Lösungen

1. Programm

■ Programm laufen lassen

```
Remove["Global`*"]

volProgr[r_, α_, schritt_] := Module[{},
   h0 = 2 r / Cos[α];
   xh[h_, rR_, αα_] := rR - h Cos[αα];
   z1[x_, rR_, αα_] := -x Tan[αα] + rR Tan[αα];
   z2b[x_, rR_, αα_] := x Tan[Pi / 2 - αα] + rR Tan[αα] + b;
   solv = Solve[z1[xh[h, r, α], r, α] == z2b[xh[h, r, α], r, α], {b}] // Flatten;
   b = b /. solv;
   z2[x_, rR_, αα_, h_] = z2b[x, rR, αα] /. solv;
   IntZ[x_, h_] :=
    Integrate[1, {z, z1[x, r, α], z2[x, r, α, h]}
     , GenerateConditions → False];
   IntZY[x_, h_] :=
    Integrate[IntZ[x, h], {y, -Sqrt[r^2 - x^2], Sqrt[r^2 - x^2]}];
   IntZYX[h_] := (Integrate[IntZY[x, h], x] /. x -> r) -
     (Integrate[IntZY[x, h], x] /. x -> r - h Cos[α]);
   Print["Plot der Kurve 'Volumen als Funktion von h' "];
   Plot[Evaluate[IntZYX[h]], {h, 0, h0}];
   Print["Tabelle zur Kurve 'Volumen als Funktion von h' "];
   Prepend[Table[{h, IntZYX[h] // N}, {h, 0, h0, schritt}], {h, V}] // MatrixForm
  ];
```

■ Eingaben

```
r = 10;  schritt = 1; (* Tabelle am Schluss *)
α = 30 Degree;  α // N
```

```
0.523599
```

volProgr[r, α, schritt]

Plot der Kurve 'Volumen als Funktion von h'

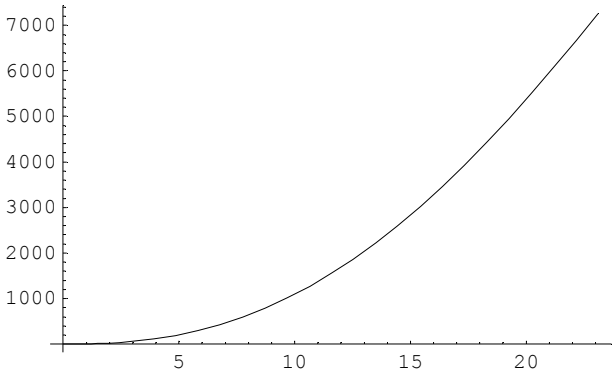

Tabelle zur Kurve 'Volumen als Funktion von h'

h	V
0	0.
1	3.80861
2	21.3392
3	58.2301
4	118.341
5	204.621
6	319.395
7	464.52
8	641.46
9	851.34
10	1094.98
11	1372.94
12	1685.47
13	2032.59
14	2414.06
15	2829.35
16	3277.68
17	3757.94
18	4268.7
19	4808.15
20	5374.01
21	5963.41
22	6572.57
23	7196.14

■ Dierekte Eingaben

■ Erst Programm oben laufen lassen!

```
volProgr[20, 36.75 Degree, 2]
```

Plot der Kurve 'Volumen als Funktion von h'

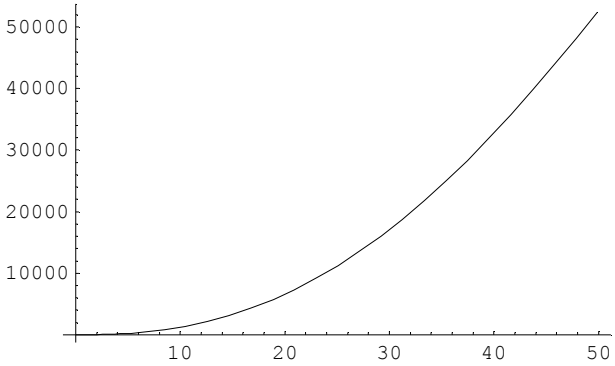

Tabelle zur Kurve 'Volumen als Funktion von h'

h	V
0	0.
2	22.6755
4	127.142
6	347.211
8	706.21
10	1222.12
12	1909.32
14	2779.45
16	3841.92
18	5104.23
20	6572.19
22	8250.01
24	10140.5
26	12244.9
28	14563.3
30	17094.3
32	19835.
34	22781.1
36	25926.9
38	29264.6
40	32784.9
42	36475.9
44	40323.2
46	44308.7
48	48408.7

2. Link

Mögliche Lösung siehe unter unter Beachtung des üblichen Passwortschutzes unter "Handouts" auf

http://rowicus.ch/Wir/Scripts/restricted/MasterIndex.html

Oder (ebenfalls Passwortschutz) unter

http://rowicus.ch/Wir/Scripts/restricted/AnwendMaterial/VivianischerKoerper.pdf

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3. a) und b)

```
Remove["Global`*"]
```

■ Flächeninhalte

```
FI[f1_,f2_,x1_,x2_]:=Integrate[f1[x]-f2[x],{x,x1,x2}]
```

■ a) Beispiel 1: Zwischen -Pi/2 und +Pi/2

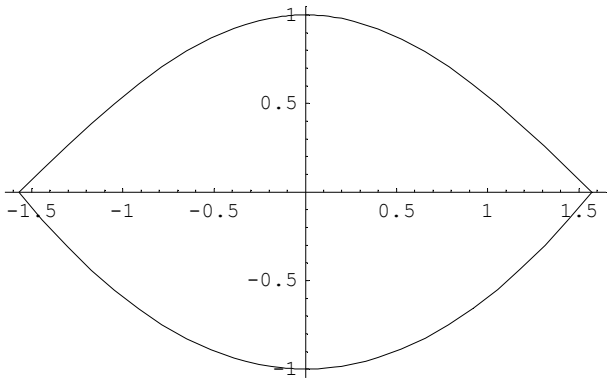
```
x1 = -Pi/2;
x2 =  Pi/2;
f1[x_]:=Cos[x];
f2[x_,a_]:= a x^2 - 1;
solv = Solve[f1[x2]==f2[x2,a],{a}]//Flatten
```

$$\left\{a \to \frac{4}{\pi^2}\right\}$$

```
f2[x_]:= a x^2 - 1/. solv; f2[x]
```

$$-1 + \frac{4\,x^2}{\pi^2}$$

```
Plot[{f1[x],f2[x]},{x,x1,x2}];
```




```
Print[FI[f1,f2,x1,x2]," = ",FI[f1,f2,x1,x2]//N]
```

$$\frac{2\,(3+\pi)}{3} = 4.0944$$

■ b) Beispiel 1: Zwischen -Pi/2 und +Pi/2

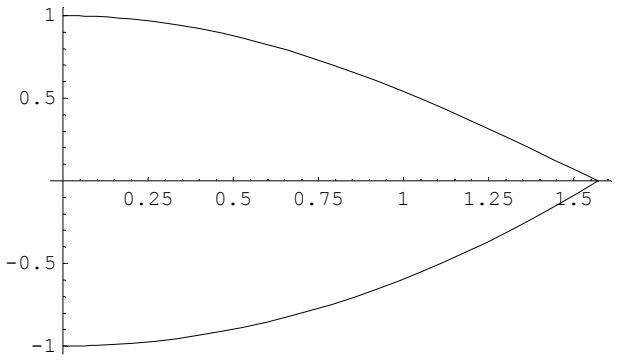
```
x1 = 0;
x2 =  Pi/2;
f1[x_]:=Cos[x];
f2[x_,a_]:= a x^2 - 1;
solv = Solve[f1[x2]==f2[x2,a],{a}]//Flatten
```

$$\left\{a \to \frac{4}{\pi^2}\right\}$$

```
f2[x_]:= a x^2 - 1/. solv; f2[x]
```

$$-1 + \frac{4\,x^2}{\pi^2}$$

```
Plot[{f1[x],f2[x]},{x,x1,x2}];
```




```
Print[FI[f1,f2,x1,x2]," = ",FI[f1,f2,x1,x2]//N]
```

$$\frac{3+\pi}{3} = 2.0472$$

■ Statische Momente

```
SMx[f1_,f2_,x1_,x2_]:=Integrate[(f1[x]-f2[x])(f1[x]+f2[x])/2,{x,x1,x2}]

SMy[f1_,f2_,x1_,x2_]:=Integrate[(f1[x]-f2[x])x,{x,x1,x2}]
```

■ a) Beispiel 1: Zwischen -Pi/2 und +Pi/2

```
x1 = -Pi/2;
x2 =  Pi/2;
f1[x_]:=Cos[x];
f2[x_,a_]:= a x^2 - 1;
solv = Solve[f1[x2]==f2[x2,a],{a}]//Flatten
```

$$\left\{a \to \frac{4}{\pi^2}\right\}$$

```
f2[x_]:= a x^2 - 1/. solv; f2[x]
```

$$-1 + \frac{4\,x^2}{\pi^2}$$

```
Print[SMx[f1,f2,x1,x2]," = ",SMx[f1,f2,x1,x2]//N]
```

$$-\frac{\pi}{60} = -0.0523599$$

```
Print[SMy[f1,f2,x1,x2]," = ",SMy[f1,f2,x1,x2]//N]
```

$$0 = 0.$$

■ b) Beispiel 2: Zwischen 0 und +Pi/2

```
x1 = 0;
x2 = Pi/2;
f1[x_]:=Cos[x];
f2[x_,a_]:= a x^2 - 1;
solv = Solve[f1[x2]==f2[x2,a],{a}]//Flatten
```

$$\left\{a \to \frac{4}{\pi^2}\right\}$$

```
f2[x_]:= a x^2 - 1/. solv; f2[x]
```

$$-1 + \frac{4\,x^2}{\pi^2}$$

```
Print[SMx[f1,f2,x1,x2]," = ",SMx[f1,f2,x1,x2]//N]
```

$$-\frac{\pi}{120} = -0.0261799$$

```
Print[SMy[f1,f2,x1,x2]," = ",SMy[f1,f2,x1,x2]//N]
```

$$\frac{1}{16}\,(-16 + 8\,\pi + \pi^2) = 1.18765$$

■ Schwerpunkt

```
SPx[f1_,f2_,x1_,x2_]:=SMx[f1,f2,x1,x2]/FI[f1,f2,x1,x2]

SPy[f1_,f2_,x1_,x2_]:=SMy[f1,f2,x1,x2]/FI[f1,f2,x1,x2]
```

■ a) Beispiel 1: Zwischen -Pi/2 und +Pi/2

```
x1 = -Pi/2;
x2 =  Pi/2;
f1[x_]:=Cos[x];
f2[x_,a_]:= a x^2 - 1;
solv = Solve[f1[x2]==f2[x2,a],{a}]//Flatten
```

$$\left\{a \to \frac{4}{\pi^2}\right\}$$

```
f2[x_]:= a x^2 - 1/. solv; f2[x]
```

$$-1 + \frac{4\,x^2}{\pi^2}$$

```
Print[SPx[f1,f2,x1,x2]," = ",SPx[f1,f2,x1,x2]//N]
```

$$-\frac{\pi}{40\,(3+\pi)} = -0.0127882$$

```
Print[SPy[f1,f2,x1,x2]," = ",SPy[f1,f2,x1,x2]//N]
```

```
0 = 0.
```

■ b) Beispiel 2: Zwischen 0 und +Pi/2

```
x1 = 0;
x2 = Pi/2;
f1[x_]:=Cos[x];
f2[x_,a_]:= a x^2 - 1;
solv = Solve[f1[x2]==f2[x2,a],{a}]//Flatten
```

$$\left\{a \to \frac{4}{\pi^2}\right\}$$

```
f2[x_]:= a x^2 - 1/. solv; f2[x]
```

$$-1 + \frac{4\,x^2}{\pi^2}$$

```
Print[SPx[f1,f2,x1,x2]," = ",SPx[f1,f2,x1,x2]//N]
```

$$-\frac{\pi}{40\,(3+\pi)} = -0.0127882$$

```
Print[SPy[f1,f2,x1,x2]," = ",SPy[f1,f2,x1,x2]//N]
```

$$\frac{3\,(-16+8\,\pi+\pi^2)}{16\,(3+\pi)} = 0.580133$$

■ Trägheitsmomente

```
TMx[f1_,f2_,x1_,x2_]:=Integrate[Integrate[y^2,{y,f1[x],f2[x]}],{x,x1,x2}]

TMy[f1_,f2_,x1_,x2_]:=Integrate[Integrate[x^2,{y,f1[x],f2[x]}],{x,x1,x2}]
```

■ a) Beispiel 1: Zwischen -Pi/2 und +Pi/2

```
x1 = -Pi/2;
x2 =  Pi/2;
f1[x_]:=Cos[x];
f2[x_,a_]:= a x^2 - 1;
solv = Solve[f1[x2]==f2[x2,a],{a}]//Flatten
```

$$\left\{a \to \frac{4}{\pi^2}\right\}$$

```
f2[x_]:= a x^2 - 1/. solv; f2[x]
```

$$-1 + \frac{4\,x^2}{\pi^2}$$

```
Print[TMx[f1,f2,x1,x2]," = ",TMx[f1,f2,x1,x2]//N]
```

$$-\frac{4}{315}\,(35+12\,\pi) = -0.923163$$

```
Print[TMy[f1,f2,x1,x2]," = ",TMy[f1,f2,x1,x2]//N]
```

$$4 - \frac{\pi^2}{2} - \frac{\pi^3}{30} = -1.96834$$

■ **b) Beispiel 2: Zwischen 0 und +Pi/2**

```
x1 = 0;
x2 = Pi/2;
f1[x_]:=Cos[x];
f2[x_,a_]:= a x^2 - 1;
solv = Solve[f1[x2]==f2[x2,a],{a}]//Flatten
```

$$\left\{a \to \frac{4}{\pi^2}\right\}$$

```
f2[x_]:= a x^2 - 1/. solv; f2[x]
```

$$-1 + \frac{4\,x^2}{\pi^2}$$

```
Print[SPx[f1,f2,x1,x2]," = ",SPx[f1,f2,x1,x2]//N]
```

$$-\frac{\pi}{40\,(3+\pi)} = -0.0127882$$

```
Print[SPy[f1,f2,x1,x2]," = ",SPy[f1,f2,x1,x2]//N]
```

$$\frac{3\,(-16+8\,\pi+\pi^2)}{16\,(3+\pi)} = 0.580133$$

---

3. c)

■ **Eigenes Projekt. Somit keine Standardlösung!**