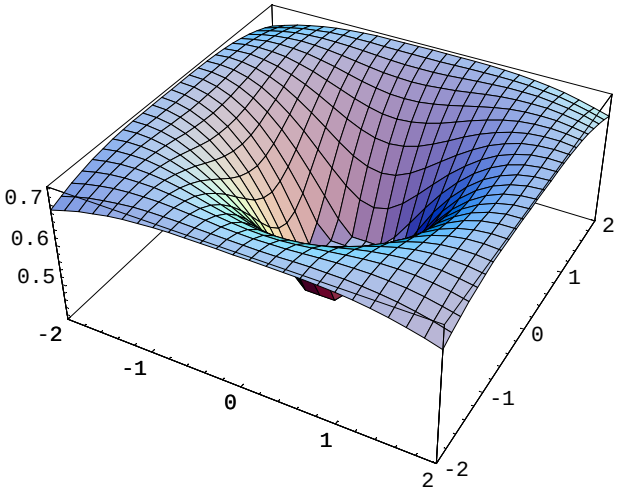
Lösungen

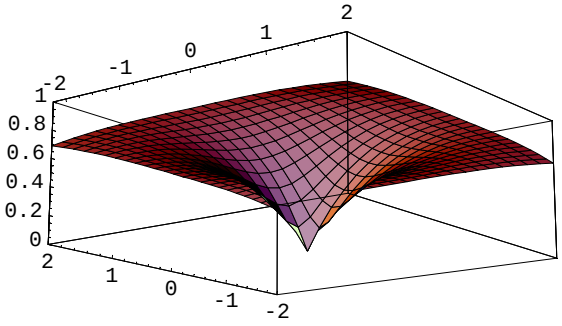
1

a

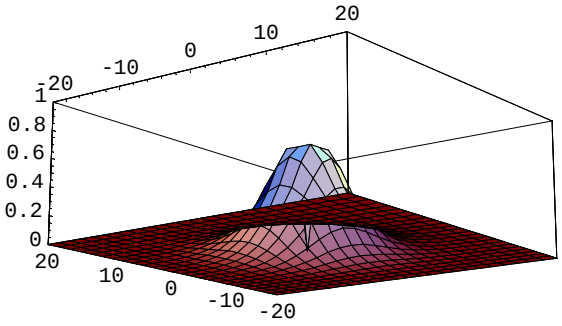
```
f[x_,y_]:= Sqrt[x^2+y^2] E^(-Sqrt[x^2+y^2]/2);
Plot3D[f[x,y],{x,-2,2},{y,-2,2}];
```



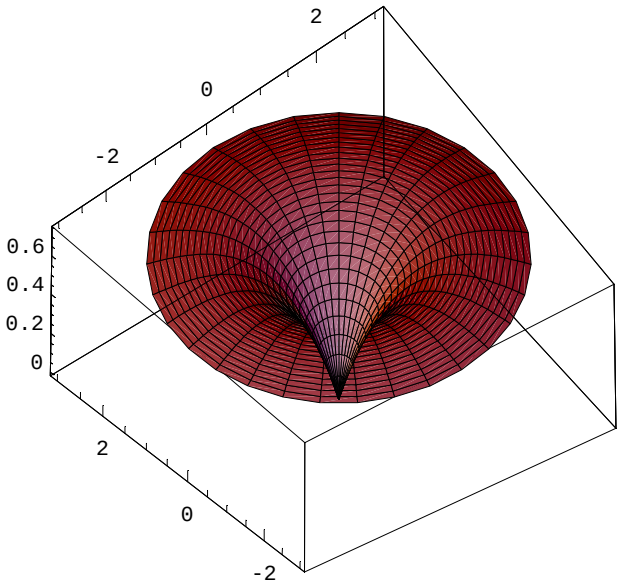

```
Plot3D[f[x,y],{x,-2,2},{y,-2,2},ViewPoint->{3.667, 2.785,
-1.120},PlotRange->{0,1}];
```



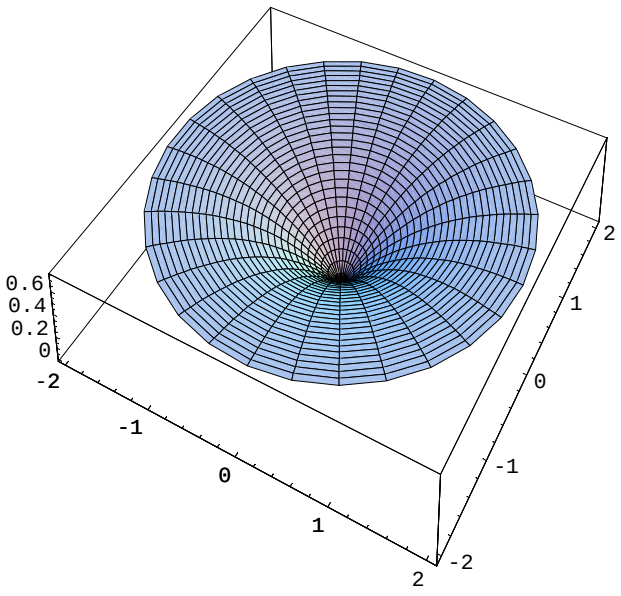
```
Plot3D[f[x,y],{x,-20,20},{y,-20,20},ViewPoint->{3.667, 2.785,
-1.120},PlotRange->{0,1}];
```




```
h[r_]:= r E^(-r/2);
ParametricPlot3D[{r Cos[t],r Sin[t], h[r]}, {r,0,3}, {t,0,2Pi}, ViewPoint->{3.667,
2.785, -1.120}, AspectRatio->1];
```




```
ParametricPlot3D[{r Cos[t],r Sin[t], h[r]}, {r,0,2}, {t,0,2Pi}, AspectRatio->1];
```

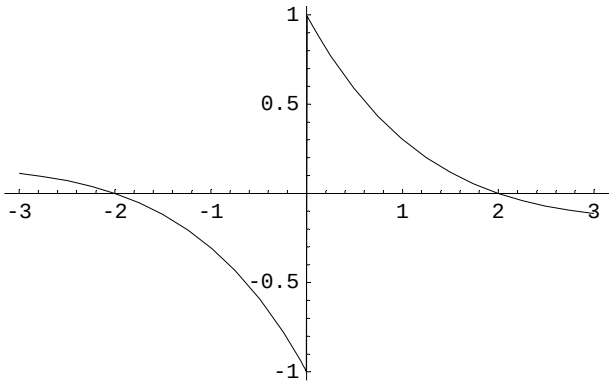


b

```
D[f[x,y],x]/.x->u
```

$$-\frac{1}{2}\, e^{-\frac{1}{2}\sqrt{u^2+y^2}}\, u + \frac{e^{-\frac{1}{2}\sqrt{u^2+y^2}}\, u}{\sqrt{u^2+y^2}}$$

```
Plot[Evaluate[D[f[x,0],x]/.x->u],{u,-3,3}];
```




```
Limit[Evaluate[D[f[x,0],x]/.x->u],u->0, Direction → 1]
```

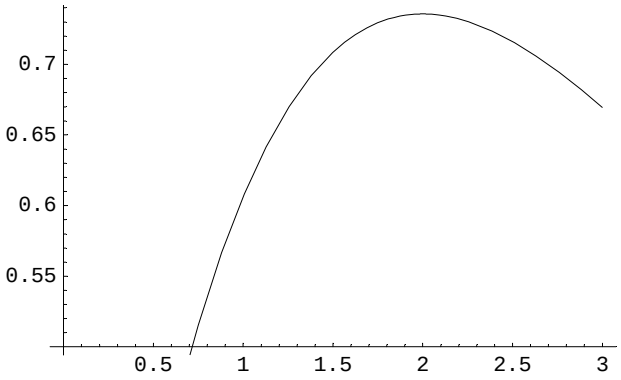
```
-1
```

```
Limit[Evaluate[D[f[x,0],x]/.x->u],u->0, Direction → -1]
```

```
1
```

c

```
Plot[h[r],{r,0,3}];
```




```
D[h[r],r]/.r->2
```

```
0
```

```
hMax=h[2]
```

$$\frac{2}{e}$$

```
hMax=h[2]//N
```

```
0.735759
```

d

```
<<Calculus`VectorAnalysis`
```

```
Drop[Grad[f[x,y], Cartesian[x, y, z]],{3}]
```

$$\left\{-\frac{1}{2}\, e^{-\frac{1}{2}\sqrt{x^2+y^2}}\, x+\frac{e^{-\frac{1}{2}\sqrt{x^2+y^2}}\, x}{\sqrt{x^2+y^2}},\ -\frac{1}{2}\, e^{-\frac{1}{2}\sqrt{x^2+y^2}}\, y+\frac{e^{-\frac{1}{2}\sqrt{x^2+y^2}}\, y}{\sqrt{x^2+y^2}}\right\}$$

```
Drop[Grad[f[x,y], Cartesian[x, y, z]],{3}] /. {x->1,y->1}
```

$$\left\{-\frac{1}{2}\, e^{-\frac{1}{\sqrt{2}}}+\frac{e^{-\frac{1}{\sqrt{2}}}}{\sqrt{2}},\ -\frac{1}{2}\, e^{-\frac{1}{\sqrt{2}}}+\frac{e^{-\frac{1}{\sqrt{2}}}}{\sqrt{2}}\right\}$$

```
N[%]
```

```
{0.102118, 0.102118}
```

e

```
Oberfl=
Integrate[Evaluate[Sqrt[D[f[x,y],x]^2+D[f[x,y],x]^2+1]/.{x->x1,y->y1}],{x1,1,2},{y1
,1,2}]
```

$$\int_1^2\int_1^2 \sqrt{1+\frac{e^{-\sqrt{x1^2+y1^2}}\, x1^2\left(-2+\sqrt{x1^2+y1^2}\right)^2}{2\left(x1^2+y1^2\right)}}\ \mathrm{d}y1\,\mathrm{d}x1$$

```
NIntegrate[Evaluate[Sqrt[D[f[x,y],x]^2+D[f[x,y],x]^2+1]/.{x->x1,y->y1}],{x1,1,2},{y
1,1,2}]
```

```
1.00133
```

```
NIntegrate[Evaluate[Sqrt[D[f[x,y],x]^2+D[f[x,y],x]^2+1]/.{x->x1,y->y1}],{x1,0,2},{y
1,1,2}]
```

```
2.00319
```

2

a

```
w=3-I; solv=Solve[(z-w)^5==6-9 I,{z}]//ComplexExpand
```

```
{{z → Root[105661 - 310320 #1 + 416840 #1^2 - 338040 #1^3 +
      183070 #1^4 - 68988 #1^5 + 18280 #1^6 - 3360 #1^7 + 410 #1^8 - 30 #1^9 + #1^10 &, 1]},
 {z → Root[105661 - 310320 #1 + 416840 #1^2 - 338040 #1^3 + 183070 #1^4 -
      68988 #1^5 + 18280 #1^6 - 3360 #1^7 + 410 #1^8 - 30 #1^9 + #1^10 &, 4]},
 {z → Root[105661 - 310320 #1 + 416840 #1^2 - 338040 #1^3 + 183070 #1^4 -
      68988 #1^5 + 18280 #1^6 - 3360 #1^7 + 410 #1^8 - 30 #1^9 + #1^10 &, 5]},
 {z → Root[105661 - 310320 #1 + 416840 #1^2 - 338040 #1^3 + 183070 #1^4 -
      68988 #1^5 + 18280 #1^6 - 3360 #1^7 + 410 #1^8 - 30 #1^9 + #1^10 &, 8]},
 {z → Root[105661 - 310320 #1 + 416840 #1^2 - 338040 #1^3 + 183070 #1^4 -
      68988 #1^5 + 18280 #1^6 - 3360 #1^7 + 410 #1^8 - 30 #1^9 + #1^10 &, 9]}}
```

```
solv//N
```

```
{{z → 1.53777 - 1.67372 i}, {z → 1.9074 + 0.182468 i},
 {z → 3.1889 - 2.59885 i}, {z → 3.78696 + 0.40453 i}, {z → 4.57897 - 1.31442 i}}
```

```
s1=z/.solv//N; Map[Abs,s1]
```

```
{2.27291, 1.9161, 4.11377, 3.80851, 4.76389}
```

1.9074+0.182468

```
solv=Solve[Reduce[(z)^5==6-9 I,{z}]/.z->z-w,{z}]
```

```
{{z → (3 - i) - (-3)^(1/5) (2 - 3 i)^(1/5)},
 {z → (3 - i) + (6 - 9 i)^(1/5)}, {z → (3 - i) + (-1)^(2/5) (6 - 9 i)^(1/5)},
 {z → (3 - i) - (-1)^(3/5) (6 - 9 i)^(1/5)}, {z → (3 - i) + (-1)^(4/5) (6 - 9 i)^(1/5)}}
```

```
solv=Solve[Reduce[(z)^5==6-9 I,{z}]/.z->z-w,{z}];
s1=z/.solv//N; Map[Abs,s1]
```

```
{2.27291, 4.76389, 3.80851, 4.11377, 1.9161}
```

$(-1)^{4/5}\ (6-9\,i)^{1/5}$

```
(-3)^(4/5)//ComplexExpand
```

$$-\frac{3^{4/5}}{4}-\frac{1}{4}\,3^{4/5}\sqrt{5}+\frac{1}{2}\,i\,3^{4/5}\sqrt{\frac{1}{2}\left(5-\sqrt{5}\right)}$$

b

```
w0=w;w1=1/Abs[w] w; w2=Conjugate[w];w3=1/w; w4=Conjugate[1/Abs[w] w];
mW={w0,w1,w2,w3,w4}
```

$$\left\{3-i,\ \frac{3-i}{\sqrt{10}},\ 3+i,\ \frac{3}{10}+\frac{i}{10},\ \frac{3+i}{\sqrt{10}}\right\}$$

```
N[%]
```

$$\{3.-1.\,i,\ 0.948683-0.316228\,i,\ 3.+1.\,i,\ 0.3+0.1\,i,\ 0.948683+0.316228\,i\}$$

```
(w0-w2)/Abs[w0-w2]
```

$$-i$$

```
N[%]
```

$$0.-1.\,i$$

```
(w1-w3)/Abs[w1-w3]
```

$$\frac{\left(-\frac{3}{10}-\frac{i}{10}\right)+\frac{3-i}{\sqrt{10}}}{\sqrt{\left(-\frac{1}{10}-\frac{1}{\sqrt{10}}\right)^2+\left(-\frac{3}{10}+\frac{3}{\sqrt{10}}\right)^2}}$$

```
N[%]
```

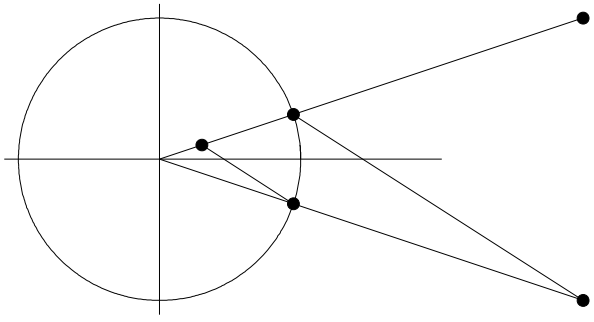
$$0.84164-0.540039\,i$$

c

```
toPoint[w_]:=Point[{Re[w],Im[w]}];
Map[toPoint,mW]
```

$$\left\{\text{Point}[\{3,-1\}],\ \text{Point}\left[\left\{\frac{3}{\sqrt{10}},-\frac{1}{\sqrt{10}}\right\}\right],\right.$$
$$\left.\text{Point}[\{3,1\}],\ \text{Point}\left[\left\{\frac{3}{10},\frac{1}{10}\right\}\right],\ \text{Point}\left[\left\{\frac{3}{\sqrt{10}},\frac{1}{\sqrt{10}}\right\}\right]\right\}$$

```
Show[Graphics[Join[{Line[{{-1.1,0},{2,0}}],Line[{{0,-1.1},{0,1.1}}],
Line[{{0,0},{3,-1}}],Line[{{0,0},{3,1}}],
Line[{toPoint[w0][[1]],toPoint[w4][[1]]}],
Line[{toPoint[w1][[1]],toPoint[w3][[1]]}]
,PointSize[0.02],Circle[{0,0},1]}, Map[toPoint,mW]]],AspectRatio->Automatic];
```



d

Winkel gespielgelt, Radius invertiert ==> r:1 = 1: (1/r)

3

```
Remove["Global`*"]
```

a

```
DSolve[{y''[x] + 2 y[x] == Cos[x]}, y[x], x ] // Simplify
```

$$\{\{y[x] \to \text{Cos}[x] + C[1]\,\text{Cos}\left[\sqrt{2}\,x\right] + C[2]\,\text{Sin}\left[\sqrt{2}\,x\right]\}\}$$

```
N[%]
```

```
{{y[x] → Cos[x] + C[1] Cos[1.41421 x] + C[2] Sin[1.41421 x]}}
```

b

```
DSolve[{y''[x]-y'[x] + 2 y[x] ==Cos[x]},y[x],x ]//Simplify
```

$$\left\{\left\{y[x] \to \frac{1}{2}\left(\text{Cos}[x] + 2\,e^{x/2}\,C[2]\,\text{Cos}\left[\frac{\sqrt{7}\,x}{2}\right] - \text{Sin}[x] + 2\,e^{x/2}\,C[1]\,\text{Sin}\left[\frac{\sqrt{7}\,x}{2}\right]\right)\right\}\right\}$$

```
N[%]
```

```
{{y[x] → 0.5 (Cos[x] + 2. 2.71828^(0.5 x) C[2] Cos[1.32288 x] -
      1. Sin[x] + 2. 2.71828^(0.5 x) C[1] Sin[1.32288 x])}}
```

c

```
DSolve[{y''[x] - y'[x] + 2 y[x] == Cos[x], y[0] == 0, y'[0] == 0}, y[x], x ] // Simplify
```

$$\left\{\left\{y[x] \to \frac{1}{14}\left(7\,\text{Cos}[x] - 7\,e^{x/2}\,\text{Cos}\left[\frac{\sqrt{7}\,x}{2}\right] - 7\,\text{Sin}[x] + 3\sqrt{7}\,e^{x/2}\,\text{Sin}\left[\frac{\sqrt{7}\,x}{2}\right]\right)\right\}\right\}$$

```
N[%]
```

```
{{y[x] → 0.0714286 (7. Cos[x] - 7. 2.71828^(0.5 x) Cos[1.32288 x] -
      7. Sin[x] + 7.93725 2.71828^(0.5 x) Sin[1.32288 x])}}
```

```
solv=DSolve[{y''[x]-y'[x]+2y[x]==Cos[x],y[0]==0,y'[0]==0},y,x ]//Simplify//Flatten
```

$$\left\{y \to \text{Function}\left[\{x\},\ \frac{1}{14}\left(-7\,e^{x/2}\,\text{Cos}\left[\frac{\sqrt{7}\,x}{2}\right] + 7\,\text{Cos}[x]\,\text{Cos}\left[\frac{\sqrt{7}\,x}{2}\right]^2 - 7\,\text{Cos}\left[\frac{\sqrt{7}\,x}{2}\right]^2\,\text{Sin}[x] + 3\sqrt{7}\,e^{x/2}\,\text{Sin}\left[\frac{\sqrt{7}\,x}{2}\right] + 7\,\text{Cos}[x]\,\text{Sin}\left[\frac{\sqrt{7}\,x}{2}\right]^2 - 7\,\text{Sin}[x]\,\text{Sin}\left[\frac{\sqrt{7}\,x}{2}\right]^2\right)\right]\right\}$$

N[%]

```
{y → Function[{x},
   0.0714286 (-7. 2.71828^(0.5 x) Cos[0.5 2.64575 x] + 7. Cos[x] Cos[0.5 2.64575 x]^2 -
      7. Cos[0.5 2.64575 x]^2 Sin[x] + 3. 2.64575 2.71828^(0.5 x) Sin[0.5 2.64575 x] +
      7. Cos[x] Sin[0.5 2.64575 x]^2 - 7. Sin[x] Sin[0.5 2.64575 x]^2)]}
```

y=y/.solv;
y[z]

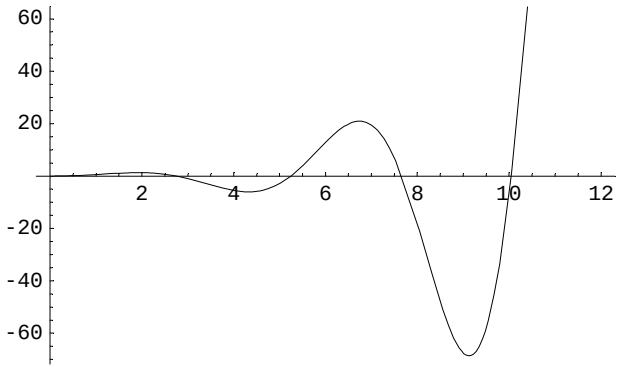
$$\frac{1}{14}\left(-7\, e^{z/2}\cos\left[\frac{\sqrt{7}\,z}{2}\right]+7\cos[z]\cos\left[\frac{\sqrt{7}\,z}{2}\right]^2-7\cos\left[\frac{\sqrt{7}\,z}{2}\right]^2\sin[z]+3\sqrt{7}\,e^{z/2}\sin\left[\frac{\sqrt{7}\,z}{2}\right]+7\cos[z]\sin\left[\frac{\sqrt{7}\,z}{2}\right]^2-7\sin[z]\sin\left[\frac{\sqrt{7}\,z}{2}\right]^2\right)$$

N[%]

```
0.0714286 (-7. 2.71828^(0.5 z) Cos[1.32288 z] + 7. Cos[z] Cos[1.32288 z]^2 -
   7. Cos[1.32288 z]^2 Sin[z] + 7.93725 2.71828^(0.5 z) Sin[1.32288 z] +
   7. Cos[z] Sin[1.32288 z]^2 - 7. Sin[z] Sin[1.32288 z]^2)
```

d

pl=Plot[y[z],{z,0,12}];



e

FindRoot[y[x],{x,3}]

```
{x → 2.75558}
```

y[2.75558]

1.56584×10^{-6}

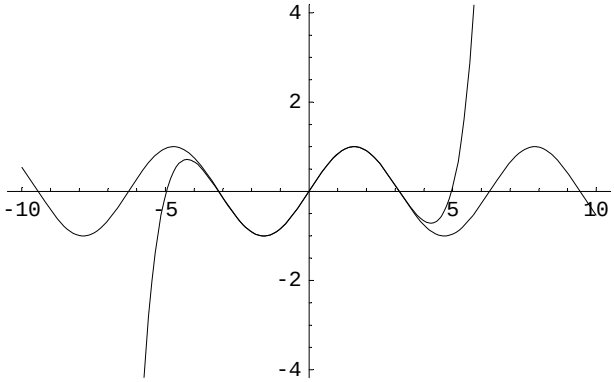
4

a

```
s1=Normal[Series[Sin[x],{x,0,10}]]
```

$$x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \frac{x^9}{362880}$$

```
Plot[{s1,Sin[x]},{x,-10,10}];
```

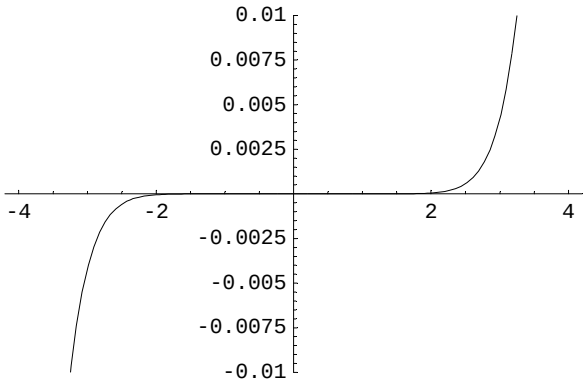



```
s2=Normal[Series[Cos[x],{x,0,10}]]
```

$$1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \frac{x^8}{40320} - \frac{x^{10}}{3628800}$$

b

```
Plot[s1-Sin[x],{x,-4,4},PlotRange->{-0.01,0.01}]
```



- Graphics -

c

D[s1,x]

$$1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \frac{x^8}{40320}$$

d

s1+s2

$$1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} - \frac{x^6}{720} - \frac{x^7}{5040} + \frac{x^8}{40320} + \frac{x^9}{362880} - \frac{x^{10}}{3628800}$$

Sqrt[2] Sin[x+Pi/4]//TrigExpand

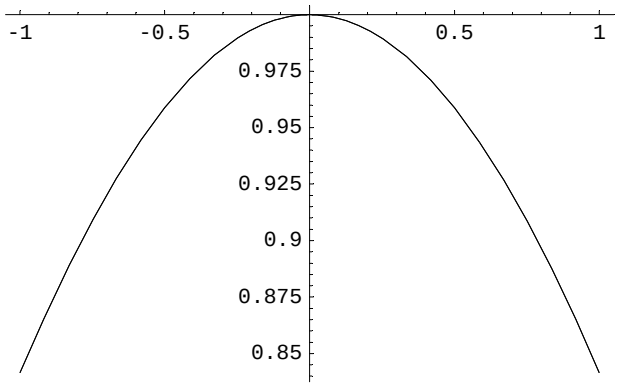
```
Cos[x] + Sin[x]
```

s3=Normal[Series[Sqrt[2] Sin[x+Pi/4],{x,0,10}]]

$$1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} - \frac{x^6}{720} - \frac{x^7}{5040} + \frac{x^8}{40320} + \frac{x^9}{362880} - \frac{x^{10}}{3628800}$$

e

Plot[{Sin[x]/x,s1/x},{x,-1,1}]



- Graphics **-**

f

Integrate[s1/x,x]

$$x - \frac{x^3}{18} + \frac{x^5}{600} - \frac{x^7}{35280} + \frac{x^9}{3265920}$$

NIntegrate[Simplify[s1/x],{x,-1,1}]

```
1.89217
```

```
NIntegrate[Sin[x]/x,{x,-1,1}]
```

```
1.89217
```

5

a

```
M=Transpose[{{1,1,2},{4,-1,4},{-1,1,-2}}]; M//MatrixForm
```

$$\begin{pmatrix} 1 & 4 & -1 \\ 1 & -1 & 1 \\ 2 & 4 & -2 \end{pmatrix}$$

b

```
Eigensystem[M]
```

$$\left\{\left\{\frac{1}{2}\left(-1-\sqrt{33}\right),\ \frac{1}{2}\left(-1+\sqrt{33}\right),\ -1\right\},\right.$$
$$\left.\left\{\left\{1,\ \frac{1}{8}\left(-1-\sqrt{33}\right),\ 1\right\},\ \left\{1,\ \frac{1}{8}\left(-1+\sqrt{33}\right),\ 1\right\},\ \{-4,\ 3,\ 4\}\right\}\right\}$$

```
Eigensystem[M]//N
```

```
{{-3.37228, 2.37228, -1.}, {{1., -0.84307, 1.}, {1., 0.59307, 1.}, {-4., 3., 4.}}}
```

```
Remove[u]
```

```
u[x_]:=x/Norm[x]
```

```
u /@ Eigenvectors[M]
```

$$\left\{\left\{\frac{1}{\sqrt{2+\frac{1}{64}\left(1+\sqrt{33}\right)^2}},\ \frac{-1-\sqrt{33}}{8\sqrt{2+\frac{1}{64}\left(1+\sqrt{33}\right)^2}},\ \frac{1}{\sqrt{2+\frac{1}{64}\left(1+\sqrt{33}\right)^2}}\right\},\right.$$
$$\left\{\frac{1}{\sqrt{2+\frac{1}{64}\left(-1+\sqrt{33}\right)^2}},\ \frac{-1+\sqrt{33}}{8\sqrt{2+\frac{1}{64}\left(-1+\sqrt{33}\right)^2}},\ \frac{1}{\sqrt{2+\frac{1}{64}\left(-1+\sqrt{33}\right)^2}}\right\},$$
$$\left.\left\{-\frac{4}{\sqrt{41}},\ \frac{3}{\sqrt{41}},\ \frac{4}{\sqrt{41}}\right\}\right\}$$

```
%//N
```

```
{{0.607371, -0.512056, 0.607371},
 {0.652088, 0.386734, 0.652088}, {-0.624695, 0.468521, 0.624695}}
```

c

Apply[Times,Eigenvalues[M]]

$$-\frac{1}{4}\left(-1-\sqrt{33}\right)\left(-1+\sqrt{33}\right)$$

Apply[Times,Eigenvalues[M]]//Expand

8

Det[M]

8

d

Det[M-λ IdentityMatrix[3]]

$$8+7\,\lambda-2\,\lambda^2-\lambda^3$$

Apply[Plus,Eigenvalues[M]]

$$-1+\frac{1}{2}\left(-1-\sqrt{33}\right)+\frac{1}{2}\left(-1+\sqrt{33}\right)$$

Apply[Plus,Eigenvalues[M]]//Expand

−2

e

Sum[Eigensystem[M][[2]][[k]]/Norm[Eigensystem[M][[2]][[k]]], {k,1,3}]

$$\left\{-\frac{4}{\sqrt{41}}+\frac{1}{\sqrt{2+\frac{1}{64}\left(-1+\sqrt{33}\right)^2}}+\frac{1}{\sqrt{2+\frac{1}{64}\left(1+\sqrt{33}\right)^2}},\right.$$

$$\frac{3}{\sqrt{41}}+\frac{-1+\sqrt{33}}{8\sqrt{2+\frac{1}{64}\left(-1+\sqrt{33}\right)^2}}+\frac{-1-\sqrt{33}}{8\sqrt{2+\frac{1}{64}\left(1+\sqrt{33}\right)^2}},$$

$$\left.\frac{4}{\sqrt{41}}+\frac{1}{\sqrt{2+\frac{1}{64}\left(-1+\sqrt{33}\right)^2}}+\frac{1}{\sqrt{2+\frac{1}{64}\left(1+\sqrt{33}\right)^2}}\right\}$$

Sum[Eigensystem[M][[2]][[k]]/Norm[Eigensystem[M][[2]][[k]]], {k,1,3}]//N

{0.634763, 0.343199, 1.88415}

```
Sum[1/Eigensystem[M][[1]][[k]]
Eigensystem[M][[2]][[k]]/Norm[Eigensystem[M][[2]][[k]]], {k,1,3}]//Simplify
```

$$\left\{\frac{4}{\sqrt{41}}+\frac{8\sqrt{\frac{2}{81-\sqrt{33}}}}{-1+\sqrt{33}}-\frac{8\sqrt{\frac{2}{81+\sqrt{33}}}}{1+\sqrt{33}},\right.$$
$$\left.-\frac{3}{\sqrt{41}}+\sqrt{\frac{2}{81-\sqrt{33}}}+\sqrt{\frac{2}{81+\sqrt{33}}},-\frac{4}{\sqrt{41}}+\frac{8\sqrt{\frac{2}{81-\sqrt{33}}}}{-1+\sqrt{33}}-\frac{8\sqrt{\frac{2}{81+\sqrt{33}}}}{1+\sqrt{33}}\right\}$$

```
Sum[1/Eigensystem[M][[1]][[k]]
Eigensystem[M][[2]][[k]]/Norm[Eigensystem[M][[2]][[k]]], {k,1,3}]//N
```

```
{0.719466, -0.153657, -0.529924}
```

```
M.Sum[1/Eigensystem[M][[1]][[k]]
Eigensystem[M][[2]][[k]]/Norm[Eigensystem[M][[2]][[k]]], {k,1,3}]//Simplify
```

$$\left\{-\frac{4}{\sqrt{41}}+\frac{1}{2}\sqrt{\frac{1}{51}\left(81-\sqrt{33}\right)}+\frac{1}{2}\sqrt{\frac{1}{51}\left(81+\sqrt{33}\right)},\ \frac{1}{656\sqrt{102}}\right.$$
$$\left(48\sqrt{4182}-41\sqrt{162-2\sqrt{33}}-41\sqrt{66\left(81-\sqrt{33}\right)}-41\sqrt{2\left(81+\sqrt{33}\right)}+41\sqrt{66\left(81+\sqrt{33}\right)}\right),$$
$$\left.\frac{4}{\sqrt{41}}+\frac{1}{2}\sqrt{\frac{1}{51}\left(81-\sqrt{33}\right)}+\frac{1}{2}\sqrt{\frac{1}{51}\left(81+\sqrt{33}\right)}\right\}$$

```
M.Sum[1/Eigensystem[M][[1]][[k]]
Eigensystem[M][[2]][[k]]/Norm[Eigensystem[M][[2]][[k]]] ,{k,1,3}]//N
```

```
{0.634763, 0.343199, 1.88415}
```

6

a

```
p1={0,0,4}; p2={0,6,0}; p3={3,0,0}; q1={2,8,0};
```

```
θ[λ_, μ_] := p1 + λ (p2 - p1) + μ (p3 - p1);
n = Cross[(p2 - p1) , (p3 - p1)];
s[t_] := q1 + t n;
{θ[λ, μ], n, s[t]}
```

```
{{3 μ, 6 λ, 4 - 4 λ - 4 μ}, {-24, -12, -18}, {2 - 24 t, 8 - 12 t, -18 t}}
```

```
solvθs = Solve[θ[λ,μ]==s[t],{λ,μ,t}]//Flatten
```

$$\left\{\lambda\to\frac{104}{87},\ \mu\to\frac{10}{87},\ t\to\frac{2}{29}\right\}$$

```
θs = s[t] /. solvθs
```

$$\left\{\frac{10}{29},\ \frac{208}{29},\ -\frac{36}{29}\right\}$$

```
N[%]
```

```
{0.344828, 7.17241, -1.24138}
```

q2 = q1 + 2 (*e*s-q1)

$\left\{-\frac{38}{29}, \frac{184}{29}, -\frac{72}{29}\right\}$

N[%]

{−1.31034, 6.34483, −2.48276}

m=(q1+q2)/2

$\left\{\frac{10}{29}, \frac{208}{29}, -\frac{36}{29}\right\}$

2 (q1 + q2)

N[%]

{0.344828, 7.17241, −1.24138}

b

V = 1/6 Det[{p1-q1,p2-q1,p3-q1}]

12

c

Flae = 1/2 Norm[Cross[p2-p1,p3-p1]]

$3\sqrt{29}$

N[%]

16.1555

d

ArcCos[(q2-p1).(q1-p1)/(Norm[(q2-p1)] Norm[(q1-p1)])]

$\text{ArcCos}\left[\frac{179}{203}\right]$

N[%]

0.491187

$\mathbf{ArcCos}\left[\frac{\mathbf{179}}{\mathbf{203}}\right] \Big/ (\mathbf{2\,Pi})\ \mathbf{360}\ \mathbf{//\ N}$

28.143

$\mathbf{ArcCos}\left[\frac{\mathbf{179}}{\mathbf{203}}\right] \Big/ \mathbf{Degree}\ \mathbf{//\ N}$

28.143

e

```
d = 2 Norm[Cross[q1,q2]/2] / Norm[q1-q2]
```

$$2\sqrt{\frac{349}{29}}$$

```
N[%]
```

```
6.93815
```

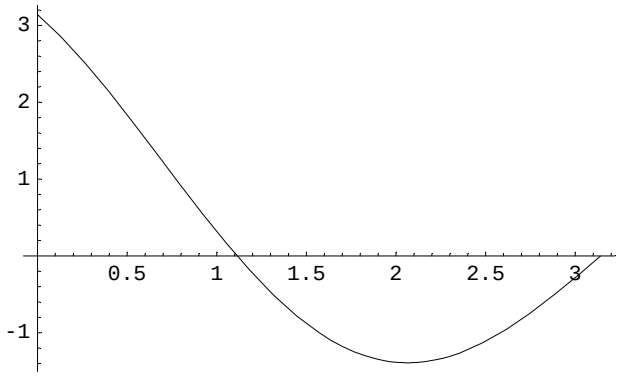
7

a

```
tr[x_]:= (Pi+ (Pi-2 x))/2 Sin[x]; tr[x]//Simplify
```

```
(π - x) Sin[x]
```

```
Plot[Evaluate[D[tr[x],x]/.x->u],{u,0,Pi}];
```




```
FindRoot[(Evaluate[D[tr[x],x]]==0) /.x->u,{u,1}]
```

```
{u → 1.11283}
```

```
(Evaluate[D[tr[x],x]]) /.x->1.11283
```

```
0.0000130208
```

```
tr[x] /.x->1.11283
```

```
1.81971
```

```
Integrate[Sin[x],{x,0,Pi}]
```

```
2
```

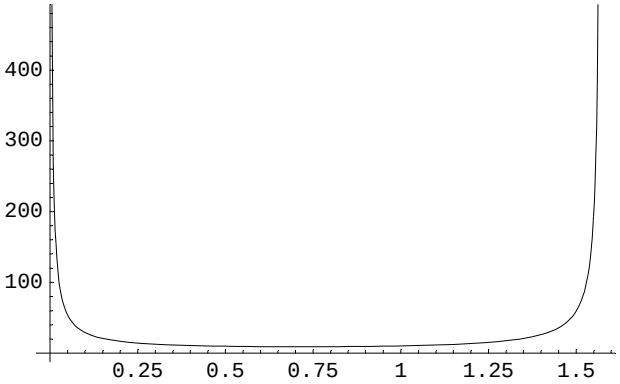
```
Integrate[Sin[x],{x,0,Pi}] - (tr[x] /.x->1.11283)
```

```
0.180294
```

b

```
h[α_] := 2.5 + 4 Tan[α];
g[α_] := 1/Tan[α] h[α];
laenge[α_] := Sqrt[h[α]^2+g[α]^2]

Plot[laenge[α],{α,0,Pi/2}];
```




```
D[laenge[α],α]
```

$$\left(8\,\text{Csc}[\alpha]^2\,(2.5+4\,\text{Tan}[\alpha])+8\,\text{Sec}[\alpha]^2\,(2.5+4\,\text{Tan}[\alpha])-2\,\text{Cot}[\alpha]\,\text{Csc}[\alpha]^2\,(2.5+4\,\text{Tan}[\alpha])^2\right)\Big/\left(2\sqrt{(2.5+4\,\text{Tan}[\alpha])^2+\text{Cot}[\alpha]^2\,(2.5+4\,\text{Tan}[\alpha])^2}\right)$$

```
fr=FindRoot[(Evaluate[D[laenge[α],α]==0] /.α->u),{u,0.75}]
```

$$\{u \to 0.707383\}$$

```
laenge[u] /. fr
```

```
9.10977
```