Lösungen

```
Remove["Global`*"]
```

1

Wir verwenden hier die Skalierung nach der Periode 2 Pi. Das vereinfacht die Rechung etwas.

```
n=10; w = 2 Pi/n;
{x[0],y[0]}={0 w,0};
{x[1],y[1]}={1 w,0.309};
{x[2],y[2]}={2 w,0.588};
{x[3],y[3]}={3 w,0.809};
{x[4],y[4]}={4 w,0.951};
{x[5],y[5]}={5 w,0.99};
{x[6],y[6]}={6 w,0.951};
{x[7],y[7]}={7 w,0.809};
{x[8],y[8]}={8 w,0.588};
{x[9],y[9]}={9 w,0.3};

p[k_]:= {x[k],y[k]};
Table[p[k],{k,0,n-1}]
```

$$\left\{\{0, 0\}, \left\{\frac{\pi}{5}, 0.309\right\}, \left\{\frac{2\pi}{5}, 0.588\right\}, \left\{\frac{3\pi}{5}, 0.809\right\}, \left\{\frac{4\pi}{5}, 0.951\right\}, \{\pi, 0.99\}, \left\{\frac{6\pi}{5}, 0.951\right\}, \left\{\frac{7\pi}{5}, 0.809\right\}, \left\{\frac{8\pi}{5}, 0.588\right\}, \left\{\frac{9\pi}{5}, 0.3\right\}\right\}$$

```
epi=Prepend[Map[Point,Table[p[k],{k,0,n-1}]],PointSize[0.03]]
```

$$\left\{\text{PointSize}[0.03], \text{Point}[\{0, 0\}], \text{Point}\left[\left\{\frac{\pi}{5}, 0.309\right\}\right], \text{Point}\left[\left\{\frac{2\pi}{5}, 0.588\right\}\right], \text{Point}\left[\left\{\frac{3\pi}{5}, 0.809\right\}\right], \text{Point}\left[\left\{\frac{4\pi}{5}, 0.951\right\}\right], \text{Point}[\{\pi, 0.99\}], \text{Point}\left[\left\{\frac{6\pi}{5}, 0.951\right\}\right], \text{Point}\left[\left\{\frac{7\pi}{5}, 0.809\right\}\right], \text{Point}\left[\left\{\frac{8\pi}{5}, 0.588\right\}\right], \text{Point}\left[\left\{\frac{9\pi}{5}, 0.3\right\}\right]\right\}$$

```
r = E^(-I 2 Pi/n);
c[s_]:= 1/n Sum[y[k] r^(s k),{k,0,n-1}];
Table[c[s],{s,0,10}]//N
```

$$\{0.6295, -0.217264 - 0.000529007\,i, -0.0494452 - 0.000855951\,i, -0.0232856 - 0.000855951\,i, -0.0178048 - 0.000529007\,i, -0.0139, -0.0178048 + 0.000529007\,i, -0.0232856 + 0.000855951\,i, -0.0494452 + 0.000855951\,i, -0.217264 + 0.000529007\,i, 0.6295\}$$

```
?ExpToTrig
```

```
ExpToTrig[expr] converts exponentials in expr to trigonometric functions. Mehr...
```

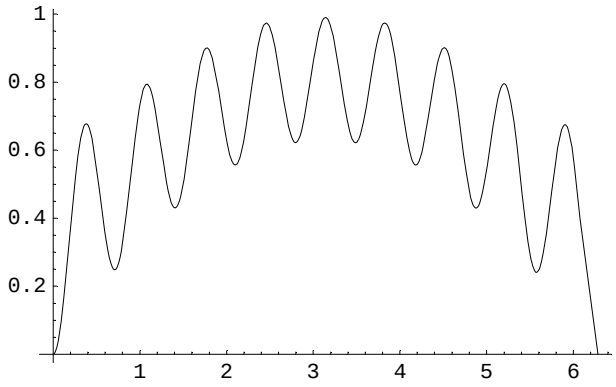
```
fS[t_]:=Sum[c[k] E^(I k t),{k,0,n-1}];
fS[t]
```

$$0.6295 - (0.217264 + 0.000529007\,i)\,e^{i\,t} - (0.0494452 + 0.000855951\,i)\,e^{2\,i\,t} - (0.0232856 + 0.000855951\,i)\,e^{3\,i\,t} - (0.0178048 + 0.000529007\,i)\,e^{4\,i\,t} - 0.0139\,e^{5\,i\,t} - (0.0178048 - 0.000529007\,i)\,e^{6\,i\,t} - (0.0232856 - 0.000855951\,i)\,e^{7\,i\,t} - (0.0494452 - 0.000855951\,i)\,e^{8\,i\,t} - (0.217264 - 0.000529007\,i)\,e^{9\,i\,t}$$

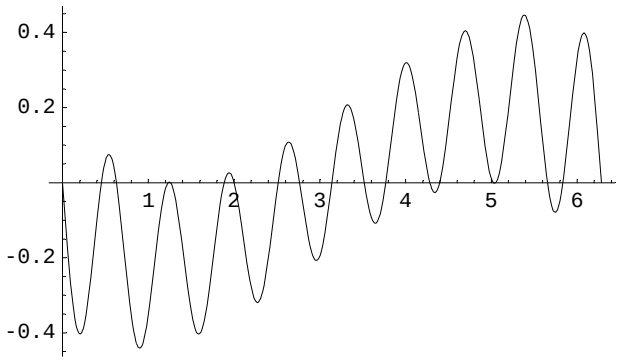
```
fS[t]//ExpToTrig
```

$$0.6295 - (0.217264 + 0.000529007\,i)\,\text{Cos}[t] - (0.0494452 + 0.000855951\,i)\,\text{Cos}[2\,t] - (0.0232856 + 0.000855951\,i)\,\text{Cos}[3\,t] - (0.0178048 + 0.000529007\,i)\,\text{Cos}[4\,t] - 0.0139\,\text{Cos}[5\,t] - (0.0178048 - 0.000529007\,i)\,\text{Cos}[6\,t] - (0.0232856 - 0.000855951\,i)\,\text{Cos}[7\,t] - (0.0494452 - 0.000855951\,i)\,\text{Cos}[8\,t] - (0.217264 - 0.000529007\,i)\,\text{Cos}[9\,t] + (0.000529007 - 0.217264\,i)\,\text{Sin}[t] + (0.000855951 - 0.0494452\,i)\,\text{Sin}[2\,t] + (0.000855951 - 0.0232856\,i)\,\text{Sin}[3\,t] + (0.000529007 - 0.0178048\,i)\,\text{Sin}[4\,t] - 0.0139\,i\,\text{Sin}[5\,t] - (0.000529007 + 0.0178048\,i)\,\text{Sin}[6\,t] - (0.000855951 + 0.0232856\,i)\,\text{Sin}[7\,t] - (0.000855951 + 0.0494452\,i)\,\text{Sin}[8\,t] - (0.000529007 + 0.217264\,i)\,\text{Sin}[9\,t]$$

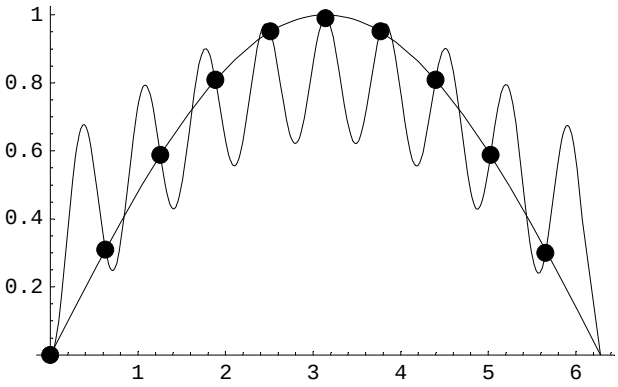
```
Plot[Re[fS[t]],{t,0,2Pi}];
```




```
Plot[Im[fS[t]],{t,0,2Pi}];
```



```
Plot[{Re[fS[t]],Sin[t/2]},{t,0,2Pi},Epilog->epi];
```



Wie man sieht, liegen die verwendeten Punkte auf der Linie von Sin[t/2]. Der Fehler (z.B. grosser Imaginäranteil stammt vermutlich davon, dass so nur wenige Koeffizienten berechnet werden können.)

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2 (Das Problem der Periodengrenzen)

```
Remove["Global`*"]
```

$$f(t) = e^{\cos(t)} + \sin(t)^2,\quad T = 2\ ,\ t_k = \frac{2\pi}{16}, k = 0, 1, ..., 15$$

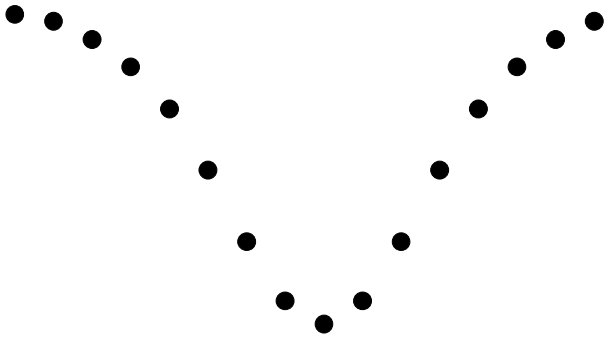
Definition der Funktion und Berechnung der Punkte

```
f[t_] := E^Cos[t] + Sin[t]^2;
n = 16; w = 2 Pi / n;
{x[k_], y[k_]} = {k 2 Pi / 16, f[k 2 Pi / n]};
p[k_] := {x[k], y[k]};
Table[p[k] // N, {k, 0, n - 1}] // TableForm
```

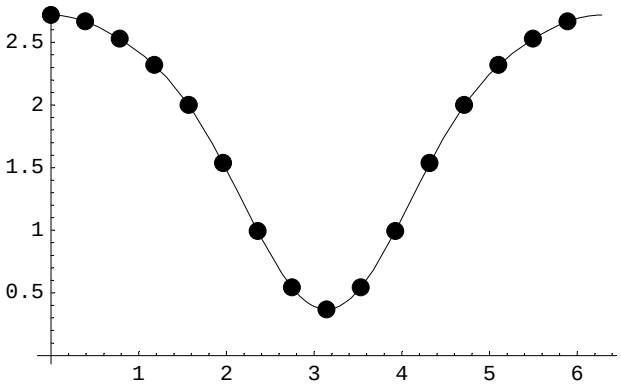
```
0.          2.71828
0.392699    2.66549
0.785398    2.52811
1.1781      2.31977
1.5708      2.
1.9635      1.53558
2.35619     0.993069
2.74889     0.543423
3.14159     0.367879
3.53429     0.543423
3.92699     0.993069
4.31969     1.53558
4.71239     2.
5.10509     2.31977
5.49779     2.52811
5.89049     2.66549
```

Plot der Funktion und der Punkte

```
epi=Prepend[Map[Point,Table[p[k],{k,0,n-1}]],PointSize[0.03]];
Show[Graphics[epi]];
```



```
Plot[f[t], {t, 0, 2 Pi}, Epilog → epi];
```



Berechnung der Koeffizienten mittels der DFT

```
r = E^(-I 2 Pi/n);
c[s_]:= 1/n Sum[y[k] r^(s k),{k,0,n-1}];
Table[c[s],{s,0,n-1}]//N
```

$$\{1.76607,\ 0.565159 - 5.55112\times10^{-17}\,i,\ -0.114252 + 5.55112\times10^{-17}\,i,\ 0.0221684 + 2.77556\times10^{-17}\,i,\ 0.00273712 + 0.\,i,\ 0.000271463 + 2.77556\times10^{-17}\,i,\ 0.0000224889 + 6.93889\times10^{-17}\,i,\ 1.60474\times10^{-6} - 4.16334\times10^{-17}\,i,\ 1.99212\times10^{-7},\ 1.60474\times10^{-6} + 4.16334\times10^{-17}\,i,\ 0.0000224889 - 6.93889\times10^{-17}\,i,\ 0.000271463 - 2.77556\times10^{-17}\,i,\ 0.00273712 + 0.\,i,\ 0.0221684 - 2.77556\times10^{-17}\,i,\ -0.114252 - 5.55112\times10^{-17}\,i,\ 0.565159 + 5.55112\times10^{-17}\,i\}$$

```
Table[c[s], {s, -n + 1, n - 1}] // N // Chop
```

$$\{0.565159,\ -0.114252,\ 0.0221684,\ 0.00273712,\ 0.000271463,\ 0.0000224889,\ 1.60474\times10^{-6},\ 1.99212\times10^{-7},\ 1.60474\times10^{-6},\ 0.0000224889,\ 0.000271463,\ 0.00273712,\ 0.0221684,\ -0.114252,\ 0.565159,\ 1.76607,\ 0.565159,\ -0.114252,\ 0.0221684,\ 0.00273712,\ 0.000271463,\ 0.0000224889,\ 1.60474\times10^{-6},\ 1.99212\times10^{-7},\ 1.60474\times10^{-6},\ 0.0000224889,\ 0.000271463,\ 0.00273712,\ 0.0221684,\ -0.114252,\ 0.565159\}$$

Berechnung der trigonometrischen Reihe durch die gegebenen Punkte (ohne Vereinfachung, aus Zeitgründen)

```
fS[t_]:=Sum[c[k] E^(I k t),{k,0,n-1}];
fS[t]
```

$$\frac{1}{16} e^{8 i t} \left(6+\frac{1}{e}+e+2 e^{-\frac{1}{\sqrt{2}}}+2 e^{\frac{1}{\sqrt{2}}}-e^{\text{Cos}\left[\frac{\pi}{8}\right]}-e^{\text{Cos}\left[\frac{3 \pi}{8}\right]}-e^{\text{Cos}\left[\frac{5 \pi}{8}\right]}-\right.$$
$$\left. e^{\text{Cos}\left[\frac{7 \pi}{8}\right]}-e^{\text{Cos}\left[\frac{9 \pi}{8}\right]}-e^{\text{Cos}\left[\frac{11 \pi}{8}\right]}-e^{\text{Cos}\left[\frac{13 \pi}{8}\right]}-e^{\text{Cos}\left[\frac{15 \pi}{8}\right]}-\text{Sin}\left[\frac{\pi}{8}\right]^2-\text{Sin}\left[\frac{3 \pi}{8}\right]^2-\right.$$
$$\left.\text{Sin}\left[\frac{5 \pi}{8}\right]^2-\text{Sin}\left[\frac{7 \pi}{8}\right]^2-\text{Sin}\left[\frac{9 \pi}{8}\right]^2-\text{Sin}\left[\frac{11 \pi}{8}\right]^2-\text{Sin}\left[\frac{13 \pi}{8}\right]^2-\text{Sin}\left[\frac{15 \pi}{8}\right]^2\right)+$$
$$\frac{1}{16}\left(6+\frac{1}{e}+e+2 e^{-\frac{1}{\sqrt{2}}}+2 e^{\frac{1}{\sqrt{2}}}+e^{\text{Cos}\left[\frac{\pi}{8}\right]}+e^{\text{Cos}\left[\frac{3 \pi}{8}\right]}+e^{\text{Cos}\left[\frac{5 \pi}{8}\right]}+e^{\text{Cos}\left[\frac{7 \pi}{8}\right]}+e^{\text{Cos}\left[\frac{9 \pi}{8}\right]}+\right.$$
$$\left. e^{\text{Cos}\left[\frac{11 \pi}{8}\right]}+e^{\text{Cos}\left[\frac{13 \pi}{8}\right]}+e^{\text{Cos}\left[\frac{15 \pi}{8}\right]}+\text{Sin}\left[\frac{\pi}{8}\right]^2+\text{Sin}\left[\frac{3 \pi}{8}\right]^2+\text{Sin}\left[\frac{5 \pi}{8}\right]^2+\right.$$
$$\left.\text{Sin}\left[\frac{7 \pi}{8}\right]^2+\text{Sin}\left[\frac{9 \pi}{8}\right]^2+\text{Sin}\left[\frac{11 \pi}{8}\right]^2+\text{Sin}\left[\frac{13 \pi}{8}\right]^2+\text{Sin}\left[\frac{15 \pi}{8}\right]^2\right)+$$
$$\frac{1}{16} e^{12 i t}\left(2+\frac{1}{e}+e-2 e^{-\frac{1}{\sqrt{2}}}-2 e^{\frac{1}{\sqrt{2}}}+i\left(e^{\text{Cos}\left[\frac{\pi}{8}\right]}+\text{Sin}\left[\frac{\pi}{8}\right]^2\right)-i\left(e^{\text{Cos}\left[\frac{3 \pi}{8}\right]}+\text{Sin}\left[\frac{3 \pi}{8}\right]^2\right)+\right.$$
$$i\left(e^{\text{Cos}\left[\frac{5 \pi}{8}\right]}+\text{Sin}\left[\frac{5 \pi}{8}\right]^2\right)-i\left(e^{\text{Cos}\left[\frac{7 \pi}{8}\right]}+\text{Sin}\left[\frac{7 \pi}{8}\right]^2\right)+i\left(e^{\text{Cos}\left[\frac{9 \pi}{8}\right]}+\text{Sin}\left[\frac{9 \pi}{8}\right]^2\right)-$$
$$\left. i\left(e^{\text{Cos}\left[\frac{11 \pi}{8}\right]}+\text{Sin}\left[\frac{11 \pi}{8}\right]^2\right)+i\left(e^{\text{Cos}\left[\frac{13 \pi}{8}\right]}+\text{Sin}\left[\frac{13 \pi}{8}\right]^2\right)-i\left(e^{\text{Cos}\left[\frac{15 \pi}{8}\right]}+\text{Sin}\left[\frac{15 \pi}{8}\right]^2\right)\right)+$$
$$\frac{1}{16} e^{4 i t}\left(2+\frac{1}{e}+e-2 e^{-\frac{1}{\sqrt{2}}}-2 e^{\frac{1}{\sqrt{2}}}-i\left(e^{\text{Cos}\left[\frac{\pi}{8}\right]}+\text{Sin}\left[\frac{\pi}{8}\right]^2\right)+i\left(e^{\text{Cos}\left[\frac{3 \pi}{8}\right]}+\text{Sin}\left[\frac{3 \pi}{8}\right]^2\right)-\right.$$
$$i\left(e^{\text{Cos}\left[\frac{5 \pi}{8}\right]}+\text{Sin}\left[\frac{5 \pi}{8}\right]^2\right)+i\left(e^{\text{Cos}\left[\frac{7 \pi}{8}\right]}+\text{Sin}\left[\frac{7 \pi}{8}\right]^2\right)-i\left(e^{\text{Cos}\left[\frac{9 \pi}{8}\right]}+\text{Sin}\left[\frac{9 \pi}{8}\right]^2\right)+$$
$$\left. i\left(e^{\text{Cos}\left[\frac{11 \pi}{8}\right]}+\text{Sin}\left[\frac{11 \pi}{8}\right]^2\right)-i\left(e^{\text{Cos}\left[\frac{13 \pi}{8}\right]}+\text{Sin}\left[\frac{13 \pi}{8}\right]^2\right)+i\left(e^{\text{Cos}\left[\frac{15 \pi}{8}\right]}+\text{Sin}\left[\frac{15 \pi}{8}\right]^2\right)\right)+$$
$$\frac{1}{16} e^{15 i t}\left(-\frac{1}{e}+e+e^{-\frac{3 i \pi}{4}}\left(\frac{1}{2}+e^{-\frac{1}{\sqrt{2}}}\right)+e^{\frac{3 i \pi}{4}}\left(\frac{1}{2}+e^{-\frac{1}{\sqrt{2}}}\right)+e^{-\frac{i \pi}{4}}\left(\frac{1}{2}+e^{\frac{1}{\sqrt{2}}}\right)+\right.$$
$$e^{\frac{i \pi}{4}}\left(\frac{1}{2}+e^{\frac{1}{\sqrt{2}}}\right)+e^{\frac{i \pi}{8}}\left(e^{\text{Cos}\left[\frac{\pi}{8}\right]}+\text{Sin}\left[\frac{\pi}{8}\right]^2\right)+e^{\frac{3 i \pi}{8}}\left(e^{\text{Cos}\left[\frac{3 \pi}{8}\right]}+\text{Sin}\left[\frac{3 \pi}{8}\right]^2\right)+$$
$$e^{\frac{5 i \pi}{8}}\left(e^{\text{Cos}\left[\frac{5 \pi}{8}\right]}+\text{Sin}\left[\frac{5 \pi}{8}\right]^2\right)+e^{\frac{7 i \pi}{8}}\left(e^{\text{Cos}\left[\frac{7 \pi}{8}\right]}+\text{Sin}\left[\frac{7 \pi}{8}\right]^2\right)+$$
$$e^{-\frac{7 i \pi}{8}}\left(e^{\text{Cos}\left[\frac{9 \pi}{8}\right]}+\text{Sin}\left[\frac{9 \pi}{8}\right]^2\right)+e^{-\frac{5 i \pi}{8}}\left(e^{\text{Cos}\left[\frac{11 \pi}{8}\right]}+\text{Sin}\left[\frac{11 \pi}{8}\right]^2\right)+$$
$$\left. e^{-\frac{3 i \pi}{8}}\left(e^{\text{Cos}\left[\frac{13 \pi}{8}\right]}+\text{Sin}\left[\frac{13 \pi}{8}\right]^2\right)+e^{-\frac{i \pi}{8}}\left(e^{\text{Cos}\left[\frac{15 \pi}{8}\right]}+\text{Sin}\left[\frac{15 \pi}{8}\right]^2\right)\right)+$$
$$\frac{1}{16} e^{i t}\left(-\frac{1}{e}+e+e^{-\frac{3 i \pi}{4}}\left(\frac{1}{2}+e^{-\frac{1}{\sqrt{2}}}\right)+e^{\frac{3 i \pi}{4}}\left(\frac{1}{2}+e^{-\frac{1}{\sqrt{2}}}\right)+e^{-\frac{i \pi}{4}}\left(\frac{1}{2}+e^{\frac{1}{\sqrt{2}}}\right)+\right.$$
$$e^{\frac{i \pi}{4}}\left(\frac{1}{2}+e^{\frac{1}{\sqrt{2}}}\right)+e^{-\frac{i \pi}{8}}\left(e^{\text{Cos}\left[\frac{\pi}{8}\right]}+\text{Sin}\left[\frac{\pi}{8}\right]^2\right)+e^{-\frac{3 i \pi}{8}}\left(e^{\text{Cos}\left[\frac{3 \pi}{8}\right]}+\text{Sin}\left[\frac{3 \pi}{8}\right]^2\right)+$$
$$e^{-\frac{5 i \pi}{8}}\left(e^{\text{Cos}\left[\frac{5 \pi}{8}\right]}+\text{Sin}\left[\frac{5 \pi}{8}\right]^2\right)+e^{-\frac{7 i \pi}{8}}\left(e^{\text{Cos}\left[\frac{7 \pi}{8}\right]}+\text{Sin}\left[\frac{7 \pi}{8}\right]^2\right)+$$
$$e^{\frac{7 i \pi}{8}}\left(e^{\text{Cos}\left[\frac{9 \pi}{8}\right]}+\text{Sin}\left[\frac{9 \pi}{8}\right]^2\right)+e^{\frac{5 i \pi}{8}}\left(e^{\text{Cos}\left[\frac{11 \pi}{8}\right]}+\text{Sin}\left[\frac{11 \pi}{8}\right]^2\right)+$$
$$\left. e^{\frac{3 i \pi}{8}}\left(e^{\text{Cos}\left[\frac{13 \pi}{8}\right]}+\text{Sin}\left[\frac{13 \pi}{8}\right]^2\right)+e^{\frac{i \pi}{8}}\left(e^{\text{Cos}\left[\frac{15 \pi}{8}\right]}+\text{Sin}\left[\frac{15 \pi}{8}\right]^2\right)\right)+$$
$$\frac{1}{16} e^{14 i t}\left(-4+\frac{1}{e}+e+e^{\frac{i \pi}{4}}\left(e^{\text{Cos}\left[\frac{\pi}{8}\right]}+\text{Sin}\left[\frac{\pi}{8}\right]^2\right)+e^{\frac{3 i \pi}{4}}\left(e^{\text{Cos}\left[\frac{3 \pi}{8}\right]}+\text{Sin}\left[\frac{3 \pi}{8}\right]^2\right)+\right.$$

$$e^{-\frac{3 i \pi}{4}} \left(e^{\cos\left[\frac{5\pi}{8}\right]} + \sin\left[\frac{5\pi}{8}\right]^2\right) + e^{-\frac{i\pi}{4}} \left(e^{\cos\left[\frac{7\pi}{8}\right]} + \sin\left[\frac{7\pi}{8}\right]^2\right) +$$
$$e^{\frac{i\pi}{4}} \left(e^{\cos\left[\frac{9\pi}{8}\right]} + \sin\left[\frac{9\pi}{8}\right]^2\right) + e^{\frac{3 i \pi}{4}} \left(e^{\cos\left[\frac{11\pi}{8}\right]} + \sin\left[\frac{11\pi}{8}\right]^2\right) +$$
$$e^{-\frac{3 i \pi}{4}} \left(e^{\cos\left[\frac{13\pi}{8}\right]} + \sin\left[\frac{13\pi}{8}\right]^2\right) + e^{-\frac{i\pi}{4}} \left(e^{\cos\left[\frac{15\pi}{8}\right]} + \sin\left[\frac{15\pi}{8}\right]^2\right)\Big) +$$

$$\frac{1}{16} e^{2 i t} \Big(-4 + \frac{1}{e} + e + e^{-\frac{i\pi}{4}} \left(e^{\cos\left[\frac{\pi}{8}\right]} + \sin\left[\frac{\pi}{8}\right]^2\right) + e^{-\frac{3 i \pi}{4}} \left(e^{\cos\left[\frac{3\pi}{8}\right]} + \sin\left[\frac{3\pi}{8}\right]^2\right) +$$
$$e^{\frac{3 i \pi}{4}} \left(e^{\cos\left[\frac{5\pi}{8}\right]} + \sin\left[\frac{5\pi}{8}\right]^2\right) + e^{\frac{i\pi}{4}} \left(e^{\cos\left[\frac{7\pi}{8}\right]} + \sin\left[\frac{7\pi}{8}\right]^2\right) +$$
$$e^{-\frac{i\pi}{4}} \left(e^{\cos\left[\frac{9\pi}{8}\right]} + \sin\left[\frac{9\pi}{8}\right]^2\right) + e^{-\frac{3 i \pi}{4}} \left(e^{\cos\left[\frac{11\pi}{8}\right]} + \sin\left[\frac{11\pi}{8}\right]^2\right) +$$
$$e^{\frac{3 i \pi}{4}} \left(e^{\cos\left[\frac{13\pi}{8}\right]} + \sin\left[\frac{13\pi}{8}\right]^2\right) + e^{\frac{i\pi}{4}} \left(e^{\cos\left[\frac{15\pi}{8}\right]} + \sin\left[\frac{15\pi}{8}\right]^2\right)\Big) +$$

$$\frac{1}{16} e^{13 i t} \Big(-\frac{1}{e} + e + e^{-\frac{i\pi}{4}} \left(\frac{1}{2} + e^{-\frac{1}{\sqrt{2}}}\right) + e^{\frac{i\pi}{4}} \left(\frac{1}{2} + e^{-\frac{1}{\sqrt{2}}}\right) + e^{-\frac{3 i \pi}{4}} \left(\frac{1}{2} + e^{\frac{1}{\sqrt{2}}}\right) + e^{\frac{3 i \pi}{4}} \left(\frac{1}{2} + e^{\frac{1}{\sqrt{2}}}\right) +$$
$$e^{\frac{3 i \pi}{8}} \left(e^{\cos\left[\frac{\pi}{8}\right]} + \sin\left[\frac{\pi}{8}\right]^2\right) + e^{-\frac{7 i \pi}{8}} \left(e^{\cos\left[\frac{3\pi}{8}\right]} + \sin\left[\frac{3\pi}{8}\right]^2\right) + e^{-\frac{i\pi}{8}} \left(e^{\cos\left[\frac{5\pi}{8}\right]} + \sin\left[\frac{5\pi}{8}\right]^2\right) +$$
$$e^{\frac{5 i \pi}{8}} \left(e^{\cos\left[\frac{7\pi}{8}\right]} + \sin\left[\frac{7\pi}{8}\right]^2\right) + e^{-\frac{5 i \pi}{8}} \left(e^{\cos\left[\frac{9\pi}{8}\right]} + \sin\left[\frac{9\pi}{8}\right]^2\right) + e^{\frac{i\pi}{8}} \left(e^{\cos\left[\frac{11\pi}{8}\right]} + \sin\left[\frac{11\pi}{8}\right]^2\right) +$$
$$e^{\frac{7 i \pi}{8}} \left(e^{\cos\left[\frac{13\pi}{8}\right]} + \sin\left[\frac{13\pi}{8}\right]^2\right) + e^{-\frac{3 i \pi}{8}} \left(e^{\cos\left[\frac{15\pi}{8}\right]} + \sin\left[\frac{15\pi}{8}\right]^2\right)\Big) +$$

$$\frac{1}{16} e^{3 i t} \Big(-\frac{1}{e} + e + e^{-\frac{i\pi}{4}} \left(\frac{1}{2} + e^{-\frac{1}{\sqrt{2}}}\right) + e^{\frac{i\pi}{4}} \left(\frac{1}{2} + e^{-\frac{1}{\sqrt{2}}}\right) + e^{-\frac{3 i \pi}{4}} \left(\frac{1}{2} + e^{\frac{1}{\sqrt{2}}}\right) +$$
$$e^{\frac{3 i \pi}{4}} \left(\frac{1}{2} + e^{\frac{1}{\sqrt{2}}}\right) + e^{-\frac{3 i \pi}{8}} \left(e^{\cos\left[\frac{\pi}{8}\right]} + \sin\left[\frac{\pi}{8}\right]^2\right) + e^{\frac{7 i \pi}{8}} \left(e^{\cos\left[\frac{3\pi}{8}\right]} + \sin\left[\frac{3\pi}{8}\right]^2\right) +$$
$$e^{\frac{i\pi}{8}} \left(e^{\cos\left[\frac{5\pi}{8}\right]} + \sin\left[\frac{5\pi}{8}\right]^2\right) + e^{-\frac{5 i \pi}{8}} \left(e^{\cos\left[\frac{7\pi}{8}\right]} + \sin\left[\frac{7\pi}{8}\right]^2\right) +$$
$$e^{\frac{5 i \pi}{8}} \left(e^{\cos\left[\frac{9\pi}{8}\right]} + \sin\left[\frac{9\pi}{8}\right]^2\right) + e^{-\frac{i\pi}{8}} \left(e^{\cos\left[\frac{11\pi}{8}\right]} + \sin\left[\frac{11\pi}{8}\right]^2\right) +$$
$$e^{-\frac{7 i \pi}{8}} \left(e^{\cos\left[\frac{13\pi}{8}\right]} + \sin\left[\frac{13\pi}{8}\right]^2\right) + e^{\frac{3 i \pi}{8}} \left(e^{\cos\left[\frac{15\pi}{8}\right]} + \sin\left[\frac{15\pi}{8}\right]^2\right)\Big) +$$

$$\frac{1}{16} e^{11 i t} \Big(-\frac{1}{e} + e + e^{-\frac{i\pi}{4}} \left(\frac{1}{2} + e^{-\frac{1}{\sqrt{2}}}\right) + e^{\frac{i\pi}{4}} \left(\frac{1}{2} + e^{-\frac{1}{\sqrt{2}}}\right) + e^{-\frac{3 i \pi}{4}} \left(\frac{1}{2} + e^{\frac{1}{\sqrt{2}}}\right) +$$
$$e^{\frac{3 i \pi}{4}} \left(\frac{1}{2} + e^{\frac{1}{\sqrt{2}}}\right) + e^{\frac{5 i \pi}{8}} \left(e^{\cos\left[\frac{\pi}{8}\right]} + \sin\left[\frac{\pi}{8}\right]^2\right) + e^{-\frac{i\pi}{8}} \left(e^{\cos\left[\frac{3\pi}{8}\right]} + \sin\left[\frac{3\pi}{8}\right]^2\right) +$$
$$e^{-\frac{7 i \pi}{8}} \left(e^{\cos\left[\frac{5\pi}{8}\right]} + \sin\left[\frac{5\pi}{8}\right]^2\right) + e^{\frac{3 i \pi}{8}} \left(e^{\cos\left[\frac{7\pi}{8}\right]} + \sin\left[\frac{7\pi}{8}\right]^2\right) +$$
$$e^{-\frac{3 i \pi}{8}} \left(e^{\cos\left[\frac{9\pi}{8}\right]} + \sin\left[\frac{9\pi}{8}\right]^2\right) + e^{\frac{7 i \pi}{8}} \left(e^{\cos\left[\frac{11\pi}{8}\right]} + \sin\left[\frac{11\pi}{8}\right]^2\right) +$$
$$e^{\frac{i\pi}{8}} \left(e^{\cos\left[\frac{13\pi}{8}\right]} + \sin\left[\frac{13\pi}{8}\right]^2\right) + e^{-\frac{5 i \pi}{8}} \left(e^{\cos\left[\frac{15\pi}{8}\right]} + \sin\left[\frac{15\pi}{8}\right]^2\right)\Big) +$$

$$\frac{1}{16} e^{5 i t} \Big(-\frac{1}{e} + e + e^{-\frac{i\pi}{4}} \left(\frac{1}{2} + e^{-\frac{1}{\sqrt{2}}}\right) + e^{\frac{i\pi}{4}} \left(\frac{1}{2} + e^{-\frac{1}{\sqrt{2}}}\right) + e^{-\frac{3 i \pi}{4}} \left(\frac{1}{2} + e^{\frac{1}{\sqrt{2}}}\right) +$$
$$e^{\frac{3 i \pi}{4}} \left(\frac{1}{2} + e^{\frac{1}{\sqrt{2}}}\right) + e^{-\frac{5 i \pi}{8}} \left(e^{\cos\left[\frac{\pi}{8}\right]} + \sin\left[\frac{\pi}{8}\right]^2\right) + e^{\frac{i\pi}{8}} \left(e^{\cos\left[\frac{3\pi}{8}\right]} + \sin\left[\frac{3\pi}{8}\right]^2\right) +$$
$$e^{\frac{7 i \pi}{8}} \left(e^{\cos\left[\frac{5\pi}{8}\right]} + \sin\left[\frac{5\pi}{8}\right]^2\right) + e^{-\frac{3 i \pi}{8}} \left(e^{\cos\left[\frac{7\pi}{8}\right]} + \sin\left[\frac{7\pi}{8}\right]^2\right) +$$
$$e^{\frac{3 i \pi}{8}} \left(e^{\cos\left[\frac{9\pi}{8}\right]} + \sin\left[\frac{9\pi}{8}\right]^2\right) + e^{-\frac{7 i \pi}{8}} \left(e^{\cos\left[\frac{11\pi}{8}\right]} + \sin\left[\frac{11\pi}{8}\right]^2\right) +$$
$$e^{-\frac{i\pi}{8}} \left(e^{\cos\left[\frac{13\pi}{8}\right]} + \sin\left[\frac{13\pi}{8}\right]^2\right) + e^{\frac{5 i \pi}{8}} \left(e^{\cos\left[\frac{15\pi}{8}\right]} + \sin\left[\frac{15\pi}{8}\right]^2\right)\Big) +$$

$$\frac{1}{16} e^{10 i t} \Big(-4 + \frac{1}{e} + e + e^{\frac{3 i \pi}{4}} \left(e^{\cos\left[\frac{\pi}{8}\right]} + \sin\left[\frac{\pi}{8}\right]^2\right) + e^{\frac{i\pi}{4}} \left(e^{\cos\left[\frac{3\pi}{8}\right]} + \sin\left[\frac{3\pi}{8}\right]^2\right) +$$

$$e^{-\frac{i\pi}{4}}\left(e^{\text{Cos}\left[\frac{5\pi}{8}\right]}+\text{Sin}\left[\frac{5\pi}{8}\right]^2\right)+e^{-\frac{3i\pi}{4}}\left(e^{\text{Cos}\left[\frac{7\pi}{8}\right]}+\text{Sin}\left[\frac{7\pi}{8}\right]^2\right)+$$
$$e^{\frac{3i\pi}{4}}\left(e^{\text{Cos}\left[\frac{9\pi}{8}\right]}+\text{Sin}\left[\frac{9\pi}{8}\right]^2\right)+e^{\frac{i\pi}{4}}\left(e^{\text{Cos}\left[\frac{11\pi}{8}\right]}+\text{Sin}\left[\frac{11\pi}{8}\right]^2\right)+$$
$$e^{-\frac{i\pi}{4}}\left(e^{\text{Cos}\left[\frac{13\pi}{8}\right]}+\text{Sin}\left[\frac{13\pi}{8}\right]^2\right)+e^{-\frac{3i\pi}{4}}\left(e^{\text{Cos}\left[\frac{15\pi}{8}\right]}+\text{Sin}\left[\frac{15\pi}{8}\right]^2\right)\Big)+$$

$$\frac{1}{16}e^{6it}\Big(-4+\frac{1}{e}+e+e^{-\frac{3i\pi}{4}}\left(e^{\text{Cos}\left[\frac{\pi}{8}\right]}+\text{Sin}\left[\frac{\pi}{8}\right]^2\right)+e^{-\frac{i\pi}{4}}\left(e^{\text{Cos}\left[\frac{3\pi}{8}\right]}+\text{Sin}\left[\frac{3\pi}{8}\right]^2\right)+$$
$$e^{\frac{i\pi}{4}}\left(e^{\text{Cos}\left[\frac{5\pi}{8}\right]}+\text{Sin}\left[\frac{5\pi}{8}\right]^2\right)+e^{\frac{3i\pi}{4}}\left(e^{\text{Cos}\left[\frac{7\pi}{8}\right]}+\text{Sin}\left[\frac{7\pi}{8}\right]^2\right)+$$
$$e^{-\frac{3i\pi}{4}}\left(e^{\text{Cos}\left[\frac{9\pi}{8}\right]}+\text{Sin}\left[\frac{9\pi}{8}\right]^2\right)+e^{-\frac{i\pi}{4}}\left(e^{\text{Cos}\left[\frac{11\pi}{8}\right]}+\text{Sin}\left[\frac{11\pi}{8}\right]^2\right)+$$
$$e^{\frac{i\pi}{4}}\left(e^{\text{Cos}\left[\frac{13\pi}{8}\right]}+\text{Sin}\left[\frac{13\pi}{8}\right]^2\right)+e^{\frac{3i\pi}{4}}\left(e^{\text{Cos}\left[\frac{15\pi}{8}\right]}+\text{Sin}\left[\frac{15\pi}{8}\right]^2\right)\Big)+$$

$$\frac{1}{16}e^{9it}\Big(-\frac{1}{e}+e+e^{-\frac{3i\pi}{4}}\left(\frac{1}{2}+e^{-\frac{1}{\sqrt{2}}}\right)+e^{\frac{3i\pi}{4}}\left(\frac{1}{2}+e^{-\frac{1}{\sqrt{2}}}\right)+e^{-\frac{i\pi}{4}}\left(\frac{1}{2}+e^{\frac{1}{\sqrt{2}}}\right)+e^{\frac{i\pi}{4}}\left(\frac{1}{2}+e^{\frac{1}{\sqrt{2}}}\right)+$$
$$e^{\frac{7i\pi}{8}}\left(e^{\text{Cos}\left[\frac{\pi}{8}\right]}+\text{Sin}\left[\frac{\pi}{8}\right]^2\right)+e^{\frac{5i\pi}{8}}\left(e^{\text{Cos}\left[\frac{3\pi}{8}\right]}+\text{Sin}\left[\frac{3\pi}{8}\right]^2\right)+e^{\frac{3i\pi}{8}}\left(e^{\text{Cos}\left[\frac{5\pi}{8}\right]}+\text{Sin}\left[\frac{5\pi}{8}\right]^2\right)+$$
$$e^{\frac{i\pi}{8}}\left(e^{\text{Cos}\left[\frac{7\pi}{8}\right]}+\text{Sin}\left[\frac{7\pi}{8}\right]^2\right)+e^{-\frac{i\pi}{8}}\left(e^{\text{Cos}\left[\frac{9\pi}{8}\right]}+\text{Sin}\left[\frac{9\pi}{8}\right]^2\right)+e^{-\frac{3i\pi}{8}}\left(e^{\text{Cos}\left[\frac{11\pi}{8}\right]}+\text{Sin}\left[\frac{11\pi}{8}\right]^2\right)+$$
$$e^{-\frac{5i\pi}{8}}\left(e^{\text{Cos}\left[\frac{13\pi}{8}\right]}+\text{Sin}\left[\frac{13\pi}{8}\right]^2\right)+e^{-\frac{7i\pi}{8}}\left(e^{\text{Cos}\left[\frac{15\pi}{8}\right]}+\text{Sin}\left[\frac{15\pi}{8}\right]^2\right)\Big)+$$

$$\frac{1}{16}e^{7it}\Big(-\frac{1}{e}+e+e^{-\frac{3i\pi}{4}}\left(\frac{1}{2}+e^{-\frac{1}{\sqrt{2}}}\right)+e^{\frac{3i\pi}{4}}\left(\frac{1}{2}+e^{-\frac{1}{\sqrt{2}}}\right)+e^{-\frac{i\pi}{4}}\left(\frac{1}{2}+e^{\frac{1}{\sqrt{2}}}\right)+e^{\frac{i\pi}{4}}\left(\frac{1}{2}+e^{\frac{1}{\sqrt{2}}}\right)+$$
$$e^{-\frac{7i\pi}{8}}\left(e^{\text{Cos}\left[\frac{\pi}{8}\right]}+\text{Sin}\left[\frac{\pi}{8}\right]^2\right)+e^{-\frac{5i\pi}{8}}\left(e^{\text{Cos}\left[\frac{3\pi}{8}\right]}+\text{Sin}\left[\frac{3\pi}{8}\right]^2\right)+e^{-\frac{3i\pi}{8}}\left(e^{\text{Cos}\left[\frac{5\pi}{8}\right]}+\text{Sin}\left[\frac{5\pi}{8}\right]^2\right)+$$
$$e^{-\frac{i\pi}{8}}\left(e^{\text{Cos}\left[\frac{7\pi}{8}\right]}+\text{Sin}\left[\frac{7\pi}{8}\right]^2\right)+e^{\frac{i\pi}{8}}\left(e^{\text{Cos}\left[\frac{9\pi}{8}\right]}+\text{Sin}\left[\frac{9\pi}{8}\right]^2\right)+e^{\frac{3i\pi}{8}}\left(e^{\text{Cos}\left[\frac{11\pi}{8}\right]}+\text{Sin}\left[\frac{11\pi}{8}\right]^2\right)+$$
$$e^{\frac{5i\pi}{8}}\left(e^{\text{Cos}\left[\frac{13\pi}{8}\right]}+\text{Sin}\left[\frac{13\pi}{8}\right]^2\right)+e^{\frac{7i\pi}{8}}\left(e^{\text{Cos}\left[\frac{15\pi}{8}\right]}+\text{Sin}\left[\frac{15\pi}{8}\right]^2\right)\Big)$$

fS[t]//ExpToTrig

$$\frac{1}{16}\left(\text{Cos}[8t]+i\,\text{Sin}[8t]\right)$$
$$\Big(6+e+\text{Cosh}[1]+4\,\text{Cosh}\Big[\frac{1}{\sqrt{2}}\Big]-\text{Cosh}\Big[\text{Cos}\Big[\frac{\pi}{8}\Big]\Big]-\text{Cosh}\Big[\text{Cos}\Big[\frac{3\pi}{8}\Big]\Big]-\text{Cosh}\Big[\text{Cos}\Big[\frac{5\pi}{8}\Big]\Big]-$$
$$\text{Cosh}\Big[\text{Cos}\Big[\frac{7\pi}{8}\Big]\Big]-\text{Cosh}\Big[\text{Cos}\Big[\frac{9\pi}{8}\Big]\Big]-\text{Cosh}\Big[\text{Cos}\Big[\frac{11\pi}{8}\Big]\Big]-\text{Cosh}\Big[\text{Cos}\Big[\frac{13\pi}{8}\Big]\Big]-$$
$$\text{Cosh}\Big[\text{Cos}\Big[\frac{15\pi}{8}\Big]\Big]-\text{Sin}\Big[\frac{\pi}{8}\Big]^2-\text{Sin}\Big[\frac{3\pi}{8}\Big]^2-\text{Sin}\Big[\frac{5\pi}{8}\Big]^2-\text{Sin}\Big[\frac{7\pi}{8}\Big]^2-$$
$$\text{Sin}\Big[\frac{9\pi}{8}\Big]^2-\text{Sin}\Big[\frac{11\pi}{8}\Big]^2-\text{Sin}\Big[\frac{13\pi}{8}\Big]^2-\text{Sin}\Big[\frac{15\pi}{8}\Big]^2-\text{Sinh}[1]-$$
$$\text{Sinh}\Big[\text{Cos}\Big[\frac{\pi}{8}\Big]\Big]-\text{Sinh}\Big[\text{Cos}\Big[\frac{3\pi}{8}\Big]\Big]-\text{Sinh}\Big[\text{Cos}\Big[\frac{5\pi}{8}\Big]\Big]-\text{Sinh}\Big[\text{Cos}\Big[\frac{7\pi}{8}\Big]\Big]-$$
$$\text{Sinh}\Big[\text{Cos}\Big[\frac{9\pi}{8}\Big]\Big]-\text{Sinh}\Big[\text{Cos}\Big[\frac{11\pi}{8}\Big]\Big]-\text{Sinh}\Big[\text{Cos}\Big[\frac{13\pi}{8}\Big]\Big]-\text{Sinh}\Big[\text{Cos}\Big[\frac{15\pi}{8}\Big]\Big]\Big)+$$

$$\frac{1}{16}\left(\text{Cos}[12t]+i\,\text{Sin}[12t]\right)\Big(2+e+\text{Cosh}[1]-4\,\text{Cosh}\Big[\frac{1}{\sqrt{2}}\Big]+i\,\text{Cosh}\Big[\text{Cos}\Big[\frac{\pi}{8}\Big]\Big]-$$
$$i\,\text{Cosh}\Big[\text{Cos}\Big[\frac{3\pi}{8}\Big]\Big]+i\,\text{Cosh}\Big[\text{Cos}\Big[\frac{5\pi}{8}\Big]\Big]-i\,\text{Cosh}\Big[\text{Cos}\Big[\frac{7\pi}{8}\Big]\Big]+i\,\text{Cosh}\Big[\text{Cos}\Big[\frac{9\pi}{8}\Big]\Big]-$$
$$i\,\text{Cosh}\Big[\text{Cos}\Big[\frac{11\pi}{8}\Big]\Big]+i\,\text{Cosh}\Big[\text{Cos}\Big[\frac{13\pi}{8}\Big]\Big]-i\,\text{Cosh}\Big[\text{Cos}\Big[\frac{15\pi}{8}\Big]\Big]+i\,\text{Sin}\Big[\frac{\pi}{8}\Big]^2-i\,\text{Sin}\Big[\frac{3\pi}{8}\Big]^2+$$
$$i\,\text{Sin}\Big[\frac{5\pi}{8}\Big]^2-i\,\text{Sin}\Big[\frac{7\pi}{8}\Big]^2+i\,\text{Sin}\Big[\frac{9\pi}{8}\Big]^2-i\,\text{Sin}\Big[\frac{11\pi}{8}\Big]^2+i\,\text{Sin}\Big[\frac{13\pi}{8}\Big]^2-i\,\text{Sin}\Big[\frac{15\pi}{8}\Big]^2-$$
$$\text{Sinh}[1]+i\,\text{Sinh}\Big[\text{Cos}\Big[\frac{\pi}{8}\Big]\Big]-i\,\text{Sinh}\Big[\text{Cos}\Big[\frac{3\pi}{8}\Big]\Big]+i\,\text{Sinh}\Big[\text{Cos}\Big[\frac{5\pi}{8}\Big]\Big]-i\,\text{Sinh}\Big[\text{Cos}\Big[\frac{7\pi}{8}\Big]\Big]+$$

$$i\,\text{Sinh}\left[\text{Cos}\left[\frac{9\pi}{8}\right]\right] - i\,\text{Sinh}\left[\text{Cos}\left[\frac{11\pi}{8}\right]\right] + i\,\text{Sinh}\left[\text{Cos}\left[\frac{13\pi}{8}\right]\right] - i\,\text{Sinh}\left[\text{Cos}\left[\frac{15\pi}{8}\right]\right]\Big) +$$

$$\frac{1}{16}\,(\text{Cos}[4t] + i\,\text{Sin}[4t])\,\Big(2 + e + \text{Cosh}[1] - 4\,\text{Cosh}\left[\frac{1}{\sqrt{2}}\right] - i\,\text{Cosh}\left[\text{Cos}\left[\frac{\pi}{8}\right]\right] +$$

$$i\,\text{Cosh}\left[\text{Cos}\left[\frac{3\pi}{8}\right]\right] - i\,\text{Cosh}\left[\text{Cos}\left[\frac{5\pi}{8}\right]\right] + i\,\text{Cosh}\left[\text{Cos}\left[\frac{7\pi}{8}\right]\right] - i\,\text{Cosh}\left[\text{Cos}\left[\frac{9\pi}{8}\right]\right] +$$

$$i\,\text{Cosh}\left[\text{Cos}\left[\frac{11\pi}{8}\right]\right] - i\,\text{Cosh}\left[\text{Cos}\left[\frac{13\pi}{8}\right]\right] + i\,\text{Cosh}\left[\text{Cos}\left[\frac{15\pi}{8}\right]\right] - i\,\text{Sin}\left[\frac{\pi}{8}\right]^2 + i\,\text{Sin}\left[\frac{3\pi}{8}\right]^2 -$$

$$i\,\text{Sin}\left[\frac{5\pi}{8}\right]^2 + i\,\text{Sin}\left[\frac{7\pi}{8}\right]^2 - i\,\text{Sin}\left[\frac{9\pi}{8}\right]^2 + i\,\text{Sin}\left[\frac{11\pi}{8}\right]^2 - i\,\text{Sin}\left[\frac{13\pi}{8}\right]^2 + i\,\text{Sin}\left[\frac{15\pi}{8}\right]^2 -$$

$$\text{Sinh}[1] - i\,\text{Sinh}\left[\text{Cos}\left[\frac{\pi}{8}\right]\right] + i\,\text{Sinh}\left[\text{Cos}\left[\frac{3\pi}{8}\right]\right] - i\,\text{Sinh}\left[\text{Cos}\left[\frac{5\pi}{8}\right]\right] + i\,\text{Sinh}\left[\text{Cos}\left[\frac{7\pi}{8}\right]\right] -$$

$$i\,\text{Sinh}\left[\text{Cos}\left[\frac{9\pi}{8}\right]\right] + i\,\text{Sinh}\left[\text{Cos}\left[\frac{11\pi}{8}\right]\right] - i\,\text{Sinh}\left[\text{Cos}\left[\frac{13\pi}{8}\right]\right] + i\,\text{Sinh}\left[\text{Cos}\left[\frac{15\pi}{8}\right]\right]\Big) +$$

$$\frac{1}{16}\,\Big(6 + e + \text{Cosh}[1] + 4\,\text{Cosh}\left[\frac{1}{\sqrt{2}}\right] + \text{Cosh}\left[\text{Cos}\left[\frac{\pi}{8}\right]\right] + \text{Cosh}\left[\text{Cos}\left[\frac{3\pi}{8}\right]\right] + \text{Cosh}\left[\text{Cos}\left[\frac{5\pi}{8}\right]\right] +$$

$$\text{Cosh}\left[\text{Cos}\left[\frac{7\pi}{8}\right]\right] + \text{Cosh}\left[\text{Cos}\left[\frac{9\pi}{8}\right]\right] + \text{Cosh}\left[\text{Cos}\left[\frac{11\pi}{8}\right]\right] + \text{Cosh}\left[\text{Cos}\left[\frac{13\pi}{8}\right]\right] +$$

$$\text{Cosh}\left[\text{Cos}\left[\frac{15\pi}{8}\right]\right] + \text{Sin}\left[\frac{\pi}{8}\right]^2 + \text{Sin}\left[\frac{3\pi}{8}\right]^2 + \text{Sin}\left[\frac{5\pi}{8}\right]^2 + \text{Sin}\left[\frac{7\pi}{8}\right]^2 + \text{Sin}\left[\frac{9\pi}{8}\right]^2 +$$

$$\text{Sin}\left[\frac{11\pi}{8}\right]^2 + \text{Sin}\left[\frac{13\pi}{8}\right]^2 + \text{Sin}\left[\frac{15\pi}{8}\right]^2 - \text{Sinh}[1] + \text{Sinh}\left[\text{Cos}\left[\frac{\pi}{8}\right]\right] + \text{Sinh}\left[\text{Cos}\left[\frac{3\pi}{8}\right]\right] +$$

$$\text{Sinh}\left[\text{Cos}\left[\frac{5\pi}{8}\right]\right] + \text{Sinh}\left[\text{Cos}\left[\frac{7\pi}{8}\right]\right] + \text{Sinh}\left[\text{Cos}\left[\frac{9\pi}{8}\right]\right] + \text{Sinh}\left[\text{Cos}\left[\frac{11\pi}{8}\right]\right] +$$

$$\text{Sinh}\left[\text{Cos}\left[\frac{13\pi}{8}\right]\right] + \text{Sinh}\left[\text{Cos}\left[\frac{15\pi}{8}\right]\right]\Big) + \frac{1}{16}\,(\text{Cos}[10t] + i\,\text{Sin}[10t])$$

$$\Bigg(-4 + e + \text{Cosh}[1] - \text{Sinh}[1] - \frac{(1-i)\,(\text{Cosh}[\text{Cos}[\frac{\pi}{8}]] + \text{Sin}[\frac{\pi}{8}]^2 + \text{Sinh}[\text{Cos}[\frac{\pi}{8}]])}{\sqrt{2}} +$$

$$\frac{(1+i)\,\left(\text{Cosh}[\text{Cos}[\frac{3\pi}{8}]] + \text{Sin}[\frac{3\pi}{8}]^2 + \text{Sinh}[\text{Cos}[\frac{3\pi}{8}]]\right)}{\sqrt{2}} +$$

$$\frac{(1-i)\,\left(\text{Cosh}[\text{Cos}[\frac{5\pi}{8}]] + \text{Sin}[\frac{5\pi}{8}]^2 + \text{Sinh}[\text{Cos}[\frac{5\pi}{8}]]\right)}{\sqrt{2}} -$$

$$\frac{(1+i)\,\left(\text{Cosh}[\text{Cos}[\frac{7\pi}{8}]] + \text{Sin}[\frac{7\pi}{8}]^2 + \text{Sinh}[\text{Cos}[\frac{7\pi}{8}]]\right)}{\sqrt{2}} -$$

$$\frac{(1-i)\,\left(\text{Cosh}[\text{Cos}[\frac{9\pi}{8}]] + \text{Sin}[\frac{9\pi}{8}]^2 + \text{Sinh}[\text{Cos}[\frac{9\pi}{8}]]\right)}{\sqrt{2}} +$$

$$\frac{(1+i)\,\left(\text{Cosh}[\text{Cos}[\frac{11\pi}{8}]] + \text{Sin}[\frac{11\pi}{8}]^2 + \text{Sinh}[\text{Cos}[\frac{11\pi}{8}]]\right)}{\sqrt{2}} +$$

$$\frac{(1-i)\,\left(\text{Cosh}[\text{Cos}[\frac{13\pi}{8}]] + \text{Sin}[\frac{13\pi}{8}]^2 + \text{Sinh}[\text{Cos}[\frac{13\pi}{8}]]\right)}{\sqrt{2}} -$$

$$\frac{(1+i)\,\left(\text{Cosh}[\text{Cos}[\frac{15\pi}{8}]] + \text{Sin}[\frac{15\pi}{8}]^2 + \text{Sinh}[\text{Cos}[\frac{15\pi}{8}]]\right)}{\sqrt{2}}\Bigg) + \frac{1}{16}\,(\text{Cos}[6t] + i\,\text{Sin}[6t])$$

$$\Bigg(-4 + e + \text{Cosh}[1] - \text{Sinh}[1] - \frac{(1+i)\,(\text{Cosh}[\text{Cos}[\frac{\pi}{8}]] + \text{Sin}[\frac{\pi}{8}]^2 + \text{Sinh}[\text{Cos}[\frac{\pi}{8}]])}{\sqrt{2}} +$$

$$\frac{(1-i)\,\left(\text{Cosh}[\text{Cos}[\frac{3\pi}{8}]] + \text{Sin}[\frac{3\pi}{8}]^2 + \text{Sinh}[\text{Cos}[\frac{3\pi}{8}]]\right)}{\sqrt{2}} +$$

$$\frac{(1+i)\,\left(\text{Cosh}[\text{Cos}[\frac{5\pi}{8}]] + \text{Sin}[\frac{5\pi}{8}]^2 + \text{Sinh}[\text{Cos}[\frac{5\pi}{8}]]\right)}{\sqrt{2}} -$$

$$\frac{(1-i)\left(\text{Cosh}[\text{Cos}[\frac{7\pi}{8}]]+\text{Sin}[\frac{7\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{7\pi}{8}]]\right)}{\sqrt{2}}-$$
$$\frac{(1+i)\left(\text{Cosh}[\text{Cos}[\frac{9\pi}{8}]]+\text{Sin}[\frac{9\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{9\pi}{8}]]\right)}{\sqrt{2}}+$$
$$\frac{(1-i)\left(\text{Cosh}[\text{Cos}[\frac{11\pi}{8}]]+\text{Sin}[\frac{11\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{11\pi}{8}]]\right)}{\sqrt{2}}+$$
$$\frac{(1+i)\left(\text{Cosh}[\text{Cos}[\frac{13\pi}{8}]]+\text{Sin}[\frac{13\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{13\pi}{8}]]\right)}{\sqrt{2}}-$$
$$\left.\frac{(1-i)\left(\text{Cosh}[\text{Cos}[\frac{15\pi}{8}]]+\text{Sin}[\frac{15\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{15\pi}{8}]]\right)}{\sqrt{2}}\right)+$$
$$\frac{1}{16}(\text{Cos}[14t]+i\,\text{Sin}[14t])\left(-4+e+\text{Cosh}[1]-\text{Sinh}[1]+\right.$$
$$\frac{(1+i)(\text{Cosh}[\text{Cos}[\frac{\pi}{8}]]+\text{Sin}[\frac{\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{\pi}{8}]])}{\sqrt{2}}-$$
$$\frac{(1-i)\left(\text{Cosh}[\text{Cos}[\frac{3\pi}{8}]]+\text{Sin}[\frac{3\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{3\pi}{8}]]\right)}{\sqrt{2}}-$$
$$\frac{(1+i)\left(\text{Cosh}[\text{Cos}[\frac{5\pi}{8}]]+\text{Sin}[\frac{5\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{5\pi}{8}]]\right)}{\sqrt{2}}+$$
$$\frac{(1-i)\left(\text{Cosh}[\text{Cos}[\frac{7\pi}{8}]]+\text{Sin}[\frac{7\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{7\pi}{8}]]\right)}{\sqrt{2}}+$$
$$\frac{(1+i)\left(\text{Cosh}[\text{Cos}[\frac{9\pi}{8}]]+\text{Sin}[\frac{9\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{9\pi}{8}]]\right)}{\sqrt{2}}-$$
$$\frac{(1-i)\left(\text{Cosh}[\text{Cos}[\frac{11\pi}{8}]]+\text{Sin}[\frac{11\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{11\pi}{8}]]\right)}{\sqrt{2}}-$$
$$\frac{(1+i)\left(\text{Cosh}[\text{Cos}[\frac{13\pi}{8}]]+\text{Sin}[\frac{13\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{13\pi}{8}]]\right)}{\sqrt{2}}+$$
$$\left.\frac{(1-i)\left(\text{Cosh}[\text{Cos}[\frac{15\pi}{8}]]+\text{Sin}[\frac{15\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{15\pi}{8}]]\right)}{\sqrt{2}}\right)+\frac{1}{16}(\text{Cos}[2t]+i\,\text{Sin}[2t])$$
$$\left(-4+e+\text{Cosh}[1]-\text{Sinh}[1]+\frac{(1-i)(\text{Cosh}[\text{Cos}[\frac{\pi}{8}]]+\text{Sin}[\frac{\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{\pi}{8}]])}{\sqrt{2}}-\right.$$
$$\frac{(1+i)\left(\text{Cosh}[\text{Cos}[\frac{3\pi}{8}]]+\text{Sin}[\frac{3\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{3\pi}{8}]]\right)}{\sqrt{2}}-$$
$$\frac{(1-i)\left(\text{Cosh}[\text{Cos}[\frac{5\pi}{8}]]+\text{Sin}[\frac{5\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{5\pi}{8}]]\right)}{\sqrt{2}}+$$
$$\frac{(1+i)\left(\text{Cosh}[\text{Cos}[\frac{7\pi}{8}]]+\text{Sin}[\frac{7\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{7\pi}{8}]]\right)}{\sqrt{2}}+$$
$$\frac{(1-i)\left(\text{Cosh}[\text{Cos}[\frac{9\pi}{8}]]+\text{Sin}[\frac{9\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{9\pi}{8}]]\right)}{\sqrt{2}}-$$
$$\frac{(1+i)\left(\text{Cosh}[\text{Cos}[\frac{11\pi}{8}]]+\text{Sin}[\frac{11\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{11\pi}{8}]]\right)}{\sqrt{2}}-$$
$$\frac{(1-i)\left(\text{Cosh}[\text{Cos}[\frac{13\pi}{8}]]+\text{Sin}[\frac{13\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{13\pi}{8}]]\right)}{\sqrt{2}}+$$

$$\frac{(1+i)\left(\text{Cosh}[\text{Cos}[\frac{15\pi}{8}]]+\text{Sin}[\frac{15\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{15\pi}{8}]]\right)}{\sqrt{2}}\Bigg)+$$

$$\frac{1}{16}(\text{Cos}[15t]+i\,\text{Sin}[15t])\Bigg(e-\text{Cosh}[1]+\text{Sinh}[1]-$$

$$\frac{2\left(\frac{1}{2}+\text{Cosh}[\frac{1}{\sqrt{2}}]-\text{Sinh}[\frac{1}{\sqrt{2}}]\right)}{\sqrt{2}}+\frac{2\left(\frac{1}{2}+\text{Cosh}[\frac{1}{\sqrt{2}}]+\text{Sinh}[\frac{1}{\sqrt{2}}]\right)}{\sqrt{2}}+$$

$$\left(\text{Cos}[\frac{\pi}{8}]+i\,\text{Sin}[\frac{\pi}{8}]\right)\left(\text{Cosh}[\text{Cos}[\frac{\pi}{8}]]+\text{Sin}[\frac{\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{\pi}{8}]]\right)+$$

$$\left(\text{Cos}[\frac{3\pi}{8}]+i\,\text{Sin}[\frac{3\pi}{8}]\right)\left(\text{Cosh}[\text{Cos}[\frac{3\pi}{8}]]+\text{Sin}[\frac{3\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{3\pi}{8}]]\right)+$$

$$\left(\text{Cos}[\frac{5\pi}{8}]+i\,\text{Sin}[\frac{5\pi}{8}]\right)\left(\text{Cosh}[\text{Cos}[\frac{5\pi}{8}]]+\text{Sin}[\frac{5\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{5\pi}{8}]]\right)+$$

$$\left(\text{Cos}[\frac{7\pi}{8}]+i\,\text{Sin}[\frac{7\pi}{8}]\right)\left(\text{Cosh}[\text{Cos}[\frac{7\pi}{8}]]+\text{Sin}[\frac{7\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{7\pi}{8}]]\right)+$$

$$\left(\text{Cos}[\frac{7\pi}{8}]-i\,\text{Sin}[\frac{7\pi}{8}]\right)\left(\text{Cosh}[\text{Cos}[\frac{9\pi}{8}]]+\text{Sin}[\frac{9\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{9\pi}{8}]]\right)+$$

$$\left(\text{Cos}[\frac{5\pi}{8}]-i\,\text{Sin}[\frac{5\pi}{8}]\right)\left(\text{Cosh}[\text{Cos}[\frac{11\pi}{8}]]+\text{Sin}[\frac{11\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{11\pi}{8}]]\right)+$$

$$\left(\text{Cos}[\frac{3\pi}{8}]-i\,\text{Sin}[\frac{3\pi}{8}]\right)\left(\text{Cosh}[\text{Cos}[\frac{13\pi}{8}]]+\text{Sin}[\frac{13\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{13\pi}{8}]]\right)+$$

$$\left(\text{Cos}[\frac{\pi}{8}]-i\,\text{Sin}[\frac{\pi}{8}]\right)\left(\text{Cosh}[\text{Cos}[\frac{15\pi}{8}]]+\text{Sin}[\frac{15\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{15\pi}{8}]]\right)\Bigg)+$$

$$\frac{1}{16}(\text{Cos}[t]+i\,\text{Sin}[t])\Bigg(e-\text{Cosh}[1]+\text{Sinh}[1]-\frac{2\left(\frac{1}{2}+\text{Cosh}[\frac{1}{\sqrt{2}}]-\text{Sinh}[\frac{1}{\sqrt{2}}]\right)}{\sqrt{2}}+$$

$$\frac{2\left(\frac{1}{2}+\text{Cosh}[\frac{1}{\sqrt{2}}]+\text{Sinh}[\frac{1}{\sqrt{2}}]\right)}{\sqrt{2}}+$$

$$\left(\text{Cos}[\frac{\pi}{8}]-i\,\text{Sin}[\frac{\pi}{8}]\right)\left(\text{Cosh}[\text{Cos}[\frac{\pi}{8}]]+\text{Sin}[\frac{\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{\pi}{8}]]\right)+$$

$$\left(\text{Cos}[\frac{3\pi}{8}]-i\,\text{Sin}[\frac{3\pi}{8}]\right)\left(\text{Cosh}[\text{Cos}[\frac{3\pi}{8}]]+\text{Sin}[\frac{3\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{3\pi}{8}]]\right)+$$

$$\left(\text{Cos}[\frac{5\pi}{8}]-i\,\text{Sin}[\frac{5\pi}{8}]\right)\left(\text{Cosh}[\text{Cos}[\frac{5\pi}{8}]]+\text{Sin}[\frac{5\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{5\pi}{8}]]\right)+$$

$$\left(\text{Cos}[\frac{7\pi}{8}]-i\,\text{Sin}[\frac{7\pi}{8}]\right)\left(\text{Cosh}[\text{Cos}[\frac{7\pi}{8}]]+\text{Sin}[\frac{7\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{7\pi}{8}]]\right)+$$

$$\left(\text{Cos}[\frac{7\pi}{8}]+i\,\text{Sin}[\frac{7\pi}{8}]\right)\left(\text{Cosh}[\text{Cos}[\frac{9\pi}{8}]]+\text{Sin}[\frac{9\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{9\pi}{8}]]\right)+$$

$$\left(\text{Cos}[\frac{5\pi}{8}]+i\,\text{Sin}[\frac{5\pi}{8}]\right)\left(\text{Cosh}[\text{Cos}[\frac{11\pi}{8}]]+\text{Sin}[\frac{11\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{11\pi}{8}]]\right)+$$

$$\left(\text{Cos}[\frac{3\pi}{8}]+i\,\text{Sin}[\frac{3\pi}{8}]\right)\left(\text{Cosh}[\text{Cos}[\frac{13\pi}{8}]]+\text{Sin}[\frac{13\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{13\pi}{8}]]\right)+$$

$$\left(\text{Cos}[\frac{\pi}{8}]+i\,\text{Sin}[\frac{\pi}{8}]\right)\left(\text{Cosh}[\text{Cos}[\frac{15\pi}{8}]]+\text{Sin}[\frac{15\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{15\pi}{8}]]\right)\Bigg)+$$

$$\frac{1}{16}(\text{Cos}[13t]+i\,\text{Sin}[13t])\Bigg(e-\text{Cosh}[1]+\text{Sinh}[1]+$$

$$\frac{2\left(\frac{1}{2}+\text{Cosh}[\frac{1}{\sqrt{2}}]-\text{Sinh}[\frac{1}{\sqrt{2}}]\right)}{\sqrt{2}}-\frac{2\left(\frac{1}{2}+\text{Cosh}[\frac{1}{\sqrt{2}}]+\text{Sinh}[\frac{1}{\sqrt{2}}]\right)}{\sqrt{2}}+$$

$$\left(\text{Cos}[\frac{3\pi}{8}]+i\,\text{Sin}[\frac{3\pi}{8}]\right)\left(\text{Cosh}[\text{Cos}[\frac{\pi}{8}]]+\text{Sin}[\frac{\pi}{8}]^2+\text{Sinh}[\text{Cos}[\frac{\pi}{8}]]\right)+$$

$$\left(\text{Cos}\left[\frac{7\pi}{8}\right]-i\,\text{Sin}\left[\frac{7\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{3\pi}{8}\right]\right]+\text{Sin}\left[\frac{3\pi}{8}\right]^2+\text{Sinh}\left[\text{Cos}\left[\frac{3\pi}{8}\right]\right]\right)+$$
$$\left(\text{Cos}\left[\frac{\pi}{8}\right]-i\,\text{Sin}\left[\frac{\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{5\pi}{8}\right]\right]+\text{Sin}\left[\frac{5\pi}{8}\right]^2+\text{Sinh}\left[\text{Cos}\left[\frac{5\pi}{8}\right]\right]\right)+$$
$$\left(\text{Cos}\left[\frac{5\pi}{8}\right]+i\,\text{Sin}\left[\frac{5\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{7\pi}{8}\right]\right]+\text{Sin}\left[\frac{7\pi}{8}\right]^2+\text{Sinh}\left[\text{Cos}\left[\frac{7\pi}{8}\right]\right]\right)+$$
$$\left(\text{Cos}\left[\frac{5\pi}{8}\right]-i\,\text{Sin}\left[\frac{5\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{9\pi}{8}\right]\right]+\text{Sin}\left[\frac{9\pi}{8}\right]^2+\text{Sinh}\left[\text{Cos}\left[\frac{9\pi}{8}\right]\right]\right)+$$
$$\left(\text{Cos}\left[\frac{\pi}{8}\right]+i\,\text{Sin}\left[\frac{\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{11\pi}{8}\right]\right]+\text{Sin}\left[\frac{11\pi}{8}\right]^2+\text{Sinh}\left[\text{Cos}\left[\frac{11\pi}{8}\right]\right]\right)+$$
$$\left(\text{Cos}\left[\frac{7\pi}{8}\right]+i\,\text{Sin}\left[\frac{7\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{13\pi}{8}\right]\right]+\text{Sin}\left[\frac{13\pi}{8}\right]^2+\text{Sinh}\left[\text{Cos}\left[\frac{13\pi}{8}\right]\right]\right)+$$
$$\left(\text{Cos}\left[\frac{3\pi}{8}\right]-i\,\text{Sin}\left[\frac{3\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{15\pi}{8}\right]\right]+\text{Sin}\left[\frac{15\pi}{8}\right]^2+\text{Sinh}\left[\text{Cos}\left[\frac{15\pi}{8}\right]\right]\right)\Bigg)+$$

$$\frac{1}{16}\left(\text{Cos}[3t]+i\,\text{Sin}[3t]\right)\Bigg(e-\text{Cosh}[1]+\text{Sinh}[1]+\frac{2\left(\frac{1}{2}+\text{Cosh}\left[\frac{1}{\sqrt{2}}\right]-\text{Sinh}\left[\frac{1}{\sqrt{2}}\right]\right)}{\sqrt{2}}-$$
$$\frac{2\left(\frac{1}{2}+\text{Cosh}\left[\frac{1}{\sqrt{2}}\right]+\text{Sinh}\left[\frac{1}{\sqrt{2}}\right]\right)}{\sqrt{2}}+$$
$$\left(\text{Cos}\left[\frac{3\pi}{8}\right]-i\,\text{Sin}\left[\frac{3\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{\pi}{8}\right]\right]+\text{Sin}\left[\frac{\pi}{8}\right]^2+\text{Sinh}\left[\text{Cos}\left[\frac{\pi}{8}\right]\right]\right)+$$
$$\left(\text{Cos}\left[\frac{7\pi}{8}\right]+i\,\text{Sin}\left[\frac{7\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{3\pi}{8}\right]\right]+\text{Sin}\left[\frac{3\pi}{8}\right]^2+\text{Sinh}\left[\text{Cos}\left[\frac{3\pi}{8}\right]\right]\right)+$$
$$\left(\text{Cos}\left[\frac{\pi}{8}\right]+i\,\text{Sin}\left[\frac{\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{5\pi}{8}\right]\right]+\text{Sin}\left[\frac{5\pi}{8}\right]^2+\text{Sinh}\left[\text{Cos}\left[\frac{5\pi}{8}\right]\right]\right)+$$
$$\left(\text{Cos}\left[\frac{5\pi}{8}\right]-i\,\text{Sin}\left[\frac{5\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{7\pi}{8}\right]\right]+\text{Sin}\left[\frac{7\pi}{8}\right]^2+\text{Sinh}\left[\text{Cos}\left[\frac{7\pi}{8}\right]\right]\right)+$$
$$\left(\text{Cos}\left[\frac{5\pi}{8}\right]+i\,\text{Sin}\left[\frac{5\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{9\pi}{8}\right]\right]+\text{Sin}\left[\frac{9\pi}{8}\right]^2+\text{Sinh}\left[\text{Cos}\left[\frac{9\pi}{8}\right]\right]\right)+$$
$$\left(\text{Cos}\left[\frac{\pi}{8}\right]-i\,\text{Sin}\left[\frac{\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{11\pi}{8}\right]\right]+\text{Sin}\left[\frac{11\pi}{8}\right]^2+\text{Sinh}\left[\text{Cos}\left[\frac{11\pi}{8}\right]\right]\right)+$$
$$\left(\text{Cos}\left[\frac{7\pi}{8}\right]-i\,\text{Sin}\left[\frac{7\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{13\pi}{8}\right]\right]+\text{Sin}\left[\frac{13\pi}{8}\right]^2+\text{Sinh}\left[\text{Cos}\left[\frac{13\pi}{8}\right]\right]\right)+$$
$$\left(\text{Cos}\left[\frac{3\pi}{8}\right]+i\,\text{Sin}\left[\frac{3\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{15\pi}{8}\right]\right]+\text{Sin}\left[\frac{15\pi}{8}\right]^2+\text{Sinh}\left[\text{Cos}\left[\frac{15\pi}{8}\right]\right]\right)\Bigg)+$$

$$\frac{1}{16}\left(\text{Cos}[11t]+i\,\text{Sin}[11t]\right)\Bigg(e-\text{Cosh}[1]+\text{Sinh}[1]+$$
$$\frac{2\left(\frac{1}{2}+\text{Cosh}\left[\frac{1}{\sqrt{2}}\right]-\text{Sinh}\left[\frac{1}{\sqrt{2}}\right]\right)}{\sqrt{2}}-\frac{2\left(\frac{1}{2}+\text{Cosh}\left[\frac{1}{\sqrt{2}}\right]+\text{Sinh}\left[\frac{1}{\sqrt{2}}\right]\right)}{\sqrt{2}}+$$
$$\left(\text{Cos}\left[\frac{5\pi}{8}\right]+i\,\text{Sin}\left[\frac{5\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{\pi}{8}\right]\right]+\text{Sin}\left[\frac{\pi}{8}\right]^2+\text{Sinh}\left[\text{Cos}\left[\frac{\pi}{8}\right]\right]\right)+$$
$$\left(\text{Cos}\left[\frac{\pi}{8}\right]-i\,\text{Sin}\left[\frac{\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{3\pi}{8}\right]\right]+\text{Sin}\left[\frac{3\pi}{8}\right]^2+\text{Sinh}\left[\text{Cos}\left[\frac{3\pi}{8}\right]\right]\right)+$$
$$\left(\text{Cos}\left[\frac{7\pi}{8}\right]-i\,\text{Sin}\left[\frac{7\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{5\pi}{8}\right]\right]+\text{Sin}\left[\frac{5\pi}{8}\right]^2+\text{Sinh}\left[\text{Cos}\left[\frac{5\pi}{8}\right]\right]\right)+$$
$$\left(\text{Cos}\left[\frac{3\pi}{8}\right]+i\,\text{Sin}\left[\frac{3\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{7\pi}{8}\right]\right]+\text{Sin}\left[\frac{7\pi}{8}\right]^2+\text{Sinh}\left[\text{Cos}\left[\frac{7\pi}{8}\right]\right]\right)+$$
$$\left(\text{Cos}\left[\frac{3\pi}{8}\right]-i\,\text{Sin}\left[\frac{3\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{9\pi}{8}\right]\right]+\text{Sin}\left[\frac{9\pi}{8}\right]^2+\text{Sinh}\left[\text{Cos}\left[\frac{9\pi}{8}\right]\right]\right)+$$
$$\left(\text{Cos}\left[\frac{7\pi}{8}\right]+i\,\text{Sin}\left[\frac{7\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{11\pi}{8}\right]\right]+\text{Sin}\left[\frac{11\pi}{8}\right]^2+\text{Sinh}\left[\text{Cos}\left[\frac{11\pi}{8}\right]\right]\right)+$$

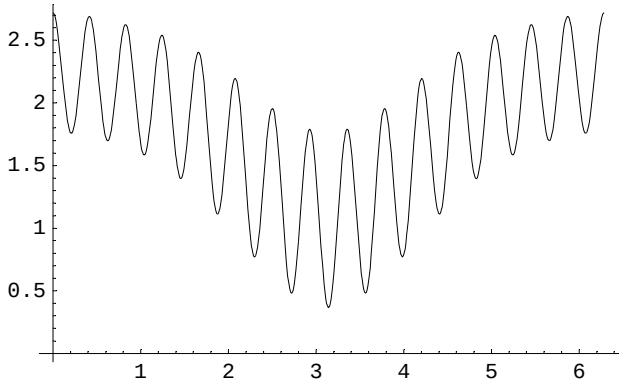
$$\left(\cos\left[\frac{\pi}{8}\right]+i\sin\left[\frac{\pi}{8}\right]\right)\left(\cosh\left[\cos\left[\frac{13\pi}{8}\right]\right]+\sin\left[\frac{13\pi}{8}\right]^2+\sinh\left[\cos\left[\frac{13\pi}{8}\right]\right]\right)+$$
$$\left(\cos\left[\frac{5\pi}{8}\right]-i\sin\left[\frac{5\pi}{8}\right]\right)\left(\cosh\left[\cos\left[\frac{15\pi}{8}\right]\right]+\sin\left[\frac{15\pi}{8}\right]^2+\sinh\left[\cos\left[\frac{15\pi}{8}\right]\right]\right)\Bigg)+$$
$$\frac{1}{16}\left(\cos[5t]+i\sin[5t]\right)\left(e-\cosh[1]+\sinh[1]+\frac{2\left(\frac{1}{2}+\cosh\left[\frac{1}{\sqrt{2}}\right]-\sinh\left[\frac{1}{\sqrt{2}}\right]\right)}{\sqrt{2}}-\right.$$
$$\frac{2\left(\frac{1}{2}+\cosh\left[\frac{1}{\sqrt{2}}\right]+\sinh\left[\frac{1}{\sqrt{2}}\right]\right)}{\sqrt{2}}+$$
$$\left(\cos\left[\frac{5\pi}{8}\right]-i\sin\left[\frac{5\pi}{8}\right]\right)\left(\cosh\left[\cos\left[\frac{\pi}{8}\right]\right]+\sin\left[\frac{\pi}{8}\right]^2+\sinh\left[\cos\left[\frac{\pi}{8}\right]\right]\right)+$$
$$\left(\cos\left[\frac{\pi}{8}\right]+i\sin\left[\frac{\pi}{8}\right]\right)\left(\cosh\left[\cos\left[\frac{3\pi}{8}\right]\right]+\sin\left[\frac{3\pi}{8}\right]^2+\sinh\left[\cos\left[\frac{3\pi}{8}\right]\right]\right)+$$
$$\left(\cos\left[\frac{7\pi}{8}\right]+i\sin\left[\frac{7\pi}{8}\right]\right)\left(\cosh\left[\cos\left[\frac{5\pi}{8}\right]\right]+\sin\left[\frac{5\pi}{8}\right]^2+\sinh\left[\cos\left[\frac{5\pi}{8}\right]\right]\right)+$$
$$\left(\cos\left[\frac{3\pi}{8}\right]-i\sin\left[\frac{3\pi}{8}\right]\right)\left(\cosh\left[\cos\left[\frac{7\pi}{8}\right]\right]+\sin\left[\frac{7\pi}{8}\right]^2+\sinh\left[\cos\left[\frac{7\pi}{8}\right]\right]\right)+$$
$$\left(\cos\left[\frac{3\pi}{8}\right]+i\sin\left[\frac{3\pi}{8}\right]\right)\left(\cosh\left[\cos\left[\frac{9\pi}{8}\right]\right]+\sin\left[\frac{9\pi}{8}\right]^2+\sinh\left[\cos\left[\frac{9\pi}{8}\right]\right]\right)+$$
$$\left(\cos\left[\frac{7\pi}{8}\right]-i\sin\left[\frac{7\pi}{8}\right]\right)\left(\cosh\left[\cos\left[\frac{11\pi}{8}\right]\right]+\sin\left[\frac{11\pi}{8}\right]^2+\sinh\left[\cos\left[\frac{11\pi}{8}\right]\right]\right)+$$
$$\left(\cos\left[\frac{\pi}{8}\right]-i\sin\left[\frac{\pi}{8}\right]\right)\left(\cosh\left[\cos\left[\frac{13\pi}{8}\right]\right]+\sin\left[\frac{13\pi}{8}\right]^2+\sinh\left[\cos\left[\frac{13\pi}{8}\right]\right]\right)+$$
$$\left(\cos\left[\frac{5\pi}{8}\right]+i\sin\left[\frac{5\pi}{8}\right]\right)\left(\cosh\left[\cos\left[\frac{15\pi}{8}\right]\right]+\sin\left[\frac{15\pi}{8}\right]^2+\sinh\left[\cos\left[\frac{15\pi}{8}\right]\right]\right)\Bigg)+$$
$$\frac{1}{16}\left(\cos[9t]+i\sin[9t]\right)\left(e-\cosh[1]+\sinh[1]-\frac{2\left(\frac{1}{2}+\cosh\left[\frac{1}{\sqrt{2}}\right]-\sinh\left[\frac{1}{\sqrt{2}}\right]\right)}{\sqrt{2}}+\right.$$
$$\frac{2\left(\frac{1}{2}+\cosh\left[\frac{1}{\sqrt{2}}\right]+\sinh\left[\frac{1}{\sqrt{2}}\right]\right)}{\sqrt{2}}+$$
$$\left(\cos\left[\frac{7\pi}{8}\right]+i\sin\left[\frac{7\pi}{8}\right]\right)\left(\cosh\left[\cos\left[\frac{\pi}{8}\right]\right]+\sin\left[\frac{\pi}{8}\right]^2+\sinh\left[\cos\left[\frac{\pi}{8}\right]\right]\right)+$$
$$\left(\cos\left[\frac{5\pi}{8}\right]+i\sin\left[\frac{5\pi}{8}\right]\right)\left(\cosh\left[\cos\left[\frac{3\pi}{8}\right]\right]+\sin\left[\frac{3\pi}{8}\right]^2+\sinh\left[\cos\left[\frac{3\pi}{8}\right]\right]\right)+$$
$$\left(\cos\left[\frac{3\pi}{8}\right]+i\sin\left[\frac{3\pi}{8}\right]\right)\left(\cosh\left[\cos\left[\frac{5\pi}{8}\right]\right]+\sin\left[\frac{5\pi}{8}\right]^2+\sinh\left[\cos\left[\frac{5\pi}{8}\right]\right]\right)+$$
$$\left(\cos\left[\frac{\pi}{8}\right]+i\sin\left[\frac{\pi}{8}\right]\right)\left(\cosh\left[\cos\left[\frac{7\pi}{8}\right]\right]+\sin\left[\frac{7\pi}{8}\right]^2+\sinh\left[\cos\left[\frac{7\pi}{8}\right]\right]\right)+$$
$$\left(\cos\left[\frac{\pi}{8}\right]-i\sin\left[\frac{\pi}{8}\right]\right)\left(\cosh\left[\cos\left[\frac{9\pi}{8}\right]\right]+\sin\left[\frac{9\pi}{8}\right]^2+\sinh\left[\cos\left[\frac{9\pi}{8}\right]\right]\right)+$$
$$\left(\cos\left[\frac{3\pi}{8}\right]-i\sin\left[\frac{3\pi}{8}\right]\right)\left(\cosh\left[\cos\left[\frac{11\pi}{8}\right]\right]+\sin\left[\frac{11\pi}{8}\right]^2+\sinh\left[\cos\left[\frac{11\pi}{8}\right]\right]\right)+$$
$$\left(\cos\left[\frac{5\pi}{8}\right]-i\sin\left[\frac{5\pi}{8}\right]\right)\left(\cosh\left[\cos\left[\frac{13\pi}{8}\right]\right]+\sin\left[\frac{13\pi}{8}\right]^2+\sinh\left[\cos\left[\frac{13\pi}{8}\right]\right]\right)+$$
$$\left(\cos\left[\frac{7\pi}{8}\right]-i\sin\left[\frac{7\pi}{8}\right]\right)\left(\cosh\left[\cos\left[\frac{15\pi}{8}\right]\right]+\sin\left[\frac{15\pi}{8}\right]^2+\sinh\left[\cos\left[\frac{15\pi}{8}\right]\right]\right)\Bigg)+$$
$$\frac{1}{16}\left(\cos[7t]+i\sin[7t]\right)\left(e-\cosh[1]+\sinh[1]-\frac{2\left(\frac{1}{2}+\cosh\left[\frac{1}{\sqrt{2}}\right]-\sinh\left[\frac{1}{\sqrt{2}}\right]\right)}{\sqrt{2}}+\right.$$
$$\frac{2\left(\frac{1}{2}+\cosh\left[\frac{1}{\sqrt{2}}\right]+\sinh\left[\frac{1}{\sqrt{2}}\right]\right)}{\sqrt{2}}+$$

$$\left(\text{Cos}\left[\frac{7\pi}{8}\right] - i\,\text{Sin}\left[\frac{7\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{\pi}{8}\right]\right] + \text{Sin}\left[\frac{\pi}{8}\right]^2 + \text{Sinh}\left[\text{Cos}\left[\frac{\pi}{8}\right]\right]\right) +$$
$$\left(\text{Cos}\left[\frac{5\pi}{8}\right] - i\,\text{Sin}\left[\frac{5\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{3\pi}{8}\right]\right] + \text{Sin}\left[\frac{3\pi}{8}\right]^2 + \text{Sinh}\left[\text{Cos}\left[\frac{3\pi}{8}\right]\right]\right) +$$
$$\left(\text{Cos}\left[\frac{3\pi}{8}\right] - i\,\text{Sin}\left[\frac{3\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{5\pi}{8}\right]\right] + \text{Sin}\left[\frac{5\pi}{8}\right]^2 + \text{Sinh}\left[\text{Cos}\left[\frac{5\pi}{8}\right]\right]\right) +$$
$$\left(\text{Cos}\left[\frac{\pi}{8}\right] - i\,\text{Sin}\left[\frac{\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{7\pi}{8}\right]\right] + \text{Sin}\left[\frac{7\pi}{8}\right]^2 + \text{Sinh}\left[\text{Cos}\left[\frac{7\pi}{8}\right]\right]\right) +$$
$$\left(\text{Cos}\left[\frac{\pi}{8}\right] + i\,\text{Sin}\left[\frac{\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{9\pi}{8}\right]\right] + \text{Sin}\left[\frac{9\pi}{8}\right]^2 + \text{Sinh}\left[\text{Cos}\left[\frac{9\pi}{8}\right]\right]\right) +$$
$$\left(\text{Cos}\left[\frac{3\pi}{8}\right] + i\,\text{Sin}\left[\frac{3\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{11\pi}{8}\right]\right] + \text{Sin}\left[\frac{11\pi}{8}\right]^2 + \text{Sinh}\left[\text{Cos}\left[\frac{11\pi}{8}\right]\right]\right) +$$
$$\left(\text{Cos}\left[\frac{5\pi}{8}\right] + i\,\text{Sin}\left[\frac{5\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{13\pi}{8}\right]\right] + \text{Sin}\left[\frac{13\pi}{8}\right]^2 + \text{Sinh}\left[\text{Cos}\left[\frac{13\pi}{8}\right]\right]\right) +$$
$$\left(\text{Cos}\left[\frac{7\pi}{8}\right] + i\,\text{Sin}\left[\frac{7\pi}{8}\right]\right)\left(\text{Cosh}\left[\text{Cos}\left[\frac{15\pi}{8}\right]\right] + \text{Sin}\left[\frac{15\pi}{8}\right]^2 + \text{Sinh}\left[\text{Cos}\left[\frac{15\pi}{8}\right]\right]\right)\Bigg)$$

Plot der trigonometrischen Reihe durch die gegebenen Punkte

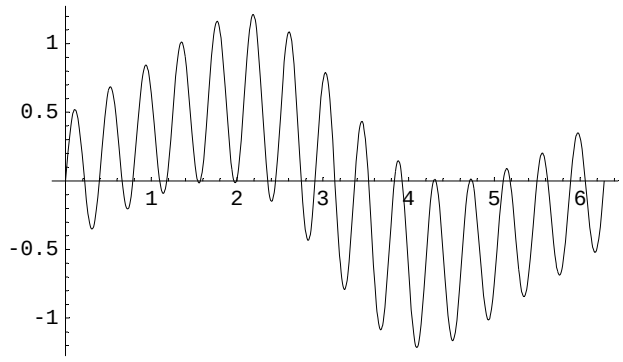
Realanteil (muss wesentlich sein)

```
Plot[Re[fS[t]],{t,0,2Pi},PlotPoints->50];
```



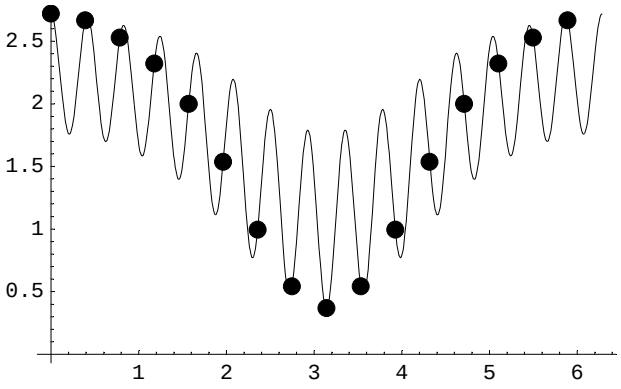
Imaginäranteil (sollte klein sein)

```
Plot[Im[fS[t]],{t,0,2Pi},PlotPoints->50];
```



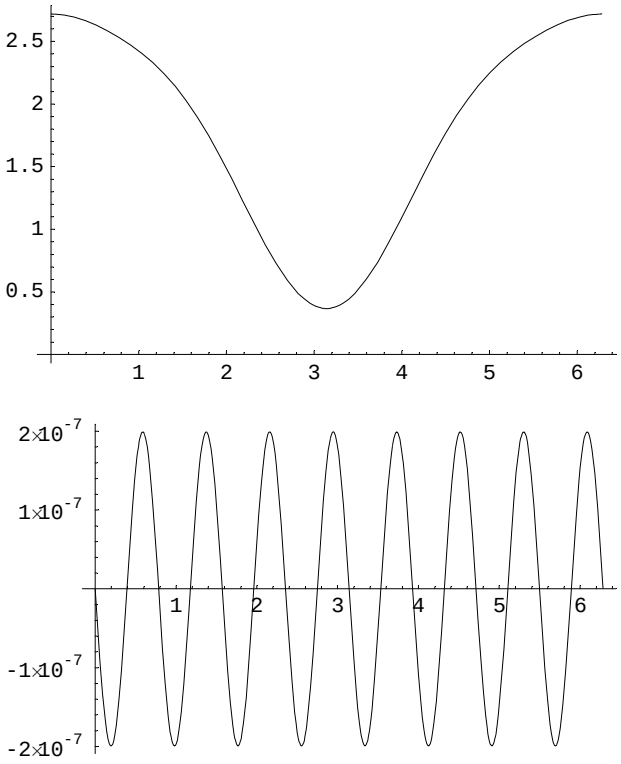
Vergleich mit den gegebenen Punkten

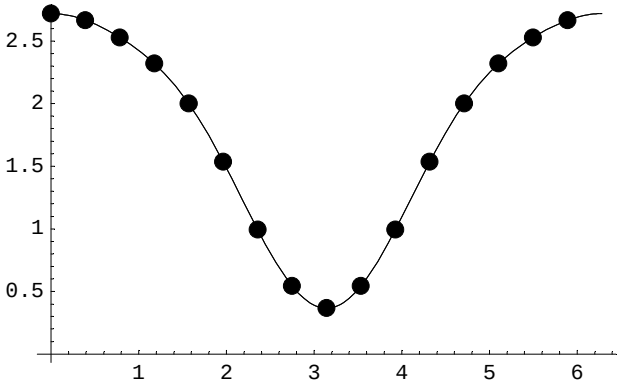
```
Plot[Re[fS[t]],{t,0,2Pi},PlotPoints->50,Epilog->epi];
```



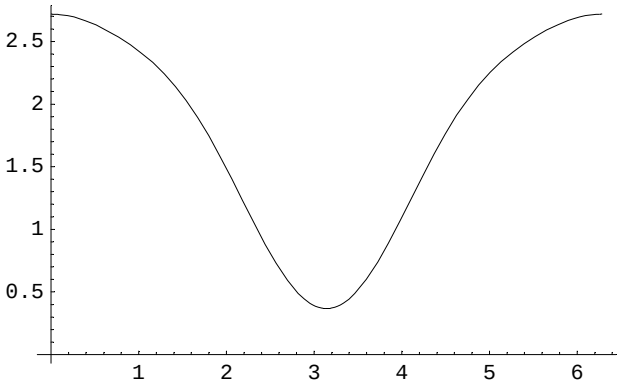
Andere Grenzen, gleiche Periode ==> besser!

```
fSsym[t_] := Sum[c[k] E^(I k t), {k, -n / 2, n - 1 - n / 2}];
Plot[Re[fSsym[t]], {t, 0, 2 Pi}, PlotPoints → 50];
Plot[Im[fSsym[t]], {t, 0, 2 Pi}, PlotPoints → 50];
Plot[{f[t], Re[fSsym[t]]}, {t, 0, 2 Pi}, PlotPoints → 50, Epilog → epi];
```

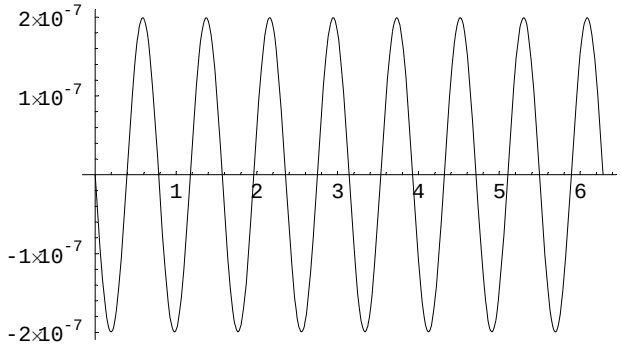




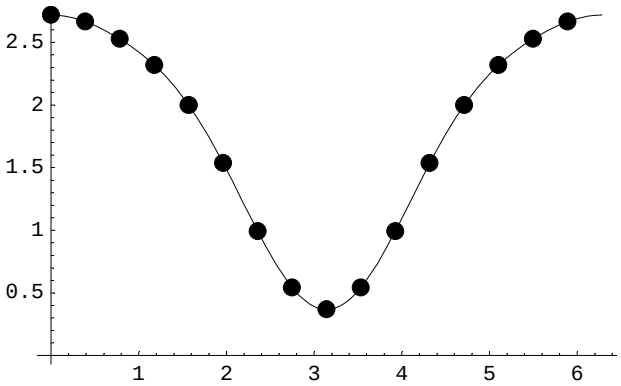
Passt so viel besser:



Sehr kleine Ausschläge:



Funktion, Approximation und Punkte:



3 Hier sind die x-Werte unregelmässig verteilt! (Kommentar siehe am Schlusse!)

```
Remove["Global`*"]
```

Wir verwenden hier die Skalierung nach der Periode 2 Pi. Das vereinfacht die Rechung etwas.

```
 n=20; w = 2 Pi/10;
{x[0],y[0]}={0.0 w, 14.2};
{x[1],y[1]}={0.05 w,12.8};
{x[2],y[2]}={1.12 w,12.9};
{x[3],y[3]}={1.6 w,7.4};
{x[4],y[4]}={2.3 w,6.7};
{x[5],y[5]}={2.8 w,9.5};
{x[6],y[6]}={3.0 w,9.8};
{x[7],y[7]}={3.9 w,9.3};
{x[8],y[8]}={4.7 w,10.2};
{x[9],y[9]}={5.4 w,11.5};
{x[10],y[10]}={5.6 w,15.2};
{x[11],y[11]}={6.6 w,15.6};
{x[12],y[12]}={6.9 w,16.7};
{x[13],y[13]}={7.3 w,16.4};
{x[14],y[14]}={7.7 w,16.7};
{x[15],y[15]}={8.2 w,14.1};
{x[16],y[16]}={8.5 w,13.2};
{x[17],y[17]}={8.8  w,10.5};
{x[18],y[18]}={9.1  w,11.8};
{x[19],y[19]}={9.9 w,13.9};

p[k_]:= {x[k],y[k]};
Table[p[k],{k,0,n-1}]
```

```
{{0., 14.2}, {0.0314159, 12.8}, {0.703717, 12.9}, {1.00531, 7.4}, {1.44513, 6.7},
 {1.75929, 9.5}, {1.88496, 9.8}, {2.45044, 9.3}, {2.9531, 10.2}, {3.39292, 11.5},
 {3.51858, 15.2}, {4.1469, 15.6}, {4.3354, 16.7}, {4.58673, 16.4}, {4.83805, 16.7},
 {5.15221, 14.1}, {5.34071, 13.2}, {5.5292, 10.5}, {5.7177, 11.8}, {6.22035, 13.9}}
```

```
epi=Prepend[Map[Point,Table[p[k],{k,0,n-1}]],PointSize[0.03]]
```

```
{PointSize[0.03], Point[{0., 14.2}], Point[{0.0314159, 12.8}],
 Point[{0.703717, 12.9}], Point[{1.00531, 7.4}], Point[{1.44513, 6.7}],
 Point[{1.75929, 9.5}], Point[{1.88496, 9.8}], Point[{2.45044, 9.3}],
 Point[{2.9531, 10.2}], Point[{3.39292, 11.5}], Point[{3.51858, 15.2}],
 Point[{4.1469, 15.6}], Point[{4.3354, 16.7}], Point[{4.58673, 16.4}],
 Point[{4.83805, 16.7}], Point[{5.15221, 14.1}], Point[{5.34071, 13.2}],
 Point[{5.5292, 10.5}], Point[{5.7177, 11.8}], Point[{6.22035, 13.9}]}
```

```
r = E^(-I 2 Pi/n);
c[s_]:= 1/n Sum[y[k] r^(s k),{k,0,n-1}];
Table[c[s],{s,0,n-1}]//N
```

```
{12.42, -0.489225 + 1.51885 i, 0.712943 - 0.651536 i, 0.610124 + 0.000912226 i,
 0.347254 - 0.229439 i, -0.27 - 0.02 i, -0.147943 + 0.179236 i,
 -0.108189 + 0.274974 i, 0.0677458 + 0.0947781 i, 0.0072899 - 0.0729668 i,
 0.32, 0.0072899 + 0.0729668 i, 0.0677458 - 0.0947781 i, -0.108189 - 0.274974 i,
 -0.147943 - 0.179236 i, -0.27 + 0.02 i, 0.347254 + 0.229439 i,
 0.610124 - 0.000912226 i, 0.712943 + 0.651536 i, -0.489225 - 1.51885 i}
```

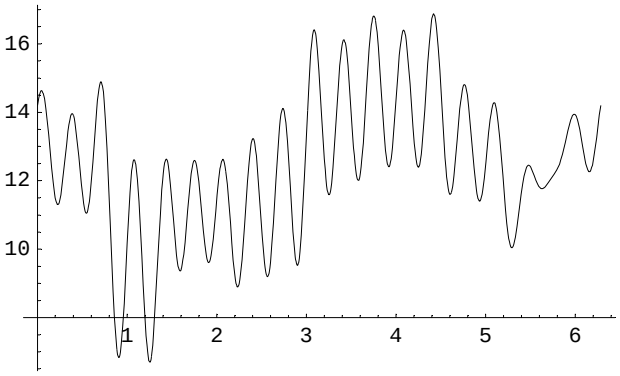
```
fS[t_]:=Sum[c[k] E^(I k t),{k,0,n-1}];
fS[t]
```

```
12.42 - (0.489225 - 1.51885 i) e^(i t) + (0.712943 - 0.651536 i) e^(2 i t) +
 (0.610124 + 0.000912226 i) e^(3 i t) + (0.347254 - 0.229439 i) e^(4 i t) -
 (0.27 + 0.02 i) e^(5 i t) - (0.147943 - 0.179236 i) e^(6 i t) - (0.108189 - 0.274974 i) e^(7 i t) +
 (0.0677458 + 0.0947781 i) e^(8 i t) + (0.0072899 - 0.0729668 i) e^(9 i t) + 0.32 e^(10 i t) +
 (0.0072899 + 0.0729668 i) e^(11 i t) + (0.0677458 - 0.0947781 i) e^(12 i t) -
 (0.108189 + 0.274974 i) e^(13 i t) - (0.147943 + 0.179236 i) e^(14 i t) - (0.27 - 0.02 i) e^(15 i t) +
 (0.347254 + 0.229439 i) e^(16 i t) + (0.610124 - 0.000912226 i) e^(17 i t) +
 (0.712943 + 0.651536 i) e^(18 i t) - (0.489225 + 1.51885 i) e^(19 i t)
```

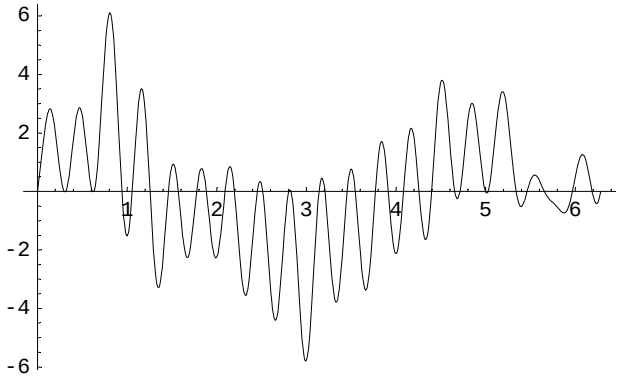
```
fS[t]//ExpToTrig
```

```
12.42 - (0.489225 - 1.51885 i) Cos[t] + (0.712943 - 0.651536 i) Cos[2 t] +
 (0.610124 + 0.000912226 i) Cos[3 t] + (0.347254 - 0.229439 i) Cos[4 t] -
 (0.27 + 0.02 i) Cos[5 t] - (0.147943 - 0.179236 i) Cos[6 t] -
 (0.108189 - 0.274974 i) Cos[7 t] + (0.0677458 + 0.0947781 i) Cos[8 t] +
 (0.0072899 - 0.0729668 i) Cos[9 t] + 0.32 Cos[10 t] + (0.0072899 + 0.0729668 i) Cos[11 t] +
 (0.0677458 - 0.0947781 i) Cos[12 t] - (0.108189 + 0.274974 i) Cos[13 t] -
 (0.147943 + 0.179236 i) Cos[14 t] - (0.27 - 0.02 i) Cos[15 t] +
 (0.347254 + 0.229439 i) Cos[16 t] + (0.610124 - 0.000912226 i) Cos[17 t] +
 (0.712943 + 0.651536 i) Cos[18 t] - (0.489225 + 1.51885 i) Cos[19 t] -
 (1.51885 + 0.489225 i) Sin[t] + (0.651536 + 0.712943 i) Sin[2 t] -
 (0.000912226 - 0.610124 i) Sin[3 t] + (0.229439 + 0.347254 i) Sin[4 t] +
 (0.02 - 0.27 i) Sin[5 t] - (0.179236 + 0.147943 i) Sin[6 t] -
 (0.274974 + 0.108189 i) Sin[7 t] - (0.0947781 - 0.0677458 i) Sin[8 t] +
 (0.0729668 + 0.0072899 i) Sin[9 t] + 0.32 i Sin[10 t] - (0.0729668 - 0.0072899 i) Sin[11 t] +
 (0.0947781 + 0.0677458 i) Sin[12 t] + (0.274974 - 0.108189 i) Sin[13 t] +
 (0.179236 - 0.147943 i) Sin[14 t] - (0.02 + 0.27 i) Sin[15 t] -
 (0.229439 - 0.347254 i) Sin[16 t] + (0.000912226 + 0.610124 i) Sin[17 t] -
 (0.651536 - 0.712943 i) Sin[18 t] + (1.51885 - 0.489225 i) Sin[19 t]
```

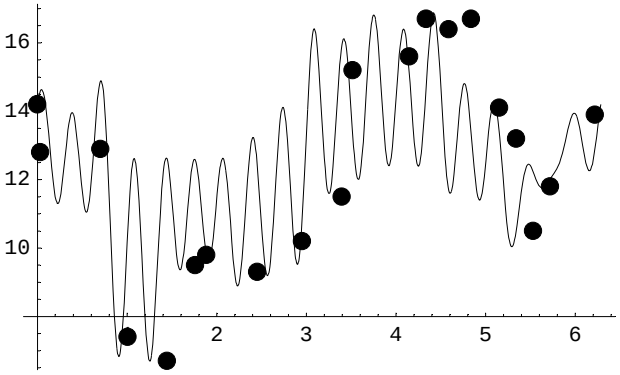
```
Plot[Re[fS[t]],{t,0,2Pi},PlotPoints->50];
```



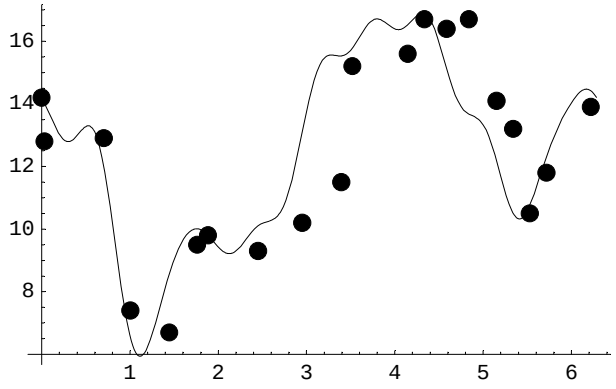
```
Plot[Im[fS[t]],{t,0,2Pi},PlotPoints->50];
```




```
Plot[Re[fS[t]], {t, 0, 2 Pi}, PlotPoints → 50, Epilog → epi];
```




```
fSsym[t_]:=Sum[c[k] E^(I k t),{k,-n/2,n-1-n/2}];
Plot[Re[fSsym[t]],{t,0,2Pi},PlotPoints->50,Epilog->epi];
```



Resultat: Das Resultat entspricht nicht dem Gewünschten. Grund: Bei der DFT geht man von gleichen Abstanden auf der x-Achse aus, z.B. t(k+1)-t(k)= 2 /n (äquidistante Messungen)

4 (Kommentar siehe am Schlusse!)

```
Remove["Global`*"]
```

Wir verwenden hier die Skalierung nach der Periode 2 Pi. Das vereinfacht die Rechung etwas.

```
 n=20; w = 2 Pi/n;
{x[0],y[0]}={0 w, 14.2};
{x[1],y[1]}={1 w,12.8};
{x[2],y[2]}={2 w,12.9};
{x[3],y[3]}={3 w,7.4};
{x[4],y[4]}={4 w,6.7};
{x[5],y[5]}={5 w,9.5};
{x[6],y[6]}={6 w,9.8};
{x[7],y[7]}={7 w,9.3};
{x[8],y[8]}={8 w,10.2};
{x[9],y[9]}={9 w,11.5};
{x[10],y[10]}={10 w,15.2};
{x[11],y[11]}={11 w,15.6};
{x[12],y[12]}={12 w,16.7};
{x[13],y[13]}={13 w,16.4};
{x[14],y[14]}={14 w,16.7};
{x[15],y[15]}={15 w,14.1};
{x[16],y[16]}={16 w,13.2};
{x[17],y[17]}={17  w,10.5};
{x[18],y[18]}={18  w,11.8};
{x[19],y[19]}={19 w,13.9};

p[k_]:= {x[k],y[k]};
Table[p[k],{k,0,n-1}]
```

$$\left\{\{0, 14.2\}, \left\{\frac{\pi}{10}, 12.8\right\}, \left\{\frac{\pi}{5}, 12.9\right\}, \left\{\frac{3\pi}{10}, 7.4\right\}, \left\{\frac{2\pi}{5}, 6.7\right\}, \left\{\frac{\pi}{2}, 9.5\right\}, \left\{\frac{3\pi}{5}, 9.8\right\}, \left\{\frac{7\pi}{10}, 9.3\right\}, \left\{\frac{4\pi}{5}, 10.2\right\}, \left\{\frac{9\pi}{10}, 11.5\right\}, \{\pi, 15.2\}, \left\{\frac{11\pi}{10}, 15.6\right\}, \left\{\frac{6\pi}{5}, 16.7\right\}, \left\{\frac{13\pi}{10}, 16.4\right\}, \left\{\frac{7\pi}{5}, 16.7\right\}, \left\{\frac{3\pi}{2}, 14.1\right\}, \left\{\frac{8\pi}{5}, 13.2\right\}, \left\{\frac{17\pi}{10}, 10.5\right\}, \left\{\frac{9\pi}{5}, 11.8\right\}, \left\{\frac{19\pi}{10}, 13.9\right\}\right\}$$

```
epi=Prepend[Map[Point,Table[p[k],{k,0,n-1}]],PointSize[0.03]]
```

$$\left\{\text{PointSize}[0.03], \text{Point}[\{0, 14.2\}], \text{Point}\left[\left\{\frac{\pi}{10}, 12.8\right\}\right], \text{Point}\left[\left\{\frac{\pi}{5}, 12.9\right\}\right], \text{Point}\left[\left\{\frac{3\pi}{10}, 7.4\right\}\right], \text{Point}\left[\left\{\frac{2\pi}{5}, 6.7\right\}\right], \text{Point}\left[\left\{\frac{\pi}{2}, 9.5\right\}\right], \text{Point}\left[\left\{\frac{3\pi}{5}, 9.8\right\}\right], \text{Point}\left[\left\{\frac{7\pi}{10}, 9.3\right\}\right], \text{Point}\left[\left\{\frac{4\pi}{5}, 10.2\right\}\right], \text{Point}\left[\left\{\frac{9\pi}{10}, 11.5\right\}\right], \text{Point}[\{\pi, 15.2\}], \text{Point}\left[\left\{\frac{11\pi}{10}, 15.6\right\}\right], \text{Point}\left[\left\{\frac{6\pi}{5}, 16.7\right\}\right], \text{Point}\left[\left\{\frac{13\pi}{10}, 16.4\right\}\right], \text{Point}\left[\left\{\frac{7\pi}{5}, 16.7\right\}\right], \text{Point}\left[\left\{\frac{3\pi}{2}, 14.1\right\}\right], \text{Point}\left[\left\{\frac{8\pi}{5}, 13.2\right\}\right], \text{Point}\left[\left\{\frac{17\pi}{10}, 10.5\right\}\right], \text{Point}\left[\left\{\frac{9\pi}{5}, 11.8\right\}\right], \text{Point}\left[\left\{\frac{19\pi}{10}, 13.9\right\}\right]\right\}$$

```
r = E^(-I 2 Pi/n);
c[s_]:= 1/n Sum[y[k] r^(s k),{k,0,n-1}];
Table[c[s],{s,0,n-1}]//N
```

$$\{12.42, -0.489225 + 1.51885\,i, 0.712943 - 0.651536\,i, 0.610124 + 0.000912226\,i, 0.347254 - 0.229439\,i, -0.27 - 0.02\,i, -0.147943 + 0.179236\,i, -0.108189 + 0.274974\,i, 0.0677458 + 0.0947781\,i, 0.0072899 - 0.0729668\,i, 0.32, 0.0072899 + 0.0729668\,i, 0.0677458 - 0.0947781\,i, -0.108189 - 0.274974\,i, -0.147943 - 0.179236\,i, -0.27 + 0.02\,i, 0.347254 + 0.229439\,i, 0.610124 - 0.000912226\,i, 0.712943 + 0.651536\,i, -0.489225 - 1.51885\,i\}$$

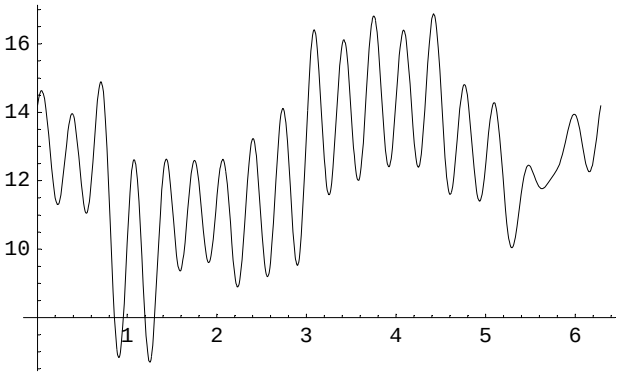
```
fS[t_]:=Sum[c[k] E^(I k t),{k,0,n-1}];
fS[t]
```

```
12.42 - (0.489225 - 1.51885 i) e^(i t) + (0.712943 - 0.651536 i) e^(2 i t) +
 (0.610124 + 0.000912226 i) e^(3 i t) + (0.347254 - 0.229439 i) e^(4 i t) -
 (0.27 + 0.02 i) e^(5 i t) - (0.147943 - 0.179236 i) e^(6 i t) - (0.108189 - 0.274974 i) e^(7 i t) +
 (0.0677458 + 0.0947781 i) e^(8 i t) + (0.0072899 - 0.0729668 i) e^(9 i t) + 0.32 e^(10 i t) +
 (0.0072899 + 0.0729668 i) e^(11 i t) + (0.0677458 - 0.0947781 i) e^(12 i t) -
 (0.108189 + 0.274974 i) e^(13 i t) - (0.147943 + 0.179236 i) e^(14 i t) - (0.27 - 0.02 i) e^(15 i t) +
 (0.347254 + 0.229439 i) e^(16 i t) + (0.610124 - 0.000912226 i) e^(17 i t) +
 (0.712943 + 0.651536 i) e^(18 i t) - (0.489225 + 1.51885 i) e^(19 i t)
```

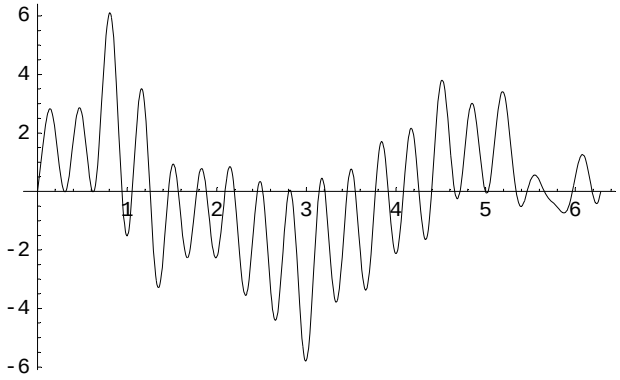
```
fS[t]//ExpToTrig
```

```
12.42 - (0.489225 - 1.51885 i) Cos[t] + (0.712943 - 0.651536 i) Cos[2 t] +
 (0.610124 + 0.000912226 i) Cos[3 t] + (0.347254 - 0.229439 i) Cos[4 t] -
 (0.27 + 0.02 i) Cos[5 t] - (0.147943 - 0.179236 i) Cos[6 t] -
 (0.108189 - 0.274974 i) Cos[7 t] + (0.0677458 + 0.0947781 i) Cos[8 t] +
 (0.0072899 - 0.0729668 i) Cos[9 t] + 0.32 Cos[10 t] + (0.0072899 + 0.0729668 i) Cos[11 t] +
 (0.0677458 - 0.0947781 i) Cos[12 t] - (0.108189 + 0.274974 i) Cos[13 t] -
 (0.147943 + 0.179236 i) Cos[14 t] - (0.27 - 0.02 i) Cos[15 t] +
 (0.347254 + 0.229439 i) Cos[16 t] + (0.610124 - 0.000912226 i) Cos[17 t] +
 (0.712943 + 0.651536 i) Cos[18 t] - (0.489225 + 1.51885 i) Cos[19 t] -
 (1.51885 + 0.489225 i) Sin[t] + (0.651536 + 0.712943 i) Sin[2 t] -
 (0.000912226 - 0.610124 i) Sin[3 t] + (0.229439 + 0.347254 i) Sin[4 t] +
 (0.02 - 0.27 i) Sin[5 t] - (0.179236 + 0.147943 i) Sin[6 t] -
 (0.274974 + 0.108189 i) Sin[7 t] - (0.0947781 - 0.0677458 i) Sin[8 t] +
 (0.0729668 + 0.0072899 i) Sin[9 t] + 0.32 i Sin[10 t] - (0.0729668 - 0.0072899 i) Sin[11 t] +
 (0.0947781 + 0.0677458 i) Sin[12 t] + (0.274974 - 0.108189 i) Sin[13 t] +
 (0.179236 - 0.147943 i) Sin[14 t] - (0.02 + 0.27 i) Sin[15 t] -
 (0.229439 - 0.347254 i) Sin[16 t] + (0.000912226 + 0.610124 i) Sin[17 t] -
 (0.651536 - 0.712943 i) Sin[18 t] + (1.51885 - 0.489225 i) Sin[19 t]
```

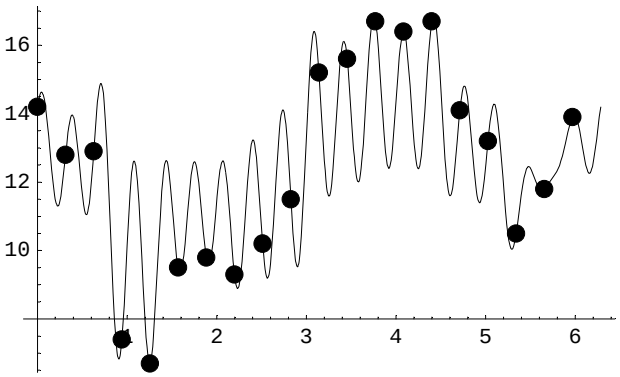
```
Plot[Re[fS[t]],{t,0,2Pi},PlotPoints->50];
```



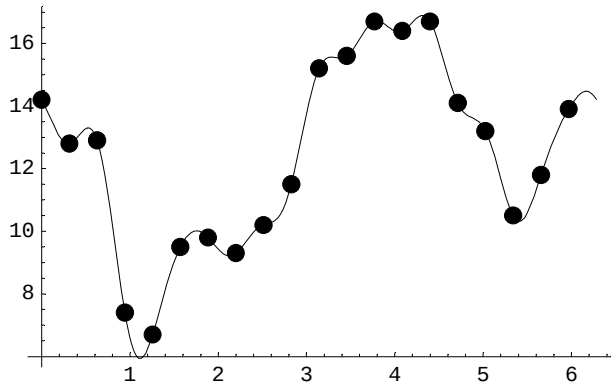
```
Plot[Im[fS[t]],{t,0,2Pi},PlotPoints->50];
```




```
Plot[Re[fS[t]],{t,0,2Pi},PlotPoints->50,Epilog->epi];
```




```
fSsym[t_] := Sum[c[k] E^(I k t), {k, -n / 2, n - 1 - n / 2}];
Plot[Re[fSsym[t]], {t, 0, 2 Pi}, PlotPoints → 50, Epilog → epi];
```



Resultat: Das Resultat entspricht nun dem Gewünschten.
Problem: Immer noch grosser Imaginäranteil infolge des frühen Abbruchs der Reihe

5 Selbststudium nach Anleitung in der Aufgabenstellung

Infolge der Natur der Aufgabe: Keine Lösung bereitstellbar.