Plot[Evaluate[A'[x]],{x,-5,5}];
```




```
FindRoot[Evaluate[A'[x]==0],{x,0.5}]
```

```
{x → 0.350753}
```

```
u[1]=0.5;
u[n_]:=u[n-1]-A'[u[n-1]]/A''[u[n-1]];
Table[u[n],{n,1,5}]//MatrixForm
```

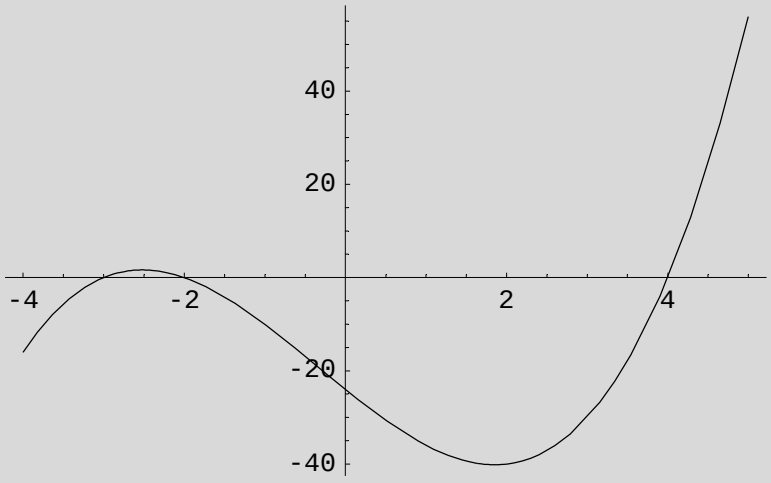
$$\begin{pmatrix} 0.5 \\ 0.357593 \\ 0.35079 \\ 0.350753 \\ 0.350753 \end{pmatrix}$$

```
Remove["Global`*"]
```

Lösung zu Aufgabe 12 a

■ **a, b**

```
f[x_]:=(x+3)(x+2)(x-4);
Plot[f[x],{x,-4,5}];
```




```
f1[x_,a_]:=a (x+3)(x+2)(x-4);
f2[x_,b_]:=x^2+b;
```

```
solv=Flatten[Solve[{f1[x,a]==f2[x,b],Evaluate[D[f1[x,a],x]=
=D[f2[x,b],x]]},{a,b}]]
```

$$\left\{b \to -\frac{48\,x + 14\,x^2 + x^4}{-14 + 2\,x + 3\,x^2},\ a \to \frac{2\,x}{-14 + 2\,x + 3\,x^2}\right\}$$

```
q=solv/.x->-2
```

$$\left\{b \to -4,\ a \to \frac{2}{3}\right\}$$

```
%//N
```

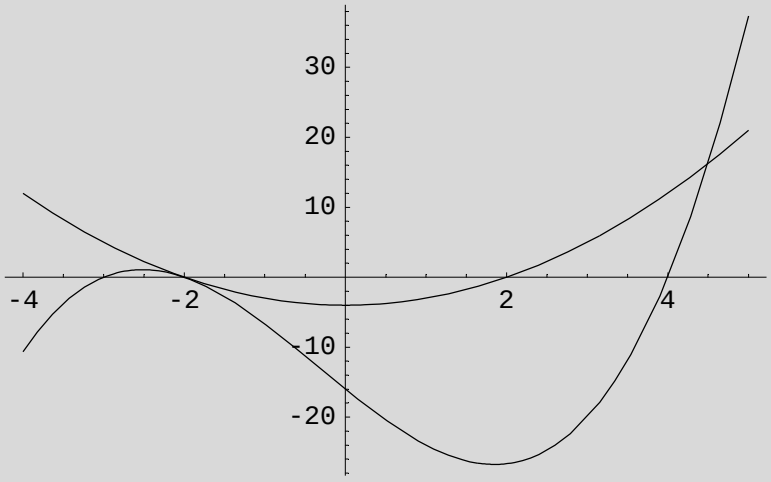
$$\{b \to -4.,\ a \to 0.666667\}$$

```
s1={-2,f1[-2]}
```

$$\{-2,\ f1[-2]\}$$

■ S2

```
f1[x_]:=a (x+3)(x+2)(x-4)/.q;
f2[x_]:=x^2+b/.q;
Plot[{f1[x],f2[x]},{x,-4,5}];
```




```
solv2=Flatten[Solve[{f1[x]==f2[x]},{x}]]
```

$$\left\{x \to -2,\ x \to -2,\ x \to \frac{9}{2}\right\}$$

```
Evaluate[f2'[x]]
```

$$2\,x$$

```
x0=x/.solv2[[3]]
```

$$\frac{9}{2}$$

```
%//N
```

$$4.5$$

```
f1[x0]
```

$$\frac{65}{4}$$

```
%//N
```

$$16.25$$

```
s2={x0,f1[x0]}
```

$$\left\{\frac{9}{2}, \frac{65}{4}\right\}$$

■ f3'

```
m=Evaluate[f1'[x]]/.x->s2[[1]]
```

$$\frac{223}{6}$$

```
f1''[x]//Simplify
```

$$\frac{4}{3} + 4\,x$$

```
solv3=Flatten[Solve[Evaluate[f1''[x]==0],{x}]]
```

$$\left\{x \to -\frac{1}{3}\right\}$$

```
%//N
```

$$\{x \to -0.333333\}$$

```
xW=x/.solv3
```

$$-\frac{1}{3}$$

```
sW={xW,f1[xW]}
```

$$\left\{-\frac{1}{3}, -\frac{1040}{81}\right\}$$

```
h[x_,a1_,b1_,c1_,d1_]:= a1 x^3+b1 x^2+c1 x+d1;
solv4=Solve[{
h[s1[[1]],a1,b1,c1,d1]==s1[[2]],
h[s2[[1]],a1,b1,c1,d1]==s2[[2]],
h[sW[[1]],a1,b1,c1,d1]==sW[[2]],
Evaluate[(D[h[x,a1,b1,c1,d1],x]/.x->s1[[1]])==m]},{a1,b1,c1
,d1}
]//Flatten
```

$$\left\{a1 \to \frac{67}{15},\ b1 \to -\frac{227}{30},\ c1 \to -\frac{467}{10},\ d1 \to -\frac{137}{5}\right\}$$

```
h[x_]:= a1 x^3+b1 x^2+c1 x+d1/.solv4;
```

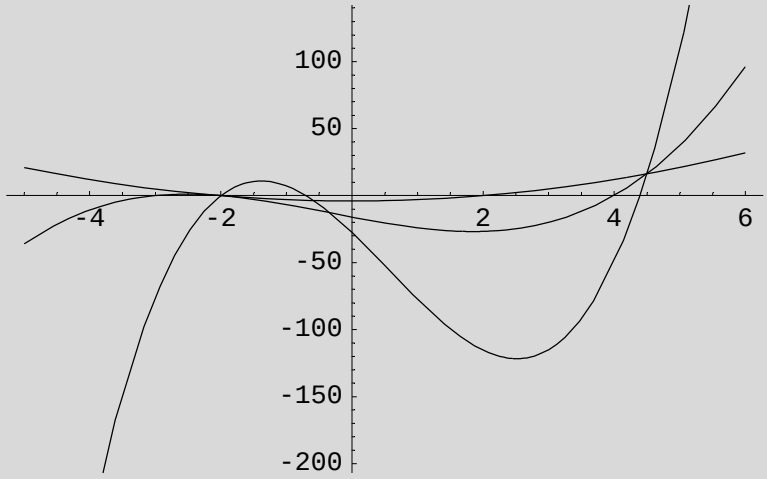
```
h'[x]/.x->s1[[2]]
```

$$-\frac{467}{10}$$

```
%//N
```

$$-46.7$$

```
Plot[{f1[x],f2[x],h[x]},{x,-5,6}];
```



■ Old

```
m=Evaluate[f2'[x]]/.solv2[[1]]
```

$$-4$$

```
f1''[x]//Simplify
```

$$\frac{4}{3}+4\,x$$

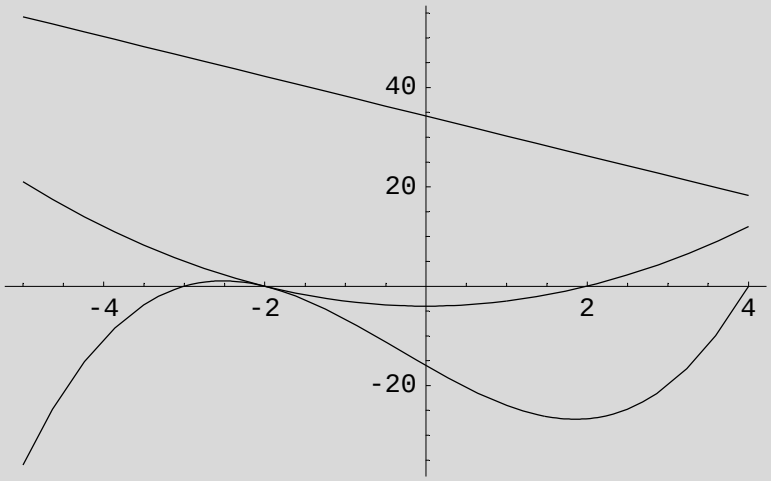
```
solv3=Flatten[Solve[Evaluate[f1''[x]==0],{x}]]
```

$$\left\{x\to-\frac{1}{3}\right\}$$

```
h[x_]:=f2[x0]+m(x-x0);
h[-1/3]
```

$$\frac{427}{12}$$

```
Plot[{f1[x],f2[x],h[x]},{x,-5,4}];
```




```
f1[-1/3]
```

$$-\frac{1040}{81}$$

```
f1[x0]
```

$$\frac{65}{4}$$

```
%//N
```

16.25

```
s1={}
```

{}

```
w={-1/3,f1[-1/3]}
```

$$\left\{-\frac{1}{3}, -\frac{1040}{81}\right\}$$

```
%//N
```

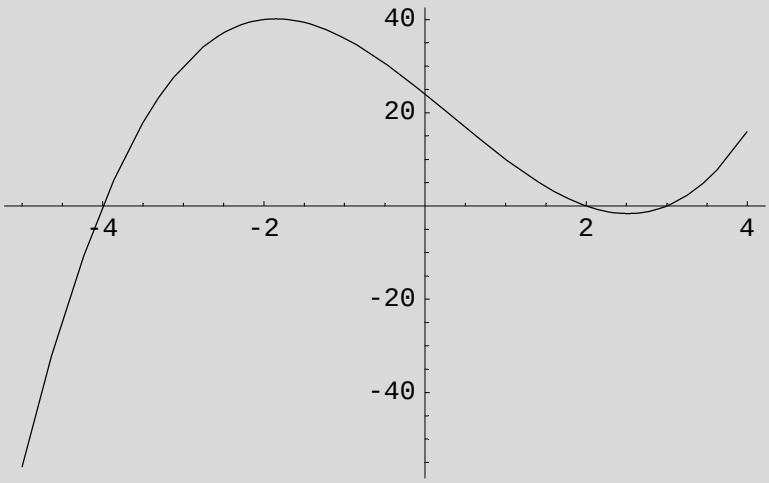
```
{-0.333333, -12.8395}
```

```
Remove["Global`*"]
```

Lösung zu Aufgabe 12 c

■ **a, b**

```
f[x_]:=(x-3)(x-2)(x+4);
Plot[f[x],{x,-5,4}];
```




```
f1[x_,a_]:=a (x-3)(x-2)(x+4);
f2[x_,b_]:=x^2+b;
```

```
solv=Flatten[Solve[{f1[x,a]==f2[x,b],Evaluate[D[f1[x,a],x]==D[f2[x,b],x]]},{a,b}]]
```

$$\left\{b \to -\frac{-48\,x+14\,x^2+x^4}{-14-2\,x+3\,x^2},\ a \to \frac{2\,x}{-14-2\,x+3\,x^2}\right\}$$

```
q=solv/.x->2
```

$$\left\{b \to -4,\ a \to -\frac{2}{3}\right\}$$

```
%//N
```

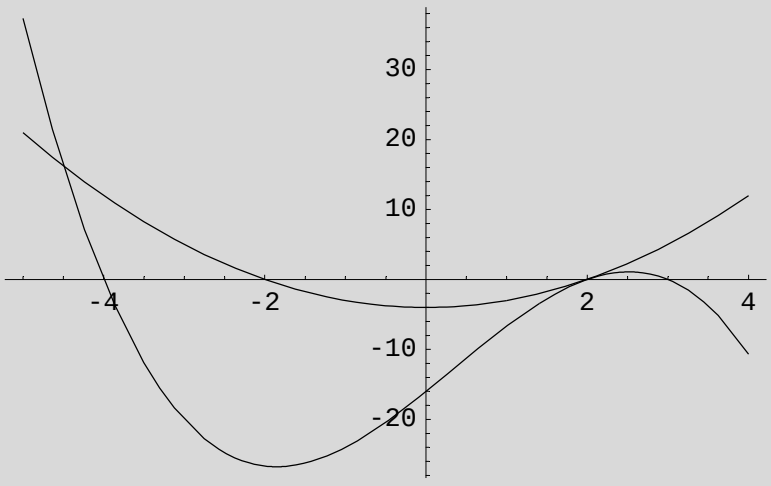
$$\{b \to -4.,\ a \to -0.666667\}$$

```
s1={-2,f1[-2]}
```

$$\{-2,\ f1[-2]\}$$

■ S2

```
f1[x_]:=a (x-3)(x-2)(x+4)/.q;
f2[x_]:=x^2+b/.q;
Plot[{f1[x],f2[x]},{x,-5,4}];
```



```
solv2=Flatten[Solve[{f1[x]==f2[x]},{x}]]
```

$$\left\{x \to -\frac{9}{2},\ x \to 2,\ x \to 2\right\}$$

```
Evaluate[f2'[x]]
```

$$2\,x$$

```
x0=x/.solv2[[1]]
```

$$-\frac{9}{2}$$

```
%//N
```

$$-4.5$$

```
f1[x0]
```

$$\frac{65}{4}$$

```
%//N
```

$$16.25$$

```
s2={x0,f1[x0]}
```

$$\left\{-\frac{9}{2},\ \frac{65}{4}\right\}$$

■ f3'

```
m=Evaluate[f1'[x]]/.x->s2[[1]]
```

$$-\frac{223}{6}$$

```
f1''[x]//Simplify
```

$$\frac{4}{3}-4\,x$$

```
solv3=Flatten[Solve[Evaluate[f1''[x]==0],{x}]]
```

$$\left\{x\to\frac{1}{3}\right\}$$

```
%//N
```

$$\{x\to 0.333333\}$$

```
xW=x/.solv3
```

$$\frac{1}{3}$$

```
sW={xW,f1[xW]}
```

$$\left\{\frac{1}{3},-\frac{1040}{81}\right\}$$

```
h[x_,a1_,b1_,c1_,d1_]:= a1 x^3+b1 x^2+c1 x+d1;
solv4=Solve[{
h[s1[[1]],a1,b1,c1,d1]==s1[[2]],
h[s2[[1]],a1,b1,c1,d1]==s2[[2]],
h[sW[[1]],a1,b1,c1,d1]==sW[[2]],
Evaluate[(D[h[x,a1,b1,c1,d1],x]/.x->s1[[1]])==m]},{a1,b1,c1
,d1}
]//Flatten
```

$$\left\{a1 \to \frac{115}{21},\ b1 \to \frac{1619}{42},\ c1 \to \frac{2155}{42},\ d1 \to -\frac{241}{7}\right\}$$

```
h[x_]:= a1 x^3+b1 x^2+c1 x+d1/.solv4;
```

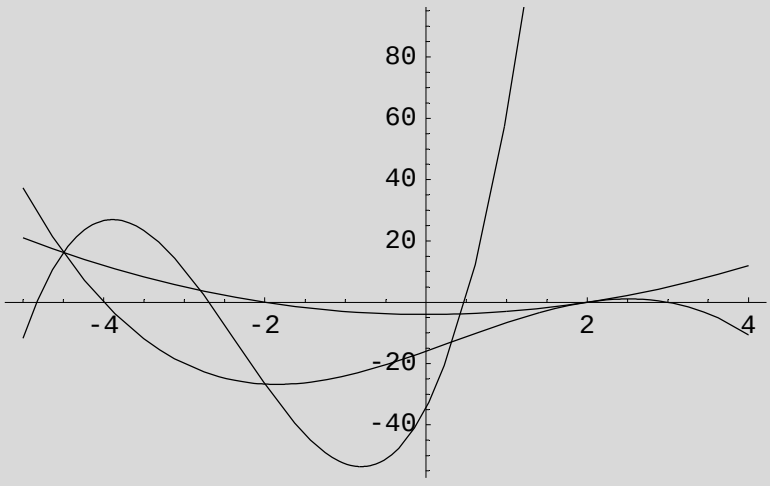
```
h'[x]/.x->s1[[2]]
```

$$\frac{406475}{42}$$

```
%//N
```

```
9677.98
```

```
Plot[{f1[x],f2[x],h[x]},{x,-5,4}];
```



■ Old

```
m=Evaluate[f2'[x]]/.solv2[[1]]
```

$$-9$$

```
f1''[x]//Simplify
```

$$\frac{4}{3} - 4\,x$$

```
%//N
```

$$1.33333 - 4.\,x$$

```
solv3=Flatten[Solve[Evaluate[f1''[x]==0],{x}]]
```

$$\left\{x \to \frac{1}{3}\right\}$$

```
%//N
```

$$\{x \to 0.333333\}$$

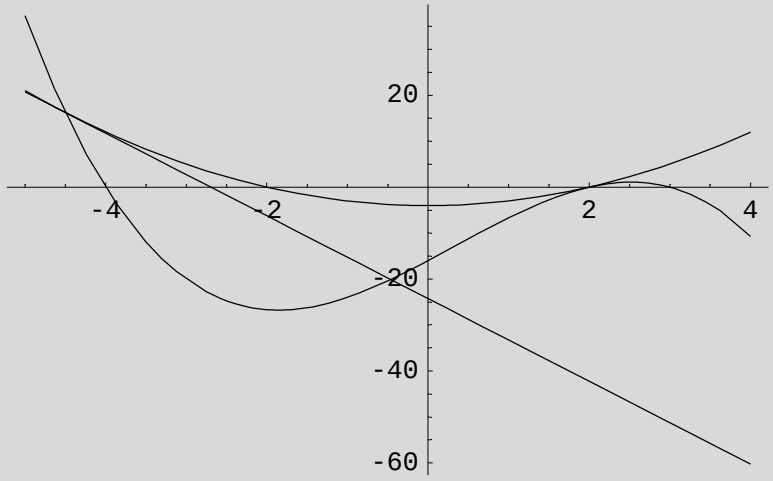
```
h[x_]:=f2[x0]+m(x-x0);
h[1/3]
```

$$-\frac{109}{4}$$

```
%//N
```

$$-27.25$$

```
Plot[{f1[x],f2[x],h[x]},{x,-5,4}];
```




```
f1[1/3]
```

$$-\frac{1040}{81}$$

```
%//N
```

```
-12.8395
```

```
s1={}
```

```
{}
```

```
w={1/3,f1[1/3]}
```

$$\left\{\frac{1}{3}, -\frac{1040}{81}\right\}$$

```
%//N
```

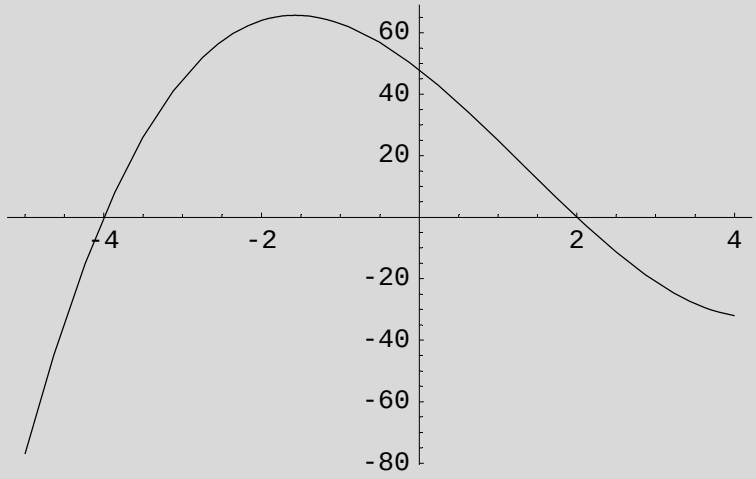
```
{0.333333, -12.8395}
```

```
Remove["Global`*"]
```

Lösung zu Aufgabe 12 x

■ a, b

```
f[x_]:=(x-6)(x-2)(x+4);
Plot[f[x],{x,-5,4}];
```




```
f1[x_,a_]:=a (x-3)(x-2)(x+4);
f2[x_,b_]:=x^2+b;
```

```
solv=Flatten[Solve[{f1[x,a]==f2[x,b],Evaluate[D[f1[x,a],x]==D[f2[x,b],x]]},{a,b}]]
```

$$\left\{b \to -\frac{-48\,x+14\,x^2+x^4}{-14-2\,x+3\,x^2},\ a \to \frac{2\,x}{-14-2\,x+3\,x^2}\right\}$$

```
q=solv/.x->2
```

$$\left\{b \to -4,\ a \to -\frac{2}{3}\right\}$$

```
%//N
```

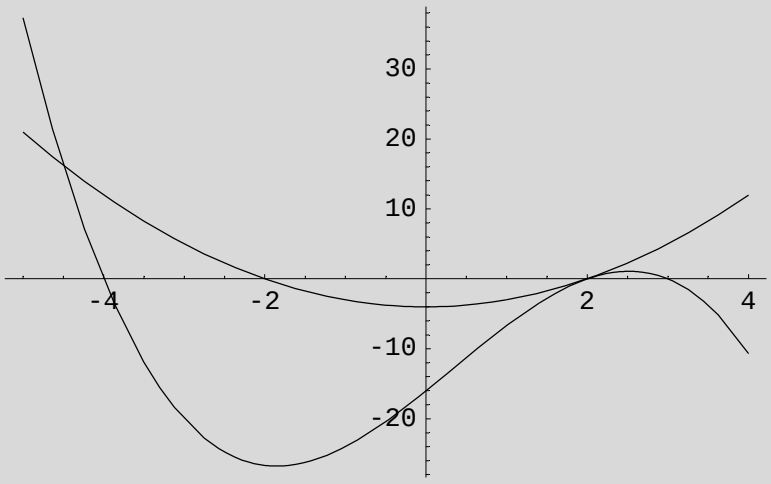
$$\{b \to -4.,\ a \to -0.666667\}$$

```
s1={-2,f1[-2]}
```

```
{-2, f1[-2]}
```

■ S2

```
f1[x_]:=a (x-3)(x-2)(x+4)/.q;
f2[x_]:=x^2+b/.q;
Plot[{f1[x],f2[x]},{x,-5,4}];
```




```
solv2=Flatten[Solve[{f1[x]==f2[x]},{x}]]
```

$$\left\{x \to -\frac{9}{2},\ x \to 2,\ x \to 2\right\}$$

```
Evaluate[f2'[x]]
```

$$2\,x$$

```
x0=x/.solv2[[1]]
```

$$-\frac{9}{2}$$

```
%//N
```

$$-4.5$$

```
f1[x0]
```

$$\frac{65}{4}$$

```
%//N
```

$$16.25$$

```
s2={x0,f1[x0]}
```

$$\left\{-\frac{9}{2}, \frac{65}{4}\right\}$$

■ f3'

```
m=Evaluate[f1'[x]]/.x->s2[[1]]
```

$$-\frac{223}{6}$$

```
f1''[x]//Simplify
```

$$\frac{4}{3} - 4\,x$$

```
solv3=Flatten[Solve[Evaluate[f1''[x]==0],{x}]]
```

$$\left\{x \to \frac{1}{3}\right\}$$

```
%//N
```

$$\{x \to 0.333333\}$$

```
xW=x/.solv3
```

$$\frac{1}{3}$$

```
sW={xW,f1[xW]}
```

$$\left\{\frac{1}{3}, -\frac{1040}{81}\right\}$$

```
h[x_,a1_,b1_,c1_,d1_]:= a1 x^3+b1 x^2+c1 x+d1;
solv4=Solve[{
h[s1[[1]],a1,b1,c1,d1]==s1[[2]],
h[s2[[1]],a1,b1,c1,d1]==s2[[2]],
h[sW[[1]],a1,b1,c1,d1]==sW[[2]],
Evaluate[(D[h[x,a1,b1,c1,d1],x]/.x->s1[[1]])==m]},{a1,b1,c1
,d1}
]//Flatten
```

$$\left\{a1 \to \frac{115}{21},\ b1 \to \frac{1619}{42},\ c1 \to \frac{2155}{42},\ d1 \to -\frac{241}{7}\right\}$$

```
h[x_]:= a1 x^3+b1 x^2+c1 x+d1/.solv4;
```

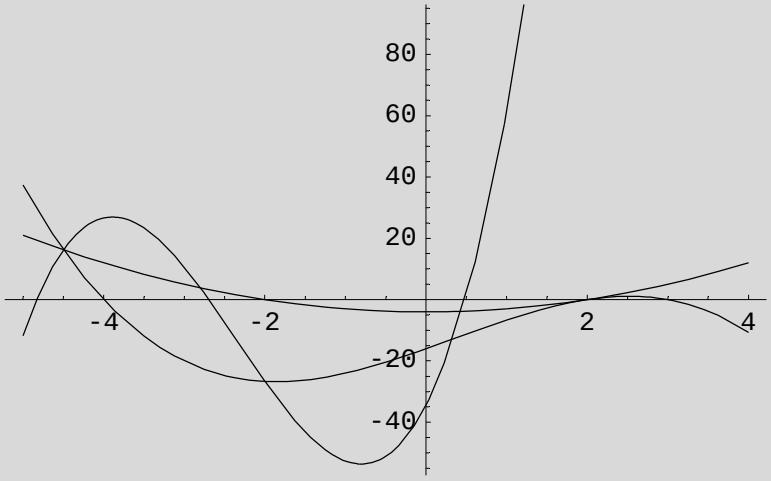
```
h'[x]/.x->s1[[2]]
```

$$\frac{406475}{42}$$

```
%//N
```

$$9677.98$$

```
Plot[{f1[x],f2[x],h[x]},{x,-5,4}];
```




```
Remove["Global`*"]
```

Lösung zu Aufgabe 13, Hyperbel

```
p[1]={3,6};
p[2]={-2,6};
p[3]={-3,-5};
p[4]={3,-4};

f[x_,y_,a_,c_,d_,e_]:=a x^2+c y^2+d x+e y + 1;

tab1=Table[f[p[k][[1]],p[k][[2]],a,c,d,e]==0,{k,1,4}]
```

$$\{1 + 9\,a + 36\,c + 3\,d + 6\,e == 0,\ 1 + 4\,a + 36\,c - 2\,d + 6\,e == 0,\ 1 + 9\,a + 25\,c - 3\,d - 5\,e == 0,\ 1 + 9\,a + 16\,c + 3\,d - 4\,e == 0\}$$

```
solv1=Flatten[Solve[tab1,{a,c,d,e}]]
```

$$\left\{a \to \frac{11}{78},\ c \to -\frac{1}{13},\ d \to -\frac{11}{78},\ e \to \frac{2}{13}\right\}$$

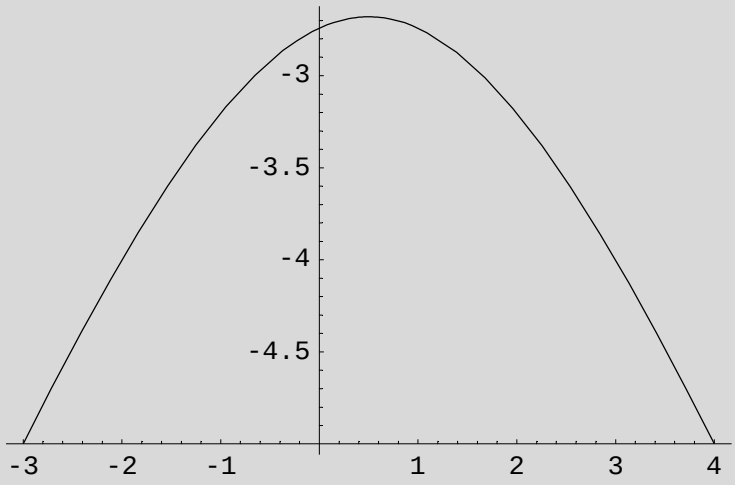
```
f[x_,y_,a_,c_,d_,e_]:=a x^2+c y^2+d x+e y + 1/.solv1;

f[x_,y_]:=f[x,y,a,c,d,e];
f[x,y]
```

$$1 - \frac{11\,x}{78} + \frac{11\,x^2}{78} + \frac{2\,y}{13} - \frac{y^2}{13}$$

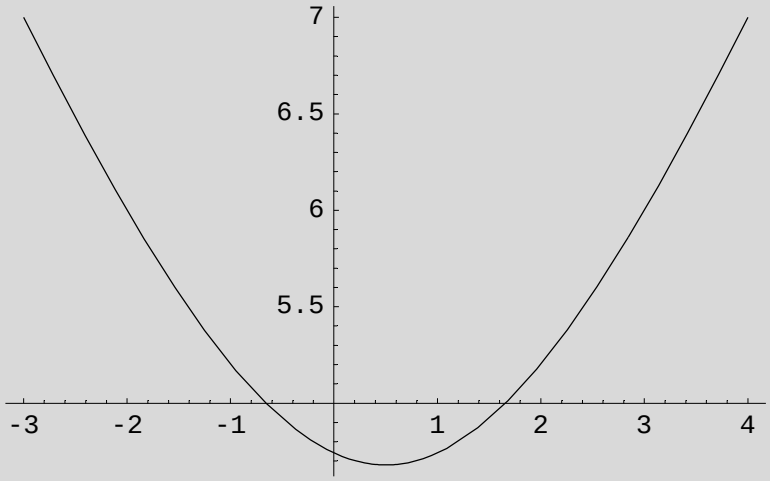
```
solv2=Flatten[Solve[f[x,y]==0,{y}]]
```

$$\left\{y \to \frac{1}{6}\left(6 - \sqrt{6}\,\sqrt{84 - 11\,x + 11\,x^2}\right),\ y \to \frac{1}{6}\left(6 + \sqrt{6}\,\sqrt{84 - 11\,x + 11\,x^2}\right)\right\}$$

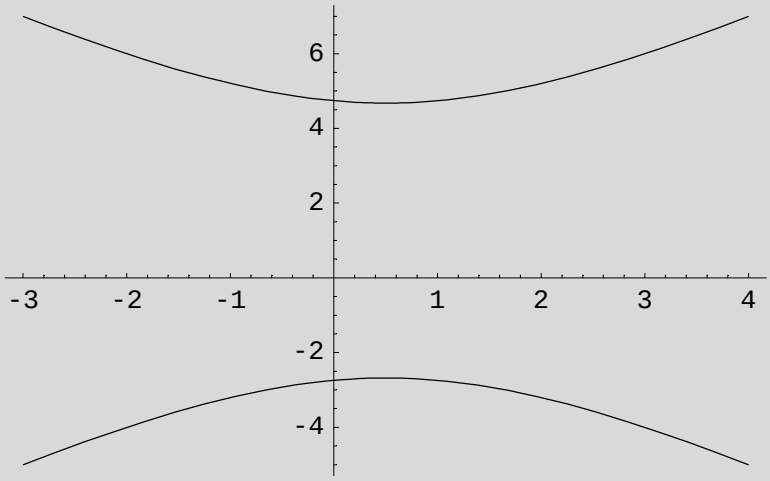
```
graf1[x_]:=y/.solv2[[1]];  pl1=
Plot[Evaluate[graf1[x]],{x,-3,4}];
```



```
graf2[x_]:=y/.solv2[[2]];  pl2=
Plot[Evaluate[graf2[x]],{x,-3,4}];
```




```
Show[pl1,pl2];
```




```
Remove["Global`*"]
```

Lösung zu Aufgabe 13, Ellipse a

```
p[1]={3,-6};
p[2]={-3,-6};
p[3]={-3,5};
p[4]={0,6};
p[5]={3,4};

f[x_,y_,a_,b_,c_,d_,e_]:=a x^2+ b x y+c y^2+d x+e y+ 1;

tab1=Table[f[p[k][[1]],p[k][[2]],a,b,c,d,e]==0,{k,1,5}]
```

$$\{1+9a-18b+36c+3d-6e==0,\ 1+9a+18b+36c-3d-6e==0,\ 1+9a-15b+25c-3d+5e==0,\ 1+36c+6e==0,\ 1+9a+12b+16c+3d+4e==0\}$$

```
solv1=Flatten[Solve[tab1,{a,b,c,d,e}]]
```

$$\left\{a\to -\frac{2}{45},\ b\to -\frac{1}{270},\ c\to -\frac{1}{45},\ d\to -\frac{1}{45},\ e\to -\frac{1}{30}\right\}$$

```
f[x_,y_,a_,b_,c_,d_,e_]:=a x^2+b x y+c y^2+d x+e y +
1/.solv1;

f[x_,y_]:=f[x,y,a,b,c,d,e];
f[x,y]
```

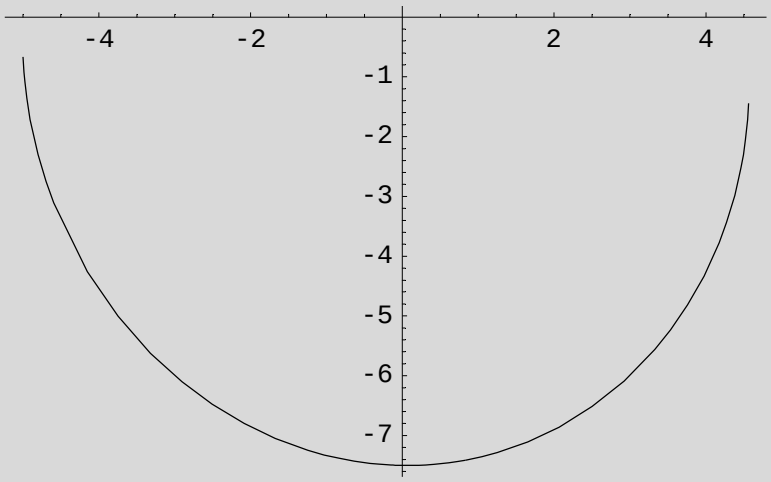
$$1-\frac{x}{45}-\frac{2x^2}{45}-\frac{y}{30}-\frac{xy}{270}-\frac{y^2}{45}$$

```
solv2=Flatten[Solve[f[x,y]==0,{y}]]
```

$$\left\{y\to \frac{1}{12}\left(-9-x-\sqrt{6561-126x-287x^2}\right),\ y\to \frac{1}{12}\left(-9-x+\sqrt{6561-126x-287x^2}\right)\right\}$$

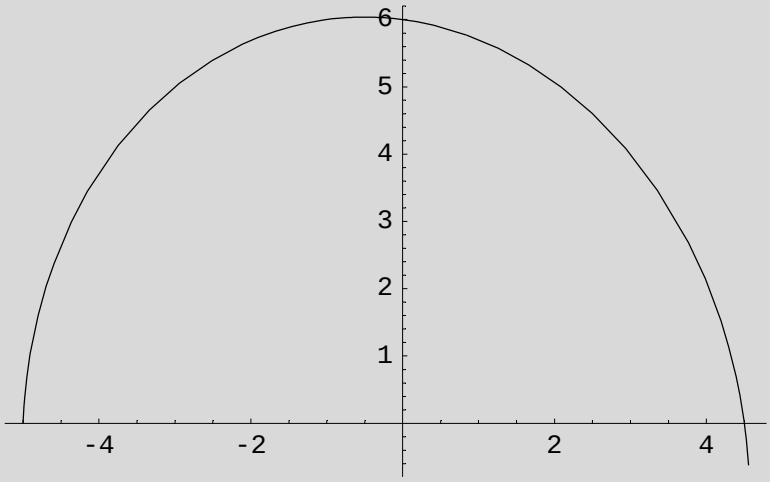
```
graf1[x_]:=y/.solv2[[1]];  pl1=
Plot[Evaluate[graf1[x]],{x,-5,5}];
```

```
Plot::plnr : 1/12 (-9 - x - Sqrt[6561 - 126 x - 287 x^2]) is not a
   machine-size real number at x = 4.600037073538462`. Mehr…
Plot::plnr : 1/12 (-9 - x - Sqrt[6561 - 126 x - 287 x^2]) is not a
   machine-size real number at x = 4.5734390401044305`. Mehr…
Plot::plnr : 1/12 (-9 - x - Sqrt[6561 - 126 x - 287 x^2]) is not a
   machine-size real number at x = 4.5672736192778185`. Mehr…
General::stop : Further output of
   Plot::plnr will be suppressed during this calculation. Mehr…
```

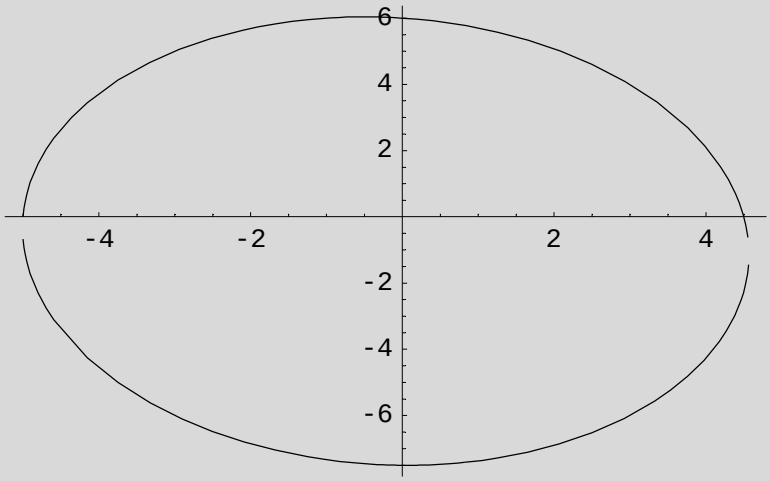


```
graf2[x_]:=y/.solv2[[2]];  pl2=
Plot[Evaluate[graf2[x]],{x,-5,5}];
```

```
– Plot::plnr : 1/12 (-9 - x + Sqrt[6561 - 126 x - 287 x^2]) is not a
    machine-size real number at x = 4.608099386246602`. Mehr…
– Plot::plnr : 1/12 (-9 - x + Sqrt[6561 - 126 x - 287 x^2]) is not a
    machine-size real number at x = 4.5794843992358905`. Mehr…
– Plot::plnr : 1/12 (-9 - x + Sqrt[6561 - 126 x - 287 x^2]) is not a
    machine-size real number at x = 4.566923486255313`. Mehr…
– General::stop : Further output of
    Plot::plnr will be suppressed during this calculation. Mehr…
```




```
Show[pl1,pl2];
```




```
D[graf2[x],x]
```

$$\frac{1}{12}\left(-1+\frac{-126-574\,x}{2\sqrt{6561-126\,x-287\,x^2}}\right)$$

```
Solve[Evaluate[D[graf2[x],x]]==0,{x}]
```

$$\left\{\left\{x \to \frac{3}{287}\left(-21-2\sqrt{182}\right)\right\}\right\}$$

```
Solve[Evaluate[D[graf2[x],x]]==0,{x}]//N
```

$$\{\{x \to -0.501549\}\}$$

Frage: x max > -0.5 ?? !!

```
Remove["Global`*"]
```

Lösung zu Aufgabe 13, Ellipse c

```
p[1]={3,6};
p[2]={-3,6};
p[3]={-3,-5};
p[4]={0,-6};
p[5]={3,-4};

f[x_,y_,a_,b_,c_,d_,e_]:=a x^2+ b x y+c y^2+d x+e y+ 1;

tab1=Table[f[p[k][[1]],p[k][[2]],a,b,c,d,e]==0,{k,1,5}]
```

$$\{1+9a+18b+36c+3d+6e==0,\ 1+9a-18b+36c-3d+6e==0,\ 1+9a+15b+25c-3d-5e==0,\ 1+36c-6e==0,\ 1+9a-12b+16c+3d-4e==0\}$$

```
solv1=Flatten[Solve[tab1,{a,b,c,d,e}]]
```

$$\left\{a \to -\frac{2}{45},\ b \to \frac{1}{270},\ c \to -\frac{1}{45},\ d \to -\frac{1}{45},\ e \to \frac{1}{30}\right\}$$

```
f[x_,y_,a_,b_,c_,d_,e_]:=a x^2+b x y+c y^2+d x+e y +
1/.solv1;

f[x_,y_]:=f[x,y,a,b,c,d,e];
f[x,y]
```

$$1 - \frac{x}{45} - \frac{2x^2}{45} + \frac{y}{30} + \frac{xy}{270} - \frac{y^2}{45}$$

```
solv2=Flatten[Solve[f[x,y]==0,{y}]]
```

$$\left\{y \to \frac{1}{12}\left(9 + x - \sqrt{6561 - 126x - 287x^2}\right),\ y \to \frac{1}{12}\left(9 + x + \sqrt{6561 - 126x - 287x^2}\right)\right\}$$

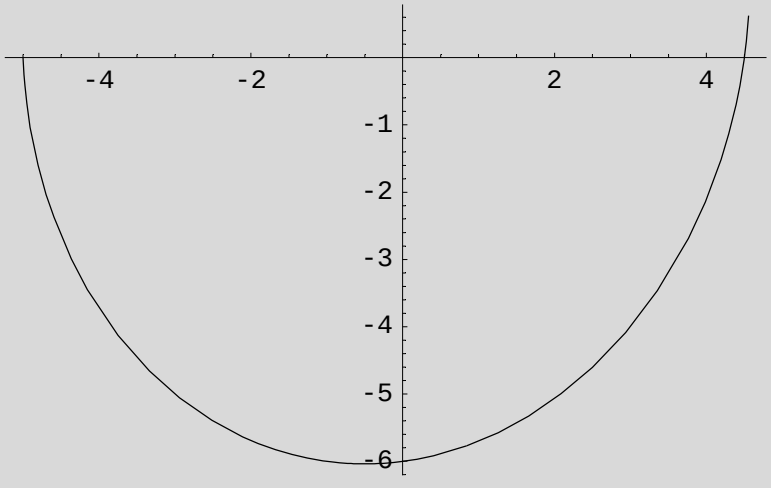
```
graf1[x_]:=y/.solv2[[1]];  pl1=
Plot[Evaluate[graf1[x]],{x,-5,5}];
```

– *Plot::plnr :* $\frac{1}{12}\left(9 + x - \sqrt{6561 - 126x - 287x^2}\right)$ *is not a machine-size real number at* $x = 4.608099386246602`$. Mehr…

– *Plot::plnr :* $\frac{1}{12}\left(9 + x - \sqrt{6561 - 126x - 287x^2}\right)$ *is not a machine-size real number at* $x = 4.5794843992358905`$. Mehr…

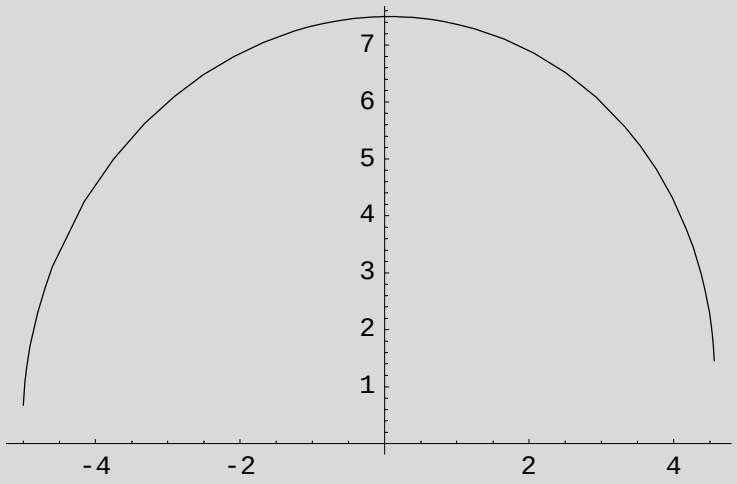
– *Plot::plnr :* $\frac{1}{12}\left(9 + x - \sqrt{6561 - 126x - 287x^2}\right)$ *is not a machine-size real number at* $x = 4.566923486255313`$. Mehr…

– *General::stop : Further output of Plot::plnr will be suppressed during this calculation.* Mehr…

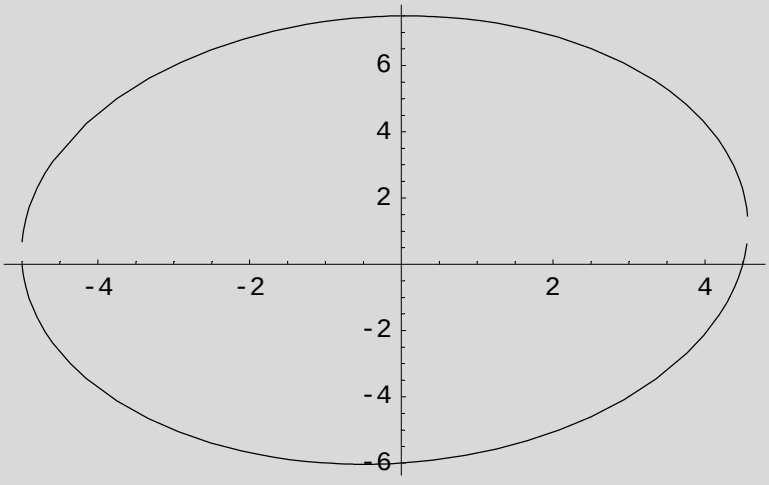


```
graf2[x_]:=y/.solv2[[2]];  pl2=
Plot[Evaluate[graf2[x]],{x,-5,5}];
```

```
– Plot::plnr : 1/12 (9 + x + Sqrt[6561 - 126 x - 287 x^2]) is not a
    machine-size real number at x = 4.600037073538462`. Mehr…
– Plot::plnr : 1/12 (9 + x + Sqrt[6561 - 126 x - 287 x^2]) is not a
    machine-size real number at x = 4.5734390401044305`. Mehr…
– Plot::plnr : 1/12 (9 + x + Sqrt[6561 - 126 x - 287 x^2]) is not a
    machine-size real number at x = 4.5672736192778185`. Mehr…
– General::stop : Further output of
    Plot::plnr will be suppressed during this calculation. Mehr…
```




```
Show[pl1,pl2];
```




```
D[graf1[x],x]
```

$$\frac{1}{12}\left(1-\frac{-126-574\,x}{2\sqrt{6561-126\,x-287\,x^2}}\right)$$

```
Solve[Evaluate[D[graf1[x],x]]==0,{x}]
```

$$\left\{\left\{x \to \frac{3}{287}\left(-21-2\sqrt{182}\right)\right\}\right\}$$

```
Solve[Evaluate[D[graf1[x],x]]==0,{x}]//N
```

$$\{\{x \to -0.501549\}\}$$

Frage: x max > -0.5 ?? Ja!!

```
Remove["Global`*"]
```

Lösung zu Aufgabe 13, Ellipse x

```
p[1]={3,8};
p[2]={-3,8};
p[3]={-3,-5};
p[4]={0,-6};
p[5]={3,-4};

f[x_,y_,a_,b_,c_,d_,e_]:=a x^2+ b x y+c y^2+d x+e y+ 1;

tab1=Table[f[p[k][[1]],p[k][[2]],a,b,c,d,e]==0,{k,1,5}]
```

$$\{1+9a+24b+64c+3d+8e==0,\ 1+9a-24b+64c-3d+8e==0,\ 1+9a+15b+25c-3d-5e==0,\ 1+36c-6e==0,\ 1+9a-12b+16c+3d-4e==0\}$$

```
solv1=Flatten[Solve[tab1,{a,b,c,d,e}]]
```

$$\left\{a \to -\frac{7}{171},\ b \to \frac{1}{342},\ c \to -\frac{1}{57},\ d \to -\frac{4}{171},\ e \to \frac{7}{114}\right\}$$

```
f[x_,y_,a_,b_,c_,d_,e_]:=a x^2+b x y+c y^2+d x+e y +
1/.solv1;

f[x_,y_]:=f[x,y,a,b,c,d,e];
f[x,y]
```

$$1 - \frac{4\,x}{171} - \frac{7\,x^2}{171} + \frac{7\,y}{114} + \frac{x\,y}{342} - \frac{y^2}{57}$$

```
solv2=Flatten[Solve[f[x,y]==0,{y}]]
```

$$\left\{y \to \frac{1}{12}\left(21 + x - \sqrt{8649 - 150\,x - 335\,x^2}\right),\ y \to \frac{1}{12}\left(21 + x + \sqrt{8649 - 150\,x - 335\,x^2}\right)\right\}$$

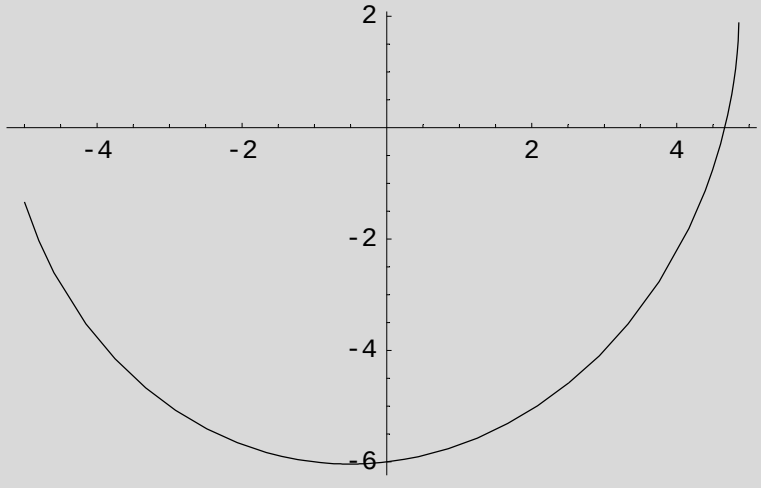
```
graf1[x_]:=y/.solv2[[1]];  pl1=
Plot[Evaluate[graf1[x]],{x,-5,5}];
```

– *Plot::plnr :* $\frac{1}{12}\left(21 + x - \sqrt{8649 - 150\,x - 335\,x^2}\right)$ *is not a machine-size real number at* $x = 4.9999995833333335`$. Mehr…

– *Plot::plnr :* $\frac{1}{12}\left(21 + x - \sqrt{8649 - 150\,x - 335\,x^2}\right)$ *is not a machine-size real number at* $x = 4.902562462467651`$. Mehr…

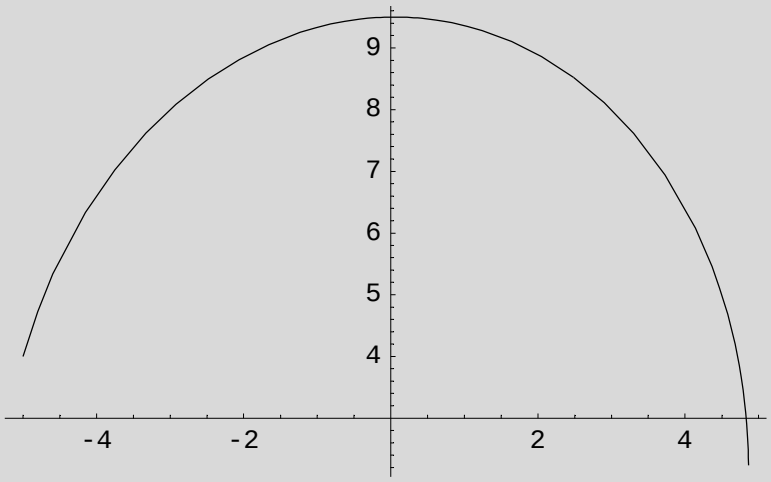
– *Plot::plnr :* $\frac{1}{12}\left(21 + x - \sqrt{8649 - 150\,x - 335\,x^2}\right)$ *is not a machine-size real number at* $x = 4.880232795478173`$. Mehr…

– *General::stop : Further output of Plot::plnr will be suppressed during this calculation.* Mehr…

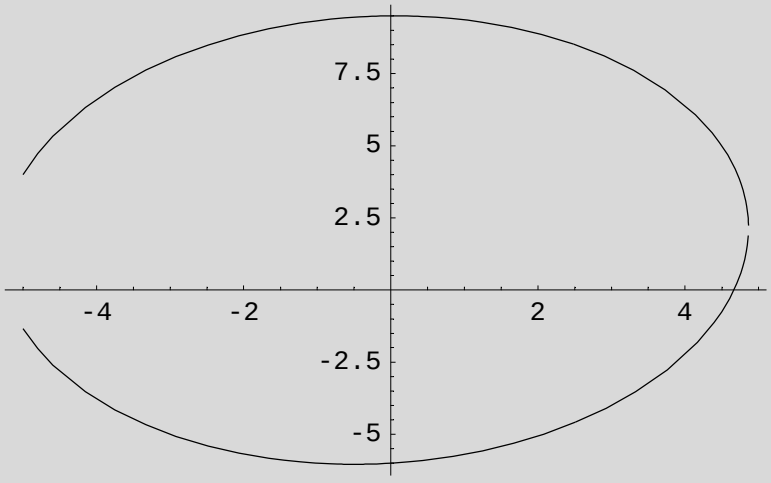


```
graf2[x_]:=y/.solv2[[2]];  pl2=
Plot[Evaluate[graf2[x]],{x,-5,5}];
```

```
– Plot::plnr : 1/12 (21 + x + Sqrt[8649 - 150 x - 335 x^2]) is not a
    machine-size real number at x = 4.984051013174095`. Mehr…
– Plot::plnr : 1/12 (21 + x + Sqrt[8649 - 150 x - 335 x^2]) is not a
    machine-size real number at x = 4.890401541198485`. Mehr…
– Plot::plnr : 1/12 (21 + x + Sqrt[8649 - 150 x - 335 x^2]) is not a
    machine-size real number at x = 4.863359774556848`. Mehr…
– General::stop : Further output of
    Plot::plnr will be suppressed during this calculation. Mehr…
```




```
Show[pl1,pl2];
```




```
D[graf1[x],x]
```

$$\frac{1}{12}\left(1-\frac{-150-670\,x}{2\sqrt{8649-150\,x-335\,x^2}}\right)$$

```
Solve[Evaluate[D[graf1[x],x]]==0,{x}]
```

$$\left\{\left\{x \to \frac{3}{335}\left(-25-8\sqrt{15}\right)\right\}\right\}$$

```
Solve[Evaluate[D[graf1[x],x]]==0,{x}]//N
```

```
{{x → -0.501348}}
```

Frage: x max > -0.5 ??

```
Remove["Global`*"]
```

Lösung zu Aufgabe 14 a

■ **a**

```
Remove[a,b,c]
```

```
a[n_]:=n^4;
b[n_]:=a[n+1]-a[n];
c[n_]:=b[n+1]-b[n];
```

```
{b[n],c[n]}//Simplify
```

$$\left\{-n^4+(1+n)^4,\ 2\,(7+12\,n+6\,n^2)\right\}$$

```
Table[c[n],{n,1,10}]
```

```
{50, 110, 194, 302, 434, 590, 770, 974, 1202, 1454}
```

```
c[9000]
```

```
972216014
```

```
c[9000]//N
```

$$9.72216 \times 10^{8}$$

■ b

```
Solve[(y-4)/8==4/(x-8),{y}]
```

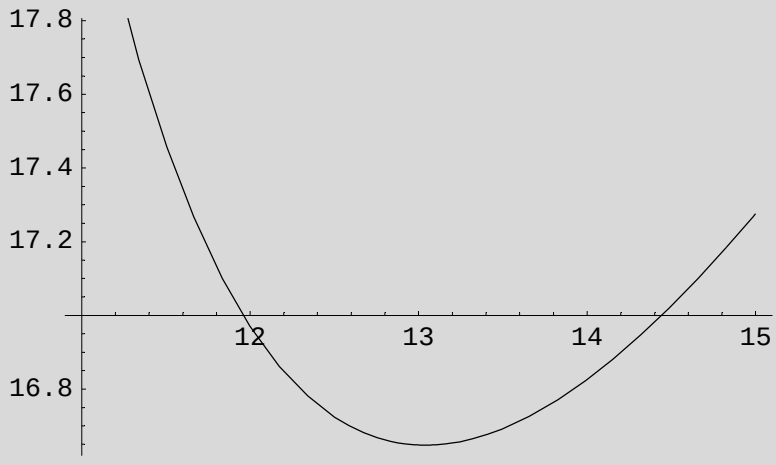
$$\left\{\left\{y \to \frac{4\,x}{-8+x}\right\}\right\}$$

```
Solve[(y-4)/8==4/(x-8),{x}]
```

$$\left\{\left\{x \to \frac{8\,y}{-4+y}\right\}\right\}$$

```
la[x_]:=Sqrt[x^2+(4 x/(x-8))^2]
```

```
Plot[la[x],{x,11,15}];
```




```
la'[x]//Simplify
```

$$\frac{x\,(-640+192\,x-24\,x^2+x^3)}{(-8+x)^3\sqrt{\left(1+\frac{16}{(-8+x)^2}\right)\,x^2}}$$

```
Solve[Evaluate[la'[x]==0],{x}]
```

$$\{\{x \to 4\,(2+2^{1/3})\},\ \{x \to 8-2\,2^{1/3}\left(1-i\sqrt{3}\right)\},\ \{x \to 8-2\,2^{1/3}\left(1+i\sqrt{3}\right)\}\}$$

```
Solve[Evaluate[la'[x]==0],{x}]//N
```

$$\{\{x \to 13.0397\},\ \{x \to 5.48016+4.36449\,i\},\ \{x \to 5.48016-4.36449\,i\}\}$$

```
Remove["Global`*"]
```

Lösung zu Aufgabe 14 c

■ a

```
Remove[a,b,c]
```

```
a[n_]:=n^4;
b[n_]:=a[n+1]-a[n];
c[n_]:=b[n+1]-b[n];
```

```
{b[n],c[n]}//Simplify
```

$$\{-n^4+(1+n)^4,\ 2\,(7+12\,n+6\,n^2)\}$$

```
Table[c[n],{n,1,10}]
```

$$\{50,\ 110,\ 194,\ 302,\ 434,\ 590,\ 770,\ 974,\ 1202,\ 1454\}$$

```
c[10000]
```

$$1200240014$$

```
c[10000]//N
```

$$1.20024 \times 10^{9}$$

■ b

```
Solve[(y-3)/6==3/(x-6),{y}]
```

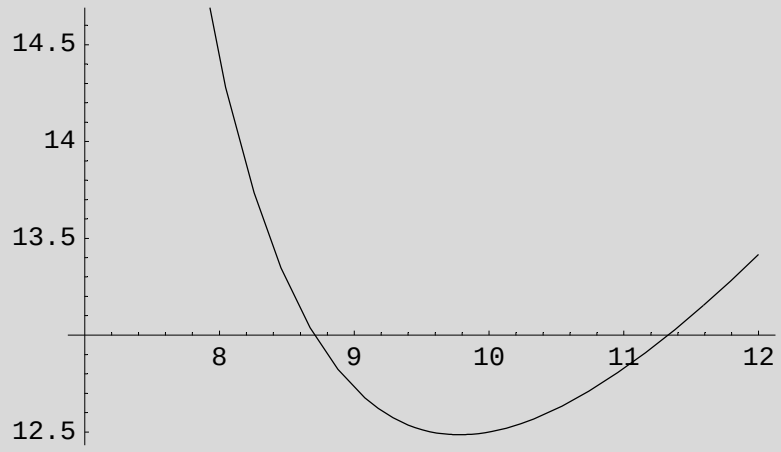
$$\left\{\left\{y \to \frac{3\,x}{-6+x}\right\}\right\}$$

```
Solve[(y-3)/6==3/(x-6),{x}]
```

$$\left\{\left\{x \to \frac{6\,y}{-3+y}\right\}\right\}$$

```
la[x_]:=Sqrt[x^2+(3x/(x-6))^2]
```

```
Plot[la[x],{x,7,12}];
```




```
la'[x]//Simplify
```

$$\frac{x\,\left(-270+108\,x-18\,x^2+x^3\right)}{(-6+x)^3\,\sqrt{\left(1+\frac{9}{(-6+x)^2}\right)\,x^2}}$$

```
Solve[Evaluate[la'[x]==0],{x}]
```

$$\left\{\{x \to 3\,(2+2^{1/3})\},\ \left\{x \to 6-\frac{3\left(1-i\sqrt{3}\right)}{2^{2/3}}\right\},\ \left\{x \to 6-\frac{3\left(1+i\sqrt{3}\right)}{2^{2/3}}\right\}\right\}$$

```
Solve[Evaluate[la'[x]==0],{x}]//N
```

```
{{x → 9.77976}, {x → 4.11012 + 3.27337 i},
  {x → 4.11012 - 3.27337 i}}
```

```
Remove["Global`*"]
```

Lösung zu Aufgabe 14 x

■ a

```
Remove[a,b,c]
```

```
a[n_]:=n^4;
b[n_]:=a[n]-a[n+1];
c[n_]:= -2(b[n+1]-b[n]);
```

```
{b[n],c[n]}//Simplify
```

$$\{n^4-(1+n)^4,\ 4\,(7+12\,n+6\,n^2)\}$$

```
Table[c[n],{n,1,10}]
```

```
{100, 220, 388, 604, 868, 1180, 1540, 1948, 2404, 2908}
```

```
c[20000]
```

```
9600960028
```

```
c[20000]//N
```

$$9.60096 \times 10^{9}$$

```
Remove["Global`*"]
```

Lösung zu Aufgabe 15

■ a

```
f[x_] := 6 x^3 + 5 x^2 - 6 x^1 + x^0 - 2 / x^2
```

i

```
D[f[x], x]
```

$$-6 + \frac{4}{x^3} + 10\,x + 18\,x^2$$

ii

```
D[f[x], x] /. x → 1
```

```
26
```

iii

```
ArcTan[D[f[x], x]] /. x → 1
```

```
ArcTan[26]
```

```
N[%]
```

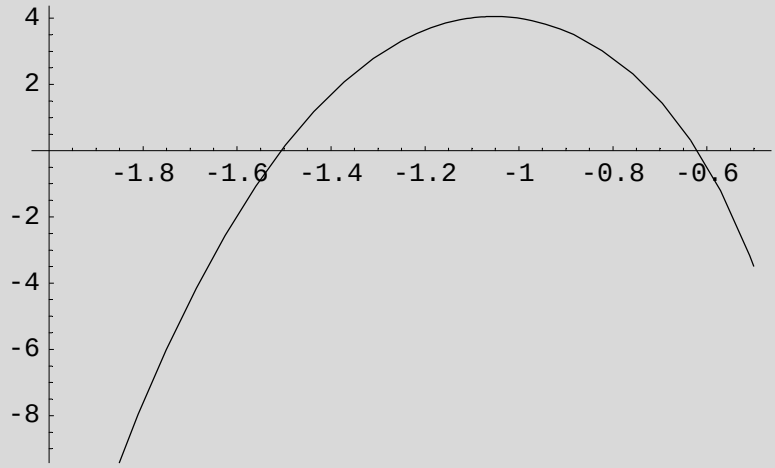
```
1.53235
```

```
% / Degree
```

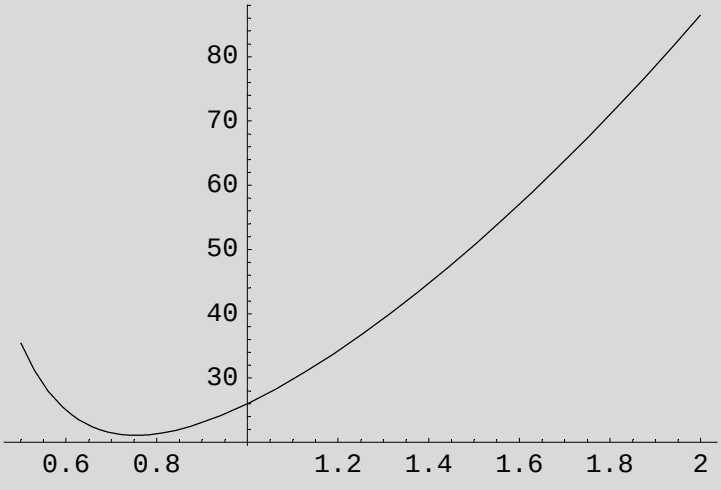
```
87.7974
```

iv

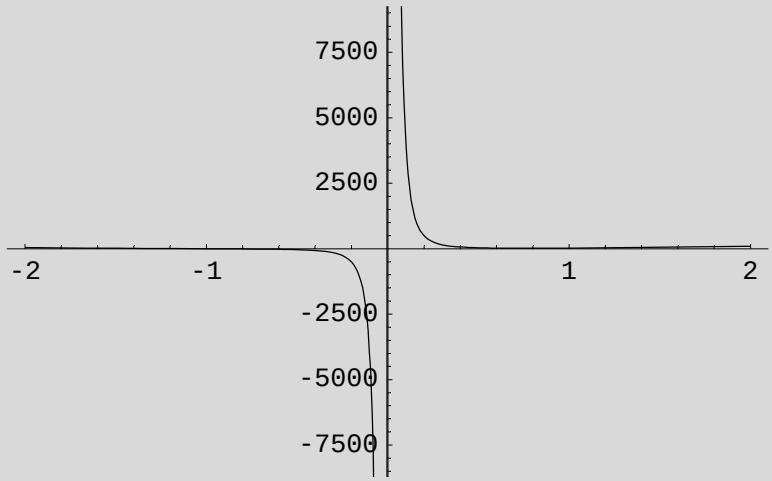
```
Plot[6 x^3 + 5 x^2 - 6 x^1 + x^0 - 2 / x^2, {x, -2, -0.5}];
```




```
Plot[-6 + 4/x^3 + 10 x + 18 x^2, {x, 0.5, 2}];
```



```
Plot[-6 + 4/x^3 + 10 x + 18 x^2, {x, -2, 2}];
```




```
NSolve[-6 + 4/x^3 + 10 x + 18 x^2 == Tan[Pi / 6], {x}]
```

```
{{x → -1.06826}, {x → 0.574461 + 0.385596 i},
 {x → 0.574461 - 0.385596 i}, {x → -0.318111 + 0.577385 i},
 {x → -0.318111 - 0.577385 i}}
```

v

```
D[f[x], {x, 2}]
```

$$10 - \frac{12}{x^4} + 36\,x$$

■ b

```
f[x_] := (E^x + Cos[x])^x + ((Sin[x / 3 - 3])^2) / (Log[x] - x^2)
```

```
D[f[x], x]
```

$$-\frac{2\,\text{Cos}\left[3-\frac{x}{3}\right]\,\text{Sin}\left[3-\frac{x}{3}\right]}{3\,\left(-x^2+\text{Log}[x]\right)}-\frac{\left(\frac{1}{x}-2\,x\right)\,\text{Sin}\left[3-\frac{x}{3}\right]^2}{\left(-x^2+\text{Log}[x]\right)^2}+$$
$$\left(e^x+\text{Cos}[x]\right)^x\left(\text{Log}\left[e^x+\text{Cos}[x]\right]+\frac{x\,\left(e^x-\text{Sin}[x]\right)}{e^x+\text{Cos}[x]}\right)$$

```
D[f[x], x] // Simplify
```

$$\frac{\text{Sin}\left[6-\frac{2x}{3}\right]}{3\left(x^2-\text{Log}[x]\right)}-\frac{\left(\frac{1}{x}-2x\right)\text{Sin}\left[3-\frac{x}{3}\right]^2}{\left(x^2-\text{Log}[x]\right)^2}+\left(e^x+\text{Cos}[x]\right)^x\left(\text{Log}\left[e^x+\text{Cos}[x]\right]+\frac{x\left(e^x-\text{Sin}[x]\right)}{e^x+\text{Cos}[x]}\right)$$

```
D[(E^x + Cos[x])^x, x]
```

$$\left(e^x+\text{Cos}[x]\right)^x\left(\text{Log}\left[e^x+\text{Cos}[x]\right]+\frac{x\left(e^x-\text{Sin}[x]\right)}{e^x+\text{Cos}[x]}\right)$$

```
D[f[x], x] /. x → (1.)
```

```
5.66414
```

```
Remove["Global`*"]
```

Lösung zu Aufgabe 16

```
p[1] = {0, 0}; p[2] = {1, 1}; p[3] = {2, 0}; p[4] = {3, 1};
p[5] = {4, -1}; p[6] = {6, 0}; p[7] = {7, 3};
```

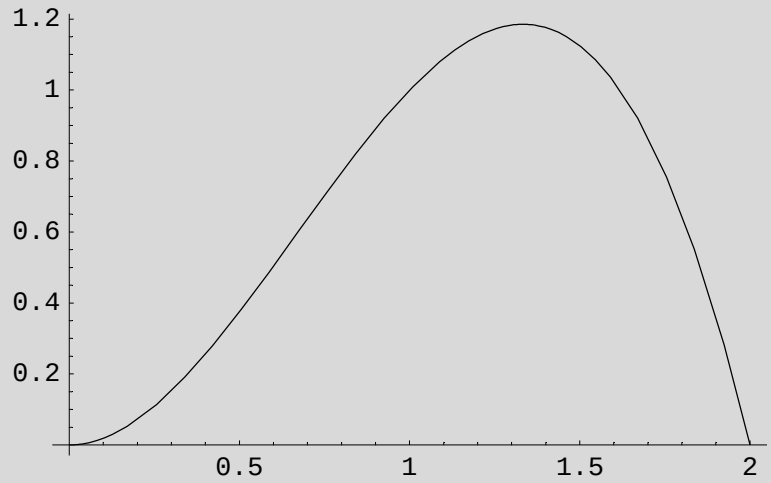
```
f1[x_] := a1 x^3 + b1 x^2 + c1 x + d1;
f2[x_] := a2 x^3 + b2 x^2 + c2 x + d2;
f3[x_] := a3 x^3 + b3 x^2 + c3 x + d3;
```

```
s1 =
 Flatten[Solve[Evaluate[{(D[f1[x], x] == 0 /. x → 0), f1[0] == 0,
     f1[1] == 1, f1[2] == 0}], {a1, b1, c1, d1}]]
```

```
{a1 → -1, b1 → 2, c1 → 0, d1 → 0}
```

```
f1[x_] := (a1 x^3 + b1 x^2 + c1 x + d1 /. s1)
```

```
pl1 = Plot[Evaluate[f1[x]], {x, 0, 2}];
```




```
s2 =
 Flatten[Solve[Evaluate[{(D[f1[x], x] == D[f2[x], x] /. x → 2),
     f2[2] == 0, f2[3] == 1, f2[4] == -1}], {a2, b2, c2, d2}]]
```

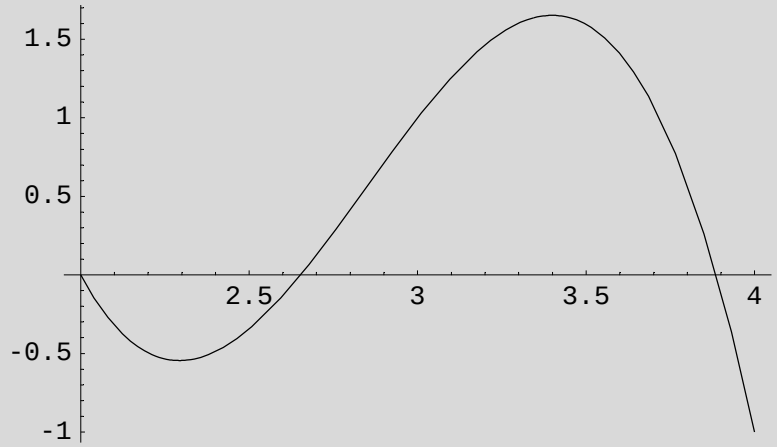
$$\left\{a2 \to -\frac{13}{4},\ b2 \to \frac{111}{4},\ c2 \to -76,\ d2 \to 67\right\}$$

```
N[%]
```

$$\{a2 \to -3.25,\ b2 \to 27.75,\ c2 \to -76.,\ d2 \to 67.\}$$

```
f2[x_] := (a2 x^3 + b2 x^2 + c2 x + d2 /. s2)
```

```
pl2 = Plot[Evaluate[f2[x]], {x, 2, 4}];
```




```
s3 =
 Flatten[Solve[Evaluate[{(D[f2[x], x] == D[f3[x], x] /. x → 4),
     f3[4] == -1, f3[6] == 0, f3[7] == 3}], {a3, b3, c3, d3}]]
```

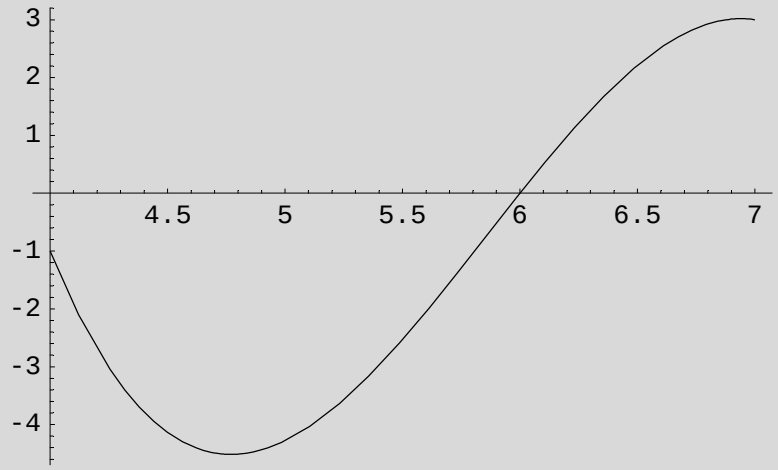
$$\left\{a3 \to -\frac{53}{36},\ b3 \to \frac{931}{36},\ c3 \to -\frac{1316}{9},\ d3 \to \frac{793}{3}\right\}$$

```
N[%]
```

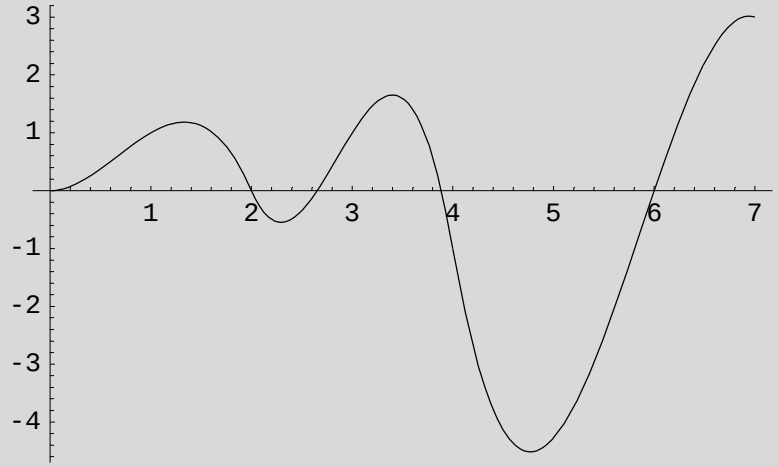
```
{a3 → -1.47222, b3 → 25.8611, c3 → -146.222, d3 → 264.333}
```

```
f3[x_] := (a3 x^3 + b3 x^2 + c3 x + d3 /. s3)
```

```
pl3 = Plot[Evaluate[f3[x]], {x, 4, 7}];
```




```
Show[pl1, pl2, pl3];
```




```
D[f1[x], x] /. x → 2
```

```
-4
```

```
D[f2[x], x] /. x → 4
```

```
-10
```

```
D[f3[x], x] /. x → 7
```

$$-\frac{7}{12}$$

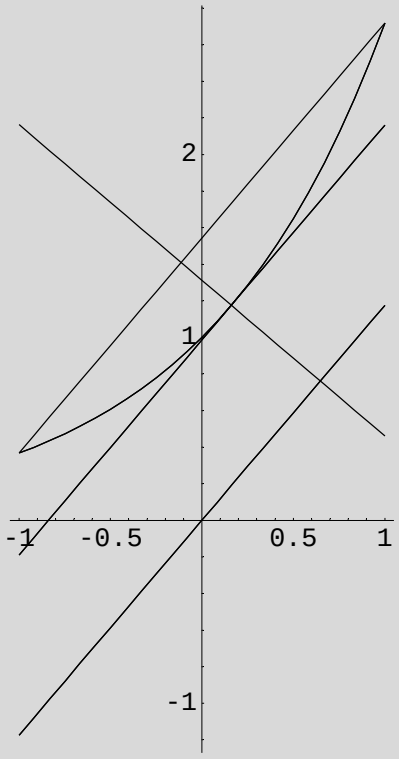
```
N[%]
```

```
-0.583333
```

```
Remove["Global`*"]
```

Lösung zu Aufgabe 17

```
f[x_] := E^x;
Plot[{f[x], -(x - 0.1614393539403694) 2 / (E - 1 / E) +
    E^0.1614393539403694, x (E - 1 / E) / 2 + 0.985477463224239,
   E^x, x (E - 1 / E) / 2, x (E - 1 / E) / 2 + 0.985477463224239,
   x (E - 1 / E) / 2, x (E - 1 / E) / 2 + E - (E - 1 / E) / 2},
  {x, -1, 1}, AspectRatio → Automatic];
```



```
(* Senkrechte *)
-(x - 0.1614393539403694) 2 / (E - 1 / E) +
   E^0.1614393539403694 // Expand // N
```

```
1.31257 - 0.850918 x
```

```
(* Parallele zur Tangente *)
```

```
x (E - 1 / E) / 2 + 0.985477463224239 // Expand // N
```

```
0.985477 + 1.1752 x
```

```
fl1 =
 Solve[1.3125728575551499` - 0.8509181282393216` x == 0, {x}] //
  Flatten
```

```
{x → 1.54254}
```

```
x1 = x /. fl1
```

```
1.54254
```

```
fl2 =
 Solve[0.985477463224239` + 1.1752011936438014` x == 0, {x}] //
  Flatten
```

```
{x → -0.838561}
```

```
x2 = x /. fl2
```

```
-0.838561
```

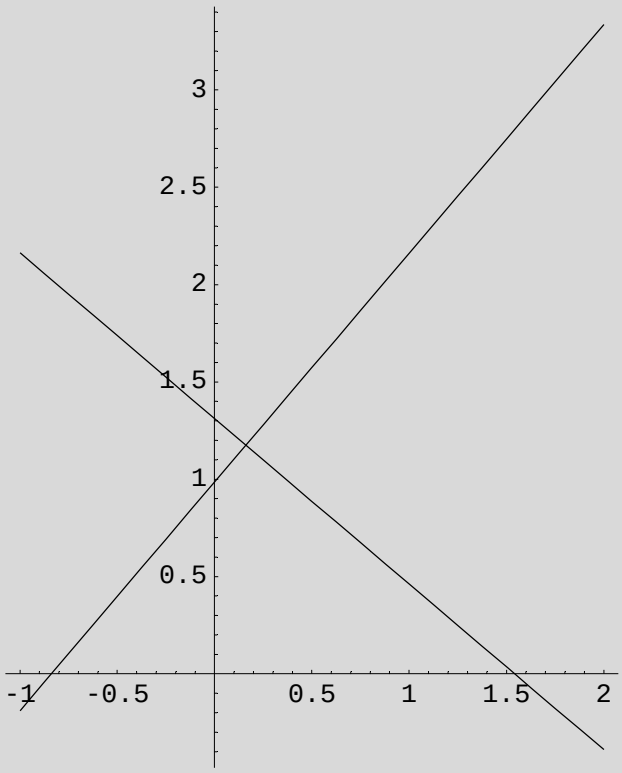
```
basis = x1 - x2
```

```
2.3811
```

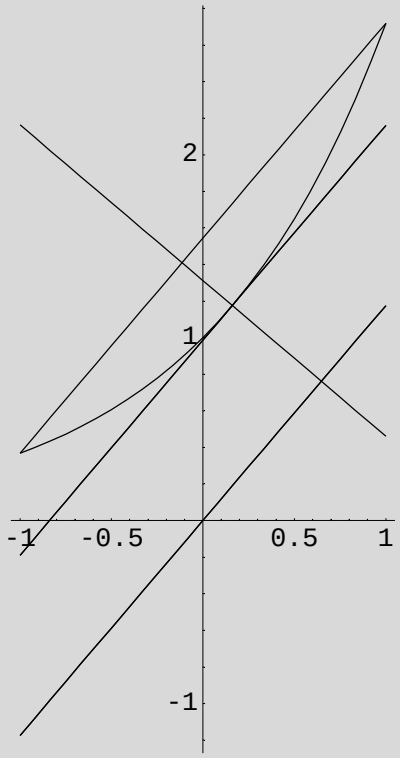
```
(1.5425371889432675 - (-0.8385606384288044`))
 E^0.1614393539403694 / 2
```

```
1.39913
```

```
Plot[{x (E - 1 / E) / 2 +
     (-1 + e^2 + Log[1/2 (-1/e + e)] - e^2 Log[1/2 (-1/e + e)]) / (2 e),
   -(x - 0.1614393539403694) 2 / (E - 1 / E) +
    E^0.1614393539403694}, {x, -1, 2}, AspectRatio → Automatic];
```



```
Plot[{E^x, x (E - 1 / E) / 2, x (E - 1 / E) / 2 + 0.985477463224239,
   x (E - 1 / E) / 2, x (E - 1 / E) / 2 +
    (-1 + e^2 + Log[1/2 (-1/e + e)] - e^2 Log[1/2 (-1/e + e)]) / (2 e),
   -(x - 0.1614393539403694) 2 / (E - 1 / E) +
    E^0.1614393539403694, x (E - 1 / E) / 2 + E - (E - 1 / E) / 2},
  {x, -1, 1}, AspectRatio → Automatic];
```




```
fr = FindRoot[E^x == (E - 1 / E) / 2, {x, 1}]
```

```
{x → 0.161439}
```

```
x0 = x /. fr
```

```
0.161439
```

```
Log[(E - 1 / E) / 2] // N
```

```
0.161439
```

```
h = E^x0
```

```
1.1752
```

```
flaeche = basis h / 2
```

```
1.39913
```

```
Solve[0.1614393539403694 (E - 1 / E) / 2 + b ==
  E^0.1614393539403694, {b}]
```

```
{{b → 0.985477}}
```

```
Solve[
 Log[(E - 1 / E) / 2] (E - 1 / E) / 2 + b == E^Log[(E - 1 / E) / 2], {b}]
```

$$\left\{\left\{b \to \frac{-1+e^2+\text{Log}\left[\frac{1}{2}\left(-\frac{1}{e}+e\right)\right]-e^2\,\text{Log}\left[\frac{1}{2}\left(-\frac{1}{e}+e\right)\right]}{2e}\right\}\right\}$$

```
N[%]
```

```
{{b → 0.985477}}
```

Lösung zu Aufgabe 18

■ a

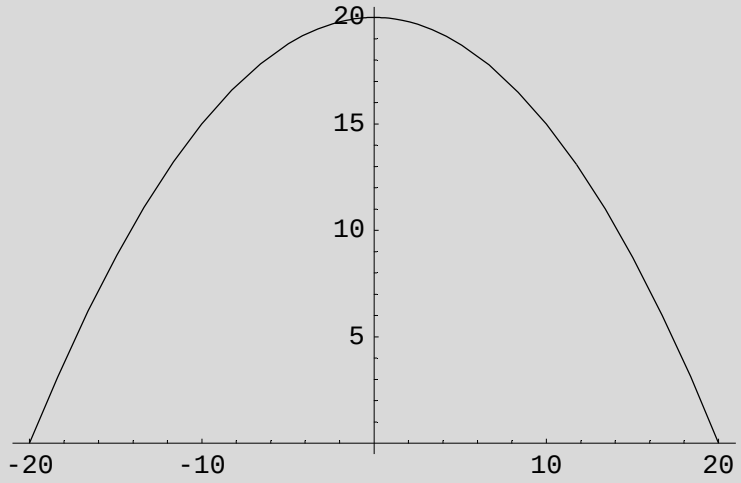
```
Remove["Global`*"]
```

```
f[a_, x_, b_] := -a x^2 + b;
bSolv =
 Flatten[Solve[{f[a, 0, b] == 20, f[a, 20, b] == 0}, {a, b}]]
```

$$\left\{a \to \frac{1}{20},\ b \to 20\right\}$$

```
f[x_] := -1 / 20 x^2 + 20;
```

```
Plot[f[x], {x, -20, 20}];
```

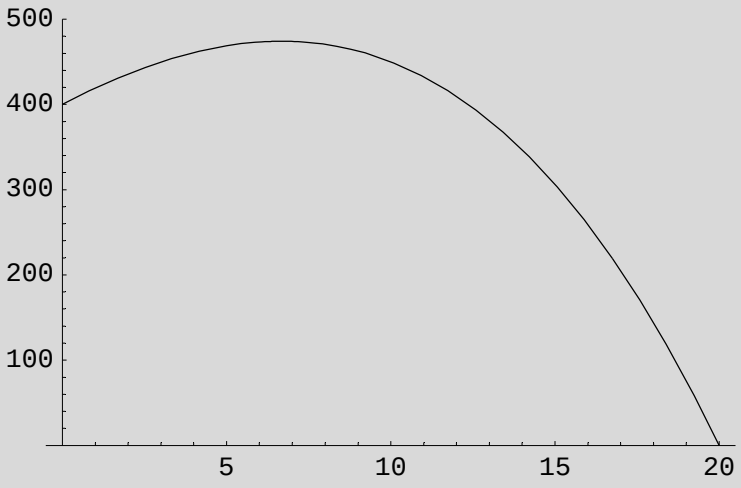


■ b

```
A[x1_] := 2 (x1 f[x1] + (20 - x1) f[x1] / 2) // Simplify; A[x1]
```

$$-\frac{1}{20}\,(-20+x1)\,(20+x1)^2$$

```
Plot[A[x1], {x1, 0, 20}, PlotRange → {0, 500}];
```



```
D[A[x1], x1] // Simplify
```

$$20 - 2\,x1 - \frac{3\,x1^2}{20}$$

```
Solve[Evaluate[D[A[x1], x1] == 0], {x1}]
```

$$\left\{\{x1 \to -20\}, \left\{x1 \to \frac{20}{3}\right\}\right\}$$

```
N[%]
```

$$\{\{x1 \to -20.\}, \{x1 \to 6.66667\}\}$$

```
(Integrate[f[x], {x, -20, 20}] - A[x1]) 20
```

$$20\left(\frac{1600}{3} + \frac{1}{20}\,(-20 + x1)\,(20 + x1)^2\right)$$

```
% // Expand
```

$$\frac{8000}{3} - 400\,x1 + 20\,x1^2 + x1^3$$

```
N[%]
```

$$2666.67 - 400.\,x1 + 20.\,x1^2 + x1^3$$

```
(Integrate[f[x], {x, -20, 20}] - A[x1]) 20 /. {x1 → 20/3}
```

$$\frac{32000}{27}$$

```
N[%]
```

$$1185.19$$

```
Remove["Global`*"]
```

Lösung zu Aufgabe 19

■ a

```
a1 = 1; a2 = -1; a3 = 2; a4 = 12;
a[n_] := α n^3 + β n^2 + γ n + δ;
solv = Solve[{a[1] == a1, a[2] == a2, a[3] == a3, a[4] == a4},
   {α, β, γ, δ}] // Flatten
```

$$\left\{\alpha \to \frac{1}{3},\ \beta \to \frac{1}{2},\ \gamma \to -\frac{35}{6},\ \delta \to 6\right\}$$

```
a[n_] := α n^3 + β n^2 + γ n + δ /. solv; a[n]
```

$$6 - \frac{35\,n}{6} + \frac{n^2}{2} + \frac{n^3}{3}$$

```
a[5]
```

```
31
```

```
s[n_] := Sum[a[k], {k, 1, n}]; s[n]
```

$$\frac{1}{12}\left(38\,n - 31\,n^2 + 4\,n^3 + n^4\right)$$

```
s[100]
```

```
8641150
```

```
s[1000] / s[999]
```

$$\frac{41832043250}{41665129497}$$

```
N[%]
```

```
1.00401
```

■ b

```
b[1] = 1; b[n_] := (n + 1) (b[n - 1] - 1)
```

```
Table[b[n], {n, 1, 50}]
```

```
{1, 0, -4, -25, -156, -1099, -8800, -79209,
 -792100, -8713111, -104557344, -1359245485,
 -19029436804, -285441552075, -4567064833216,
 -77640102164689, -1397521838964420, -26552914940323999,
 -531058298806480000, -11152224274936080021,
 -245348934048593760484, -5643025483117656491155,
 -135432611594823755787744, -3385815289870593894693625,
 -88031197536635441262034276, -2376842333489156914074925479,
 -66551585337696393594097913440,
 -1929995974793195414228839489789,
 -57899879243795862426865184693700,
 -1794896256557671735232820725504731,
 -57436680209845495527450263216151424,
 -1895410446924901352405858686132997025,
 -64443955195446645981799195328521898884,
 -2255538431840632609362971836498266460975,
 -81199383546262773937066986113937592595136,
 -3004377191211722635671478486215690926020069,
 -114166333266045460155516182476196255188762660,
 -44524869973757729460651311165716539523617437779,
 -17809947989503091784260524466286615809446975120 0,
 -73020786756962676315468150311775124818732597992 41,
 -30668730437924324052496623130945552423867691156 8164,
 -13187554088307459342573547946306587542263107197 431095,
 -58025237988552821107323610963748985185957671668 6968224,
 -26111357094848769498295624933687043333680952250 913570125,
 -12011224263630433969215987469496039933493238035 42024225796
,
 -56452754039063039655315141106631387687418218766 4751386124
   59,
 -27097321938750259034551267731183066089960745007 9080665339
   8080,
 -13277687749987626926930121188279702384080765053 8749526016
   505969,
 -66388438749938134634650605941398511920403825269 3747630082
   5298500,
 -33858103762468448663671809030113241079405950887 3811291342
   090223551}
```

```
Table[b[n], {n, 1, 50}] // N
```

```
{1., 0., -4., -25., -156., -1099., -8800., -79209.,
 -792100., -8.71311×10^6, -1.04557×10^8, -1.35925×10^9,
 -1.90294×10^10, -2.85442×10^11, -4.56706×10^12, -7.76401×10^13,
 -1.39752×10^15, -2.65529×10^16, -5.31058×10^17, -1.11522×10^19,
 -2.45349×10^20, -5.64303×10^21, -1.35433×10^23, -3.38582×10^24,
 -8.80312×10^25, -2.37684×10^27, -6.65516×10^28, -1.93×10^30,
 -5.78999×10^31, -1.7949×10^33, -5.74367×10^34, -1.89541×10^36,
 -6.4444×10^37, -2.25554×10^39, -8.11994×10^40, -3.00438×10^42,
 -1.14166×10^44, -4.45249×10^45, -1.78099×10^47, -7.30208×10^48,
 -3.06687×10^50, -1.31876×10^52, -5.80252×10^53, -2.61114×10^55,
 -1.20112×10^57, -5.64528×10^58, -2.70973×10^60,
 -1.32777×10^62, -6.63884×10^63, -3.38581×10^65}
```

■ c

```
Limit[(E^2 - E) / Sum[1 / k!, {k, 0, n}]
  (Cos[n^2] + n^2 - 2 n + 2) / (3 n^3 + 4 n^2 + 2 n - 1)
  (n + 1), n → Infinity]
```

$$\frac{1}{3}\,(-1+e)$$

```
Limit[Sum[1 / k!, {k, 0, n}], n → Infinity]
```

$$e$$