Lösungen

1

a

```
f[x_]:=100 x^100+10 x^10+5x^2-3x+1;
D[f[x],x]
```

$-3 + 10\,x + 100\,x^9 + 10000\,x^{99}$

b

```
D[f[x],x]/.x->1
```

10107

c

```
f[x_]:=a x^a+10 x^10+5x^2-3x+1;
D[f[x],x]
```

$-3 + 10\,x + 100\,x^9 + a^2\,x^{-1+a}$

```
D[f[x],x]/.x->a
```

$-3 + 10\,a + 100\,a^9 + a^{1+a}$

d

```
f[x_]:=Cos[x] Cot[x]//Simplify;
D[f[x],x]
```

$-\mathrm{Cos}[x] - \mathrm{Cot}[x]\,\mathrm{Csc}[x]$

e

```
f[x_]:=Cos[x] Cot[x]//Simplify;
D[f[x],x]/.x->Pi/4
```

$-\dfrac{1}{\sqrt{2}} - \sqrt{2}$

```
N[%]
```

-2.12132

f

```
f[x_]:=Cos[x]/x^2//Simplify;
D[f[x],x]
```

$$-\frac{2\,Cos[x]}{x^3} - \frac{Sin[x]}{x^2}$$

g

```
f[x_]:=Cos[x]/x^2//Simplify;
D[f[x],x]/.x->Pi
```

$$\frac{2}{\pi^3}$$

h

```
f[x_]:=E^(2x^2-4x+5)//Simplify;
D[f[x],x]
```

$$e^{5-4\,x+2\,x^2}\,(-4+4\,x)$$

```
f[x_]:=E^(2x^2-4x+5)//Simplify;
D[f[x],x] /.x->1
```
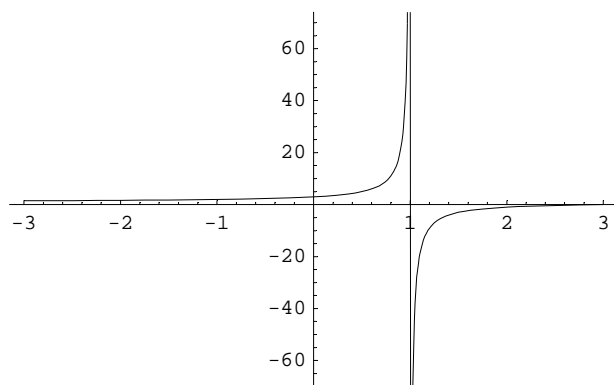
0

Winkel?

2

a

```
f[x_]:=(x-3)(x-1)/(x-1)^2;
Plot[f[x],{x,-3,3}];
```



```
f[x]//Apart
```

$$1 - \frac{2}{-1+x}$$

```
Limit[f[x],x->Infinity]
```

1

```
Solve[f[x]==0,{x}]
```

$\{\{x \rightarrow 3\}\}$

```
Solve[Denominator[f[x]]==0,{x}]
```

$\{\{x \rightarrow 1\}\}$

```
Solve[Evaluate[D[f[x],x]==0],{x}]
```
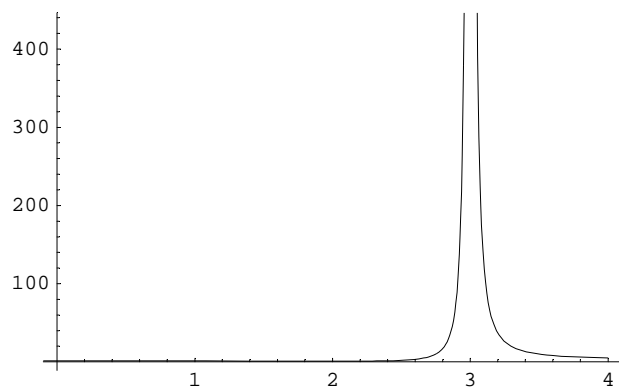
$\{\}$

```
Solve[Evaluate[D[f[x],{x,2}]==0],{x}]
```

$\{\}$

b

```
f[x_]:=1+((x-2)^2)/(x-3)^2;
Plot[f[x],{x,-3,4}];
```



```
1+((x-2)^2)/(x-3)^2//Apart
```

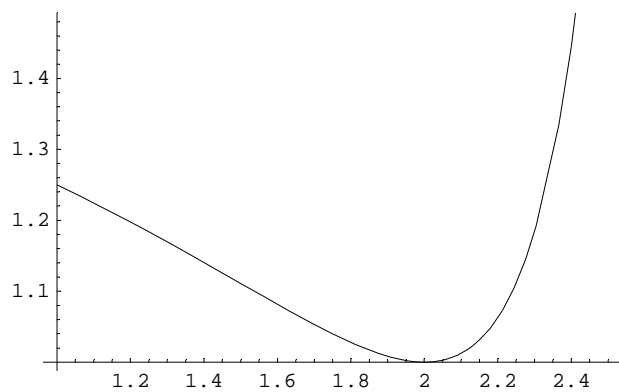$$2 + \frac{1}{(-3+x)^2} + \frac{2}{-3+x}$$

```
1+((x-2)^2)/(x-3)^2//Together
```

$$\frac{13 - 10\,x + 2\,x^2}{(-3+x)^2}$$

```
f[x_]:=1+((x-2)^2)/(x-3)^2;
D[f[x],x]//Together
```

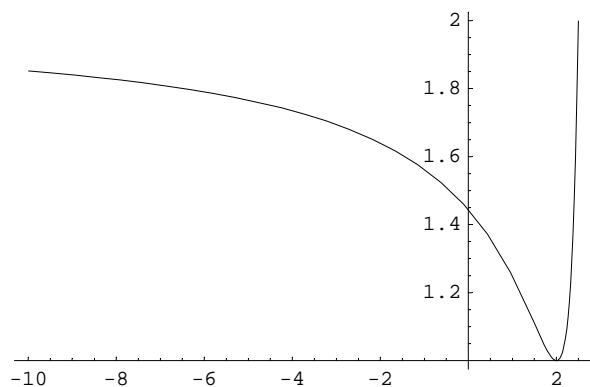$$-\frac{2\,(-2+x)}{(-3+x)^3}$$
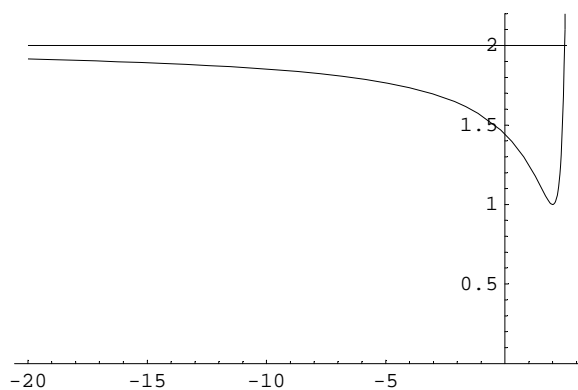
```
f[x_]:=1+((x-2)^2)/(x-3)^2;
Plot[f[x],{x,1,2.5}];
```



```
f[x_]:=1+((x-2)^2)/(x-3)^2;
D[f[x],{x,2}]//Together
```

$$\frac{2\,(-3+2\,x)}{(-3+x)^4}$$

```
f[x_]:=1+((x-2)^2)/(x-3)^2;
Plot[f[x],{x,-10,2.5}];
```



```
f[x_]:=1+((x-2)^2)/(x-3)^2;
Plot[{2,f[x]},{x,-20,2.6},PlotRange->{0,2.2}];
```



```
f[x]//Apart
```

$$2 + \frac{1}{(-3+x)^2} + \frac{2}{-3+x}$$

```
Limit[f[x],x->Infinity]
```

2

```
Limit[f[x],x-> -Infinity]
```

2

```
Solve[f[x]==0,{x}]
```

$$\left\{ \left\{ x \to \frac{5}{2} - \frac{i}{2} \right\}, \left\{ x \to \frac{5}{2} + \frac{i}{2} \right\} \right\}$$

Komplex

```
Solve[Denominator[Together[f[x]]]==0,{x}]
```

$\{\{x \to 3\}, \{x \to 3\}\}$
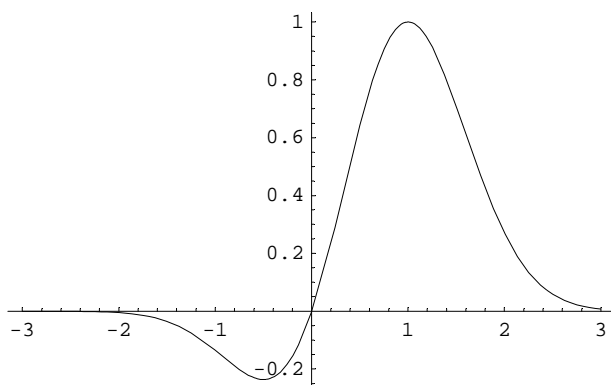
```
Solve[Evaluate[D[f[x],x]==0],{x}]
```

$\{\{x \to 2\}\}$

```
Solve[Evaluate[D[f[x],{x,2}]==0],{x}]
```

$$\left\{\left\{x \to \frac{3}{2}\right\}\right\}$$

c

```
f[x_]:=x E^(-x^2+x);
Plot[f[x],{x,-3,3}];
```

```
Limit[f[x],x->Infinity]
```

0

```
Limit[f[x],x-> -Infinity]
```

0

```
D[f[x],{x,1}]//Together
```

$-e^{x-x^2} \left(-1 - x + 2 x^2\right)$

```
D[f[x],{x,2}]//Together
```

$e^{x-x^2} \left(2 - 5 x - 4 x^2 + 4 x^3\right)$

```
Solve[f[x]==0,{x}]
```

Solve::ifun : Inverse functions are being used by Solve, so some
   solutions may not be found; use Reduce for complete solution information. Mehr…

$\{\{x \to 0\}\}$

```
Solve[Evaluate[D[f[x],x]==0],{x}]
```

Solve::ifun : Inverse functions are being used by Solve, so some
   solutions may not be found; use Reduce for complete solution information. Mehr…

$$\left\{\left\{x \to -\frac{1}{2}\right\}, \{x \to 1\}\right\}$$

```
Solve[Evaluate[D[f[x],{x,2}]==0],{x}]
```

Solve::ifun : Inverse functions are being used by Solve, so some
   solutions may not be found; use Reduce for complete solution information. Mehr…

$$\left\{\left\{x \to \frac{1}{3} + \frac{19}{6\left(-1 + 3\,\mathbb{i}\,\sqrt{762}\right)^{1/3}} + \frac{1}{6}\left(-1 + 3\,\mathbb{i}\,\sqrt{762}\right)^{1/3}\right\},\right.$$

$$\left\{x \to \frac{1}{3} - \frac{19\left(1 + \mathbb{i}\,\sqrt{3}\right)}{12\left(-1 + 3\,\mathbb{i}\,\sqrt{762}\right)^{1/3}} - \frac{1}{12}\left(1 - \mathbb{i}\,\sqrt{3}\right)\left(-1 + 3\,\mathbb{i}\,\sqrt{762}\right)^{1/3}\right\},$$

$$\left.\left\{x \to \frac{1}{3} - \frac{19\left(1 - \mathbb{i}\,\sqrt{3}\right)}{12\left(-1 + 3\,\mathbb{i}\,\sqrt{762}\right)^{1/3}} - \frac{1}{12}\left(1 + \mathbb{i}\,\sqrt{3}\right)\left(-1 + 3\,\mathbb{i}\,\sqrt{762}\right)^{1/3}\right\}\right\}$$

```
N[%]//Chop
```

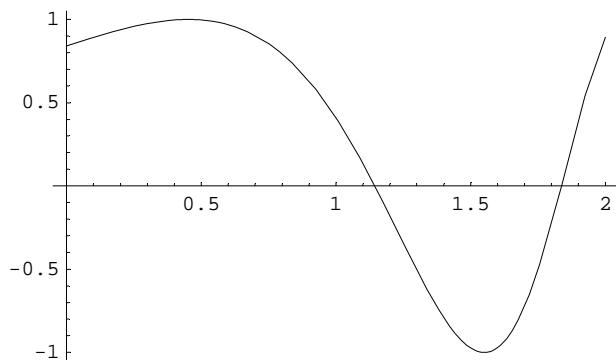$\{\{x \to 1.5887\},\ \{x \to -0.927886\},\ \{x \to 0.339181\}\}$

```
NSolve[Evaluate[D[f[x],{x,2}]==0],{x}]
```

Solve::ifun : Inverse functions are being used by Solve, so some
   solutions may not be found; use Reduce for complete solution information. Mehr…

$\{\{x \to -0.927886\},\ \{x \to 0.339181\},\ \{x \to 1.5887\}\}$

d

```
f[x_]:=Sin[(E^x)];
Plot[f[x],{x,0,2}];
```



```
Limit[f[x],x->Infinity]
```

$\text{Interval}[\{-1,\ 1\}]$

```
Limit[f[x],x-> -Infinity]
```

0

```
D[f[x],{x,1}]//Together
```

$\mathbb{e}^x\,\text{Cos}[\mathbb{e}^x]$

```
D[f[x],{x,2}]//Together
```

$-\mathbb{e}^x\left(-\text{Cos}[\mathbb{e}^x] + \mathbb{e}^x\,\text{Sin}[\mathbb{e}^x]\right)$

```
D[f[x],{x,1}]//Together
```

$e^x \, Cos[e^x]$

```
D[f[x],{x,2}]//Together
```

$-e^x \, (-Cos[e^x] + e^x \, Sin[e^x])$

```
Solve[f[x]==0,{x}]
```

Solve::ifun : Inverse functions are being used by Solve, so some
   solutions may not be found; use Reduce for complete solution information. Mehr…

$\{\{x \to -\infty\}\}$

```
FindRoot[f[x]==0,{x,1.2}]
```

$\{x \to 1.14473\}$

```
FindRoot[f[x]==0,{x,1.8}]
```

$\{x \to 1.83788\}$

```
{Log[Pi/2],Log[3Pi/2],Log[5Pi/2]}//N
```

$\{0.451583, \, 1.55019, \, 2.06102\}$

```
Solve[Evaluate[D[f[x],x]==0],{x}]
```

Solve::ifun : Inverse functions are being used by Solve, so some
   solutions may not be found; use Reduce for complete solution information. Mehr…

$\left\{\left\{x \to Log\left[\frac{\pi}{2}\right]\right\}, \, \left\{x \to i \, \pi + Log\left[\frac{\pi}{2}\right]\right\}\right\}$

```
N[%]
```

$\{\{x \to 0.451583\}, \, \{x \to 0.451583 + 3.14159 \, i\}\}$

```
Solve[Evaluate[D[f[x],{x,2}]==0],{x}]
```

Solve::tdep : The equations appear to involve the
   variables to be solved for in an essentially non-algebraic way. Mehr…

$Solve[e^x \, Cos[e^x] - e^{2\,x} \, Sin[e^x] == 0, \, \{x\}]$

```
FindRoot[Evaluate[D[f[x],{x,2}]==0],{x,0}]
```

$\{x \to -0.150435\}$

```
FindRoot[Evaluate[D[f[x],{x,2}]==0],{x,0.8}]
```

$\{x \to -0.150435\}$

```
FindRoot[Evaluate[D[f[x],{x,2}]==0],{x,1.2}]
```

$\{x \to 1.23128\}$

```
FindRoot[Evaluate[D[f[x],{x,2}]==0],{x,1.8}]
```
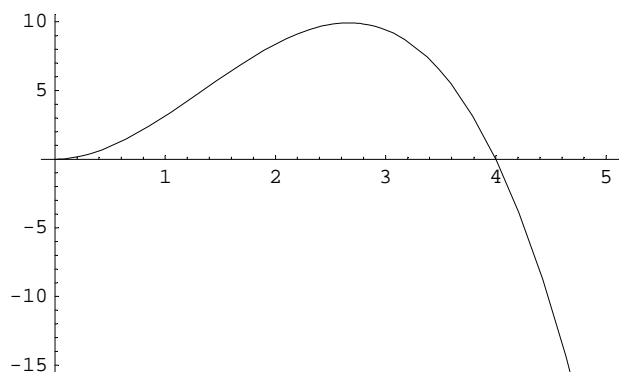
$\{x \to 1.86211\}$

3

a Höhensatz r^2 = h (2R-h)

V[h_]:=r^2 Pi h/3 /. r->Sqrt[h(4-h)]; V[h]

$\frac{1}{3}$ (4 – h) h^2 π

Plot[V[h],{h,0,5}];

D[V[h],{h,1}]//Together

$\frac{1}{3}$ (8 h π – 3 h^2 π)

Solve[Evaluate[D[V[h],{h,1}]==0],{h}]

$\left\{ \{h \to 0\}, \left\{h \to \frac{8}{3}\right\} \right\}$

N[%]

$\{\{h \to 0.\}, \{h \to 2.66667\}\}$

b Proportion (h^2+r^2) : (h-R^2)=r^2 : R^2

Solve[(h^2+r^2)/(h-2)^2==r^2/2^2, {h}]

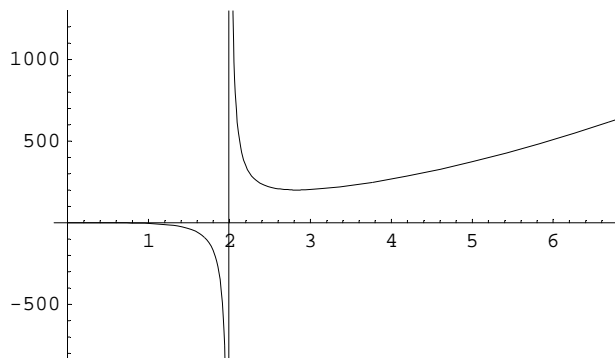$\left\{ \{h \to 0\}, \left\{h \to \frac{4\,r^2}{-4+r^2}\right\} \right\}$

```
Plot[ 4 r^2 / (-4 + r^2) , {r, 0, 10}];
```



```
V[r_]:=r^2 Pi 4 r^2/(r^2-4) ; V[r]
```

$$\frac{4 \pi r^4}{-4 + r^2}$$

```
Plot[V[r], {r, 0, 10}];
```



```
D[V[r],{r,1}]//Together
```

$$\frac{8 \left(-8 \pi r^3 + \pi r^5\right)}{\left(-4 + r^2\right)^2}$$

```
Solve[Evaluate[D[V[r],{r,1}]==0],{r}]
```

$$\left\{\{r \to 0\}, \{r \to 0\}, \{r \to 0\}, \left\{r \to -2\sqrt{2}\right\}, \left\{r \to 2\sqrt{2}\right\}\right\}$$

```
N[%]
```

$$\{\{r \to 0.\}, \{r \to 0.\}, \{r \to 0.\}, \{r \to -2.82843\}, \{r \to 2.82843\}\}$$

c

```
Remove[x, a, b, c, d]

f[x_,a_,b_,c_,d_]:=(a x^2+b x+ c)/(x + d);
f[x,a,b,c,d]
```

$$\frac{c + b x + a x^2}{d + x}$$

```
f[x,a,b,c,d]//Apart
```

$$b - a\,d + a\,x + \frac{c - b\,d + a\,d^2}{d + x}$$

```
Evaluate[D[f[x,a,b,c,d],x]==0/.x->4]
```

$$-\frac{16\,a + 4\,b + c}{(4 + d)^2} + \frac{8\,a + b}{4 + d} == 0$$

```
Evaluate[D[f[x,a,b,c,d],x]==0]
```

$$\frac{b + 2\,a\,x}{d + x} - \frac{c + b\,x + a\,x^2}{(d + x)^2} == 0$$

```
solv=Solve[{Evaluate[D[f[x,a,b,c,d],x]==0/.x->4],8+d==0, a==0.5, b-a
d==-3},{a,b,c,d}]//Flatten
```

$$\{c \rightarrow 32.,\ b \rightarrow -7.,\ a \rightarrow 0.5,\ d \rightarrow -8.\}$$

```
f[x_]:=f[x,a,b,c,d]/.solv;
f[0]
```

$-4.$