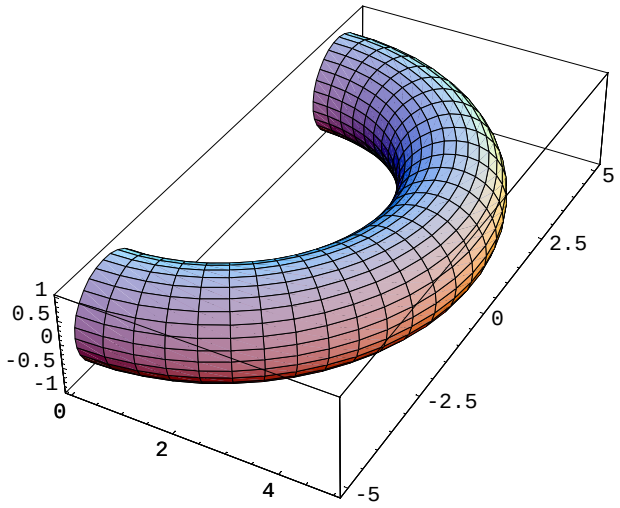
Körper

(Hilfen zu Kleinprojekt)

Volumenformen, Beispiele

```
h[u_,v_,r_]:={
(4+r Cos[u])Sin[v],
(4+r Cos[u])Cos[v],
Sin[u]};
ParametricPlot3D[h[u,v,1],{u,0,2Pi},{v,0,Pi}];
```




```
x[r_,u_,v_]:= (r Cos[u]+4) Sin[v];
y[r_,u_,v_]:= (r Cos[u]+4) Cos[v];
z[r_,u_,v_]:= r Sin[u];
j[r_,u_,v_]:= Det[{
{D[x[r,u,v],{r}],D[x[r,u,v],{u}],D[x[r,u,v],{v}]},
{D[y[r,u,v],{r}],D[y[r,u,v],{u}],D[y[r,u,v],{v}]},
{D[z[r,u,v],{r}],D[z[r,u,v],{u}],D[z[r,u,v],{v}]}
}];
j[r,u,v]//Simplify
```

```
r (4 + r Cos[u])
```

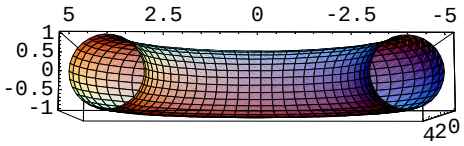
```
Integrate[j[r,u,v],{r,0,1},{u,0,2Pi},{v,0,Pi}]
```

$$4\pi^2$$

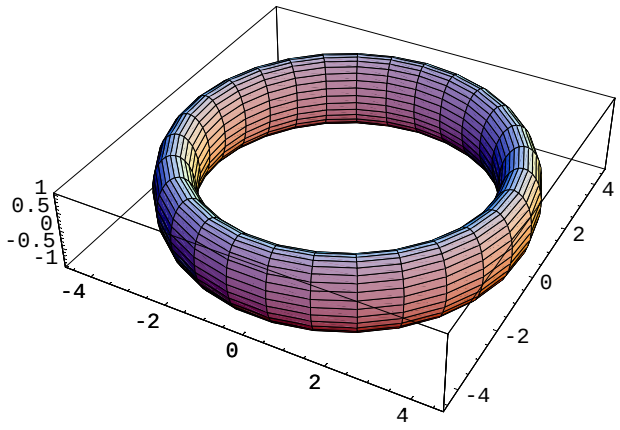
```
N[%]
```

```
39.4784
```

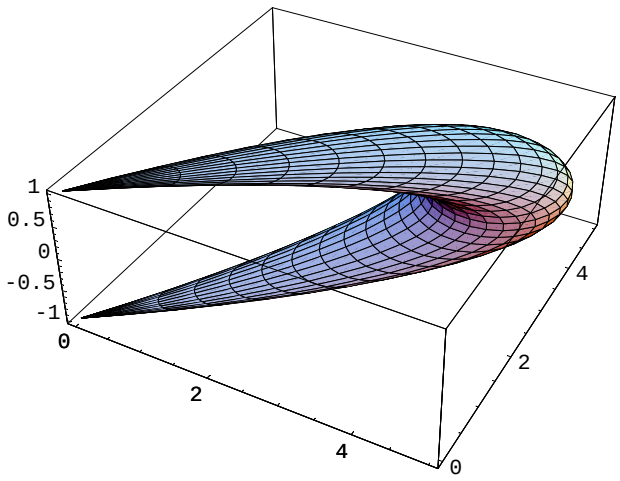
```
h[u_,v_,r_]:={
(4+r Cos[u])Sin[v],
(4+r Cos[u])Cos[v],
Sin[u]};
ParametricPlot3D[h[u,v,1],{u,0,2Pi},{v,0,Pi},ViewPoint->{-3.178, 0.064, -0.285}];
```



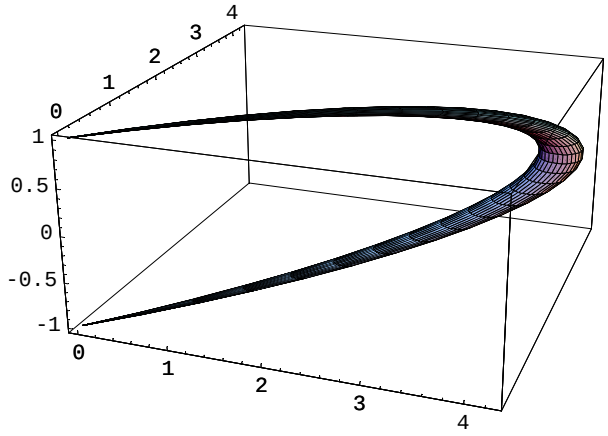

```
h[u_,v_,r_]:={
(4+r Cos[u])Sin[v],
(4+r Cos[u])Cos[v],
Sin[u]};
ParametricPlot3D[h[u,v,0.5],{u,0,2Pi},{v,0,2Pi}];
```



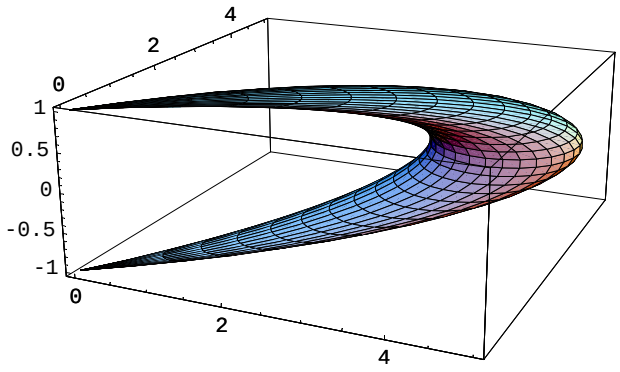

```
h[u_,v_,r_]:={
(4+r Cos[u])Sin[v],
(4+r Sin[u])Sin[v],
Cos[v]};
ParametricPlot3D[h[u,v,1],{u,0,2Pi},{v,0,Pi}];
Transpose[{h[u,v,5]}]//TeXForm;
```



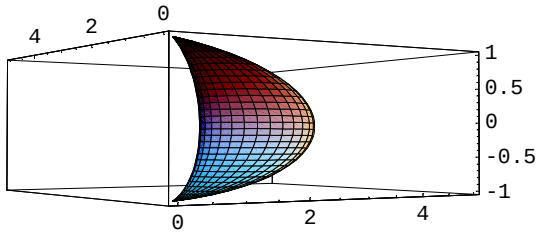
```
h[u_,v_,r_]:={
(4+r Cos[u])Sin[v],
(4+r Sin[u])Sin[v],
Cos[v]};
ParametricPlot3D[h[u,v,0.25],{u,0,2Pi},{v,0,Pi},ViewPoint->{1.010, -2.798, 1.155}];
```



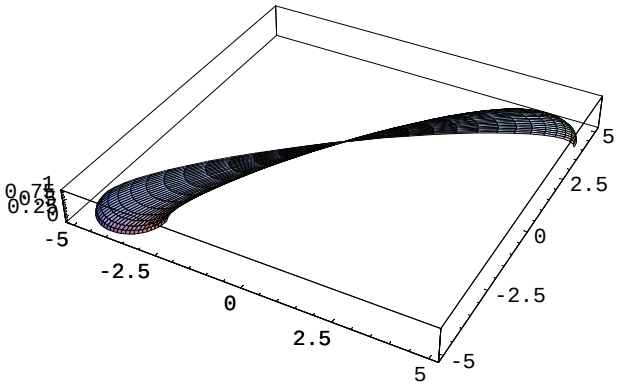

```
h[u_,v_,r_]:={
(4+r Cos[u])Sin[v],
(4+r Sin[u])Sin[v],
Cos[v]};
ParametricPlot3D[h[u,v,1],{u,0,2Pi},{v,0,Pi},ViewPoint->{1.202, -2.688, 0.979}];
```



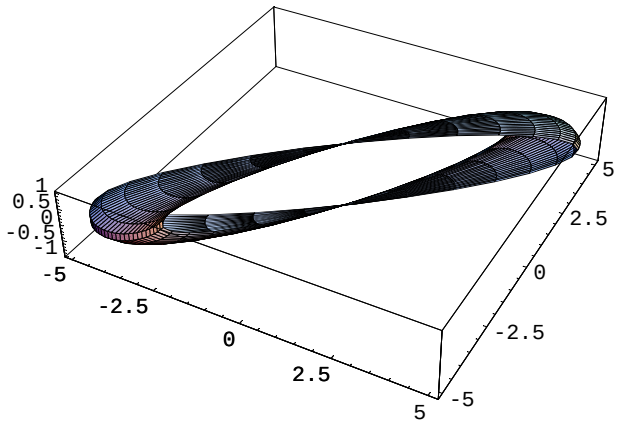

```
h[u_,v_,r_]:={
(4+r Cos[u])Sin[v],
(4+r Sin[u])Sin[v],
Cos[v]};
ParametricPlot3D[h[u,v,1],{u,0,2Pi},{v,0,Pi},ViewPoint->{-1.699, -2.596, -0.059}];
```



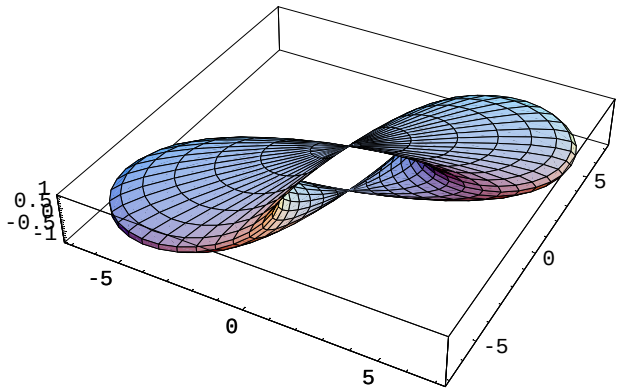
```
h[u_,v_,r_]:={
(4+r Cos[u])Cos[v],
(4+r Sin[u])Cos[v],
Sin[v]};
ParametricPlot3D[h[u,v,1],{u,0,Pi},{v,0,Pi}];
```



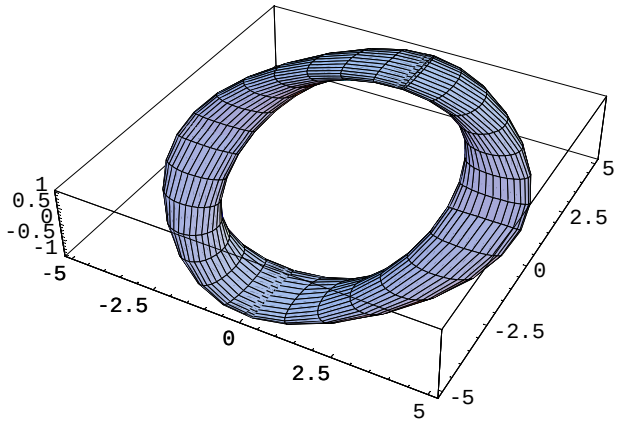

```
h[u_,v_,r_]:={
(4+r Cos[u])Cos[v],
(4+r Sin[u])Cos[v],
Sin[v]};
ParametricPlot3D[h[u,v,1],{u,0,Pi},{v,0,2Pi}];
```



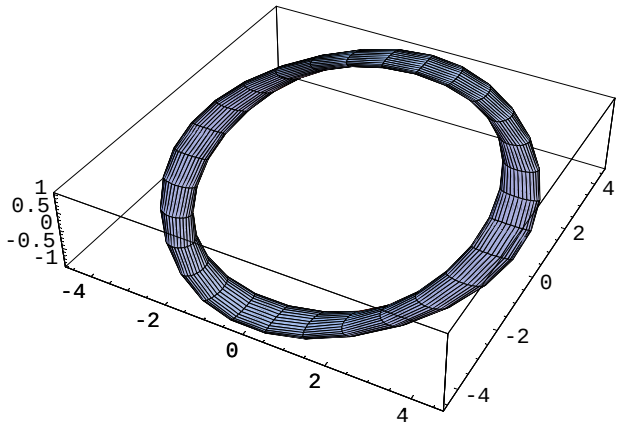

```
h[u_,v_,r_]:={
(4+r Cos[u])Cos[v],
(4+r Sin[u])Cos[v],
Sin[v]};
ParametricPlot3D[h[u,v,3],{u,0,2Pi},{v,0,2Pi}];
```



```
h[u_,v_,r_]:={
(4+r Cos[u])Cos[v],
(4+r Sin[u])Sin[v],
Sin[v]};
ParametricPlot3D[h[u,v,1],{u,0,2Pi},{v,0,2Pi}];
```



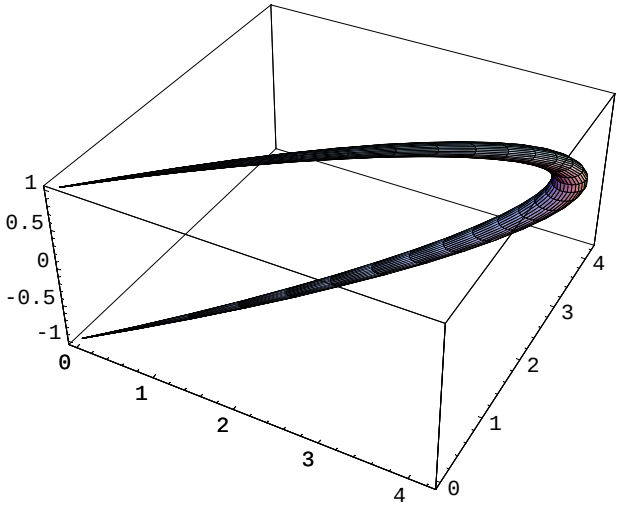

```
h[u_,v_,r_]:={
(4+r Cos[u])Sin[v],
(4+r Sin[u])Cos[v],
Cos[v]};
ParametricPlot3D[h[u,v,0.5],{u,0,2Pi},{v,0,2Pi}];
```

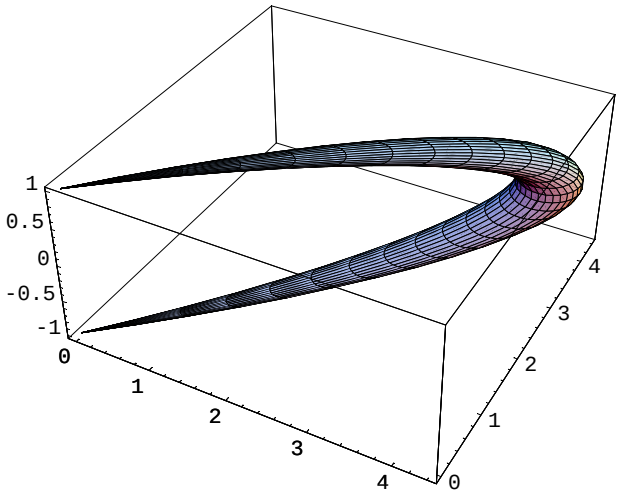


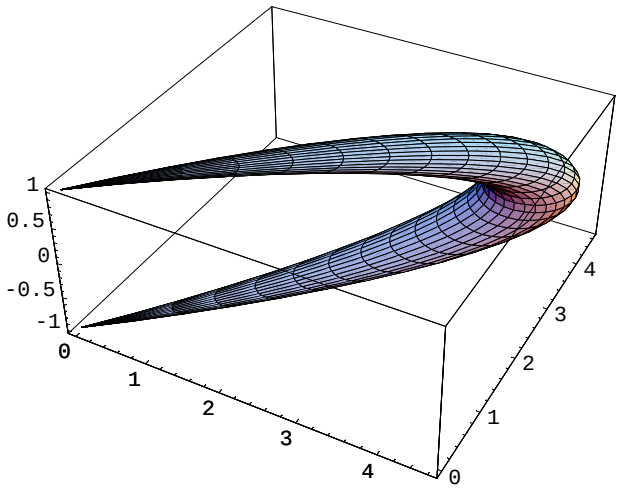
Volumenintegrale, Beispiel 1

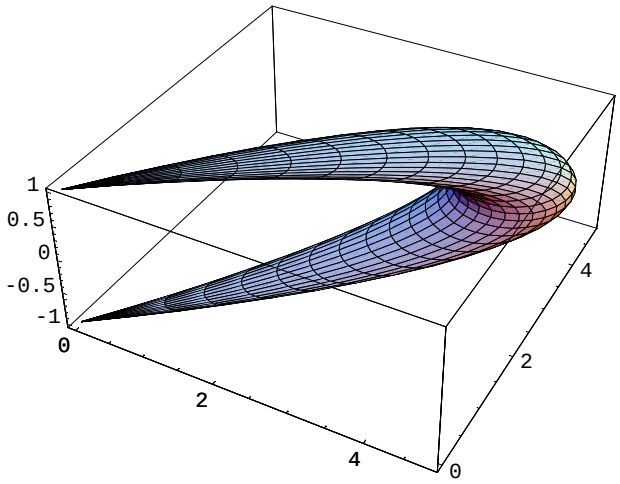
Wir studieren einen Körper, zuerst die Form durch Aufblasen

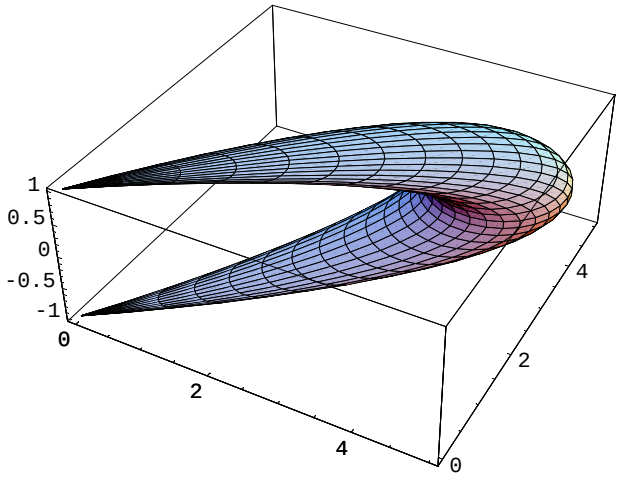
```
h[u_,v_,r_]:={
(4+r Cos[u])Sin[v],
(4+r Sin[u])Sin[v],
Cos[v]};
Table[ParametricPlot3D[h[u,v,r],{u,0,2Pi},{v,0,Pi}],{r,0.2,1,0.2}];
```


1
0.5
0
-0.5
-1
0
1
2
3
4
0
1
2
3
4


1
0.5
0
-0.5
-1
0
1
2
3
4
0
1
2
3
4


1
0.5
0
-0.5
-1
0
1
2
3
4
0
1
2
3
4





==> Durch Veränderung von r wird der Körper aufgeblasen!

Funktionaldeterminante

```
x[u_,v_,r_]:= (4+r Cos[u])Sin[v];
y[u_,v_,r_]:= (4+r Sin[u])Sin[v];
z[u_,v_,r_]:= Cos[v];
jacobi[u_,v_,r_]:={
{D[x[u,v,r],u],D[x[u,v,r],v],D[x[u,v,r],r]},
{D[y[u,v,r],u],D[y[u,v,r],v],D[y[u,v,r],r]},
{D[z[u,v,r],u],D[z[u,v,r],v],D[z[u,v,r],r]}
};
jacobi[u,v,r] //  MatrixForm
```

$$\begin{pmatrix} -r\,\text{Sin}[u]\,\text{Sin}[v] & (4+r\,\text{Cos}[u])\,\text{Cos}[v] & \text{Cos}[u]\,\text{Sin}[v] \\ r\,\text{Cos}[u]\,\text{Sin}[v] & \text{Cos}[v]\,(4+r\,\text{Sin}[u]) & \text{Sin}[u]\,\text{Sin}[v] \\ 0 & -\text{Sin}[v] & 0 \end{pmatrix}$$

```
Det[jacobi[u,v,r]]
```

$$-r\,\text{Cos}[u]^2\,\text{Sin}[v]^3 - r\,\text{Sin}[u]^2\,\text{Sin}[v]^3$$

Volumenintegral

```
Abs[Integrate[Evaluate[Det[jacobi[u,v,r]]],{r,0,1},{v,0,Pi},{u,0,2Pi}]]
```

$$\frac{4\,\pi}{3}$$

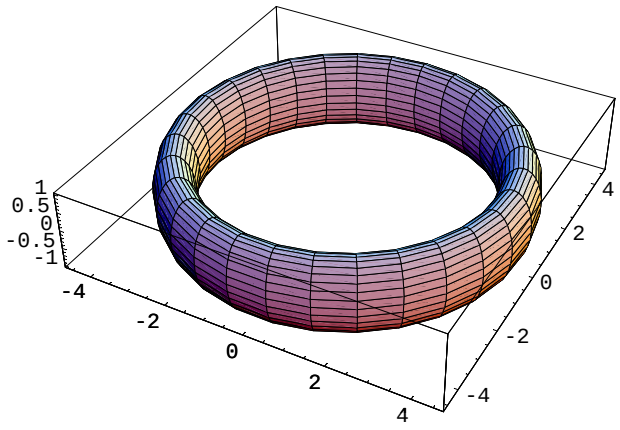
```
Abs[NIntegrate[Evaluate[Det[jacobi[u,v,r]]],{r,0,1},{v,0,Pi},{u,0,2Pi}]]
```

```
4.18879
```

Volumenintegrale, Beispiel 2

Wir studieren einen Körper, zuerst die Form durch Aufblasen

```
h[u_,v_,r_]:={
(4+r Cos[u])Sin[v],
(4+r Cos[u])Cos[v],
Sin[u]};
ParametricPlot3D[h[u,v,0.5],{u,0,2Pi},{v,0,2Pi}];
```



==> Durch Veränderung von r wird der Körper aufgeblasen!

Funktionaldeterminante

```
x[u_,v_,r_]:= (4+r Cos[u])Sin[v];
y[u_,v_,r_]:= (4+r Cos[u])Cos[v];
z[u_,v_,r_]:= Sin[u];
jacobi[u_,v_,r_]:={
{D[x[u,v,r],u],D[x[u,v,r],v],D[x[u,v,r],r]},
{D[y[u,v,r],u],D[y[u,v,r],v],D[y[u,v,r],r]},
{D[z[u,v,r],u],D[z[u,v,r],v],D[z[u,v,r],r]}
};
jacobi[u,v,r] //  MatrixForm
```

$$\begin{pmatrix} -r\,\text{Sin}[u]\,\text{Sin}[v] & (4+r\,\text{Cos}[u])\,\text{Cos}[v] & \text{Cos}[u]\,\text{Sin}[v] \\ -r\,\text{Cos}[v]\,\text{Sin}[u] & -(4+r\,\text{Cos}[u])\,\text{Sin}[v] & \text{Cos}[u]\,\text{Cos}[v] \\ \text{Cos}[u] & 0 & 0 \end{pmatrix}$$

```
Det[jacobi[u,v,r]]
```

$$4\,\text{Cos}[u]^2\,\text{Cos}[v]^2 + r\,\text{Cos}[u]^3\,\text{Cos}[v]^2 + 4\,\text{Cos}[u]^2\,\text{Sin}[v]^2 + r\,\text{Cos}[u]^3\,\text{Sin}[v]^2$$

Volumenintegral

```
Abs[Integrate[Evaluate[Det[jacobi[u,v,r]]],{r,0,1},{v,0,2Pi},{u,0,2Pi}]]
```

$$8\,\pi^2$$

```
Abs[NIntegrate[Evaluate[Det[jacobi[u,v,r]]],{r,0,1},{v,0,2Pi},{u,0,2Pi}]]
```

```
78.9568
```

Volumenintegral Formelvergleich "Querschnitt mal Umfang Schwerpunktskreis"

```
A = 1^2 Pi
```

$$\pi$$

```
Umf = 2 * 4 * Pi
```

$$8\,\pi$$

```
A*Umf
```

$$8\,\pi^2$$

==> Formel richtig!

Hinweise auf Plots und Rechnungen im Matlab / Octave

Siehe http://rowicus.ch/Wir/Matlab_Octave/FileList.html