# Lösungen Modulprüfung Mathematik B 07

## 1B

### a

```
D[Sum[1/k x^k,{k,1,100}],x]
```

$$1 + x + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} + x^{11} + x^{12} + x^{13} + x^{14} + x^{15} + x^{16} + x^{17} + x^{18} + x^{19} + x^{20} + x^{21} + x^{22} + x^{23} + x^{24} + x^{25} + x^{26} + x^{27} + x^{28} + x^{29} + x^{30} + x^{31} + x^{32} + x^{33} + x^{34} + x^{35} + x^{36} + x^{37} + x^{38} + x^{39} + x^{40} + x^{41} + x^{42} + x^{43} + x^{44} + x^{45} + x^{46} + x^{47} + x^{48} + x^{49} + x^{50} + x^{51} + x^{52} + x^{53} + x^{54} + x^{55} + x^{56} + x^{57} + x^{58} + x^{59} + x^{60} + x^{61} + x^{62} + x^{63} + x^{64} + x^{65} + x^{66} + x^{67} + x^{68} + x^{69} + x^{70} + x^{71} + x^{72} + x^{73} + x^{74} + x^{75} + x^{76} + x^{77} + x^{78} + x^{79} + x^{80} + x^{81} + x^{82} + x^{83} + x^{84} + x^{85} + x^{86} + x^{87} + x^{88} + x^{89} + x^{90} + x^{91} + x^{92} + x^{93} + x^{94} + x^{95} + x^{96} + x^{97} + x^{98} + x^{99}$$

```
D[Sum[1/k x^k,{k,1,1000}],x]/.x->1
```

```
1000
```

### b Diff., Indentität

```
(Sin[x/2]+Cos[x/2])^2 // Expand // TrigReduce
```

$$1 + \text{Sin}[x]$$

```
D[(Sin[x/2]+Cos[x/2])^2,x]==Cos[x]
```

$$2 \left(\frac{1}{2} \text{Cos}\left[\frac{x}{2}\right] - \frac{1}{2} \text{Sin}\left[\frac{x}{2}\right]\right) \left(\text{Cos}\left[\frac{x}{2}\right] + \text{Sin}\left[\frac{x}{2}\right]\right) == \text{Cos}[x]$$

```
(D[(Sin[x/2]+Cos[x/2])^2,x]// Expand // TrigReduce ) ==Cos[x]
```

```
True
```

### c Partielle Integration

```
Integrate[(x-k+1)E^x,x]
```

$$e^x \, (-k + x)$$

### d Diff, Indentität

```
D[Log[(x Sqrt[x])]-Log[(x Sqrt[x-k])],{x,1}]//Simplify
```

$$\frac{k}{2\,k\,x - 2\,x^2}$$

```
Limit[1/Evaluate[D[Log[(x Sqrt[x])]-Log[(x Sqrt[x-k])],{x,1}]//Simplify], {x ->k}]
```

```
{0}
```

```
Limit[Evaluate[1/D[Log[(x Sqrt[x])]-Log[(x Sqrt[x-k])],{x,1}]//Simplify], {x
->k^2}]
```

```
{-2 (-1 + k) k^2}
```

### e

```
esin[x_]:=E^x Sin[x]; ecos[x_]:=E^x Cos[x];
```

```
Integrate[Integrate[esin[x],x],x]
```

$$-\frac{1}{2}\,e^x \text{Cos}[x]$$

```
Integrate[Integrate[ecos[x],x],x]
```

$$\frac{1}{2}\,e^x \text{Sin}[x]$$

```
Integrate[Integrate[esin[x],x],x] == -ecos[x]/2
```

```
True
```

```
Integrate[Integrate[ecos[x],x],x] == esin[x]/2
```

```
True
```

```
{Table[D[E^x Sin[x],{x,n}],{n,1,10}]}//Transpose//MatrixForm
```

$$\begin{pmatrix} e^x \text{Cos}[x] + e^x \text{Sin}[x] \\ 2\,e^x \text{Cos}[x] \\ 2\,e^x \text{Cos}[x] - 2\,e^x \text{Sin}[x] \\ -4\,e^x \text{Sin}[x] \\ -4\,e^x \text{Cos}[x] - 4\,e^x \text{Sin}[x] \\ -8\,e^x \text{Cos}[x] \\ -8\,e^x \text{Cos}[x] + 8\,e^x \text{Sin}[x] \\ 16\,e^x \text{Sin}[x] \\ 16\,e^x \text{Cos}[x] + 16\,e^x \text{Sin}[x] \\ 32\,e^x \text{Cos}[x] \end{pmatrix}$$

---

## 2B

```
Remove["Global`*"]
```

## a

```
pA = {2,0}; pB = {7,1}; pC = {5,4};
pS = (pA+pB)/2+ 1/3 (pC-(pA+pB)/2)
```

$$\left\{\frac{14}{3}, \frac{5}{3}\right\}$$

```
pS = (pA+pB+pC)/3
```

$$\left\{\frac{14}{3}, \frac{5}{3}\right\}$$

```
N[%]
```

```
{4.66667, 1.66667}
```

```
pB-pS
```

$$\left\{\frac{7}{3}, -\frac{2}{3}\right\}$$

```
J[p_]:={-p[[2]],p[[1]]};
J[pB-pS]
```

$$\left\{\frac{2}{3}, \frac{7}{3}\right\}$$

```
d = (pB.J[pB-pS])/Norm[J[pB-pS]]
```

$$\frac{21}{\sqrt{53}}$$

```
N[%]
```

```
2.88457
```

## b

```
v3[p_]:={p[[1]],p[[2]],0};

Cross[v3[pB-pA],v3[pC-pA]]/2
```

$$\left\{0, 0, \frac{17}{2}\right\}$$

```
Norm[Cross[v3[pB-pA],v3[pC-pA]]/2] // N
```

```
8.5
```

## c

```
mD[α_]:={{Cos[α],-Sin[α]},{Sin[α],Cos[α]}};
mV[p_]:=Transpose[{p}];
```

```
mD[30 Degree] // MatrixForm
```

$$\begin{pmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$$

```
mD[30 Degree] // N // MatrixForm
```

$$\begin{pmatrix} 0.866025 & -0.5 \\ 0.5 & 0.866025 \end{pmatrix}$$

```
pS1 = mD[30 Degree].mV[pS];
```

```
pS1 // MatrixForm
```

$$\begin{pmatrix} -\frac{5}{6} + \frac{7}{\sqrt{3}} \\ \frac{7}{3} + \frac{5}{2\sqrt{3}} \end{pmatrix}$$

```
pS1 // N // MatrixForm
```

$$\begin{pmatrix} 3.20812 \\ 3.77671 \end{pmatrix}$$

```
flat[v_]:= Flatten[Transpose[v]];
pS1 = flat[pS1]
```

```
General::spell1 :
 Possible spelling error: new symbol name "flat" is similar to existing symbol "Flat". Mehr…
```

$$\left\{-\frac{5}{6} + \frac{7}{\sqrt{3}}, \frac{7}{3} + \frac{5}{2\sqrt{3}}\right\}$$

## d

```
mSp[α_]:={{Cos[2α],Sin[2α]},{Sin[2α],-Cos[2α]}};
```

```
α = ArcCos[(pB-pA).{1,0}/Norm[pB-pA]]
```

$$\text{ArcCos}\left[\frac{5}{\sqrt{26}}\right]$$

```
α // N
```

```
0.197396
```

```
α / Degree // N
```

```
11.3099
```

```
mSp[α].mV[pS-pA]+ mV[pA] // N  // MatrixForm
```

$$\begin{pmatrix} 5.10256 \\ -0.512821 \end{pmatrix}$$

```
mSp[α].mV[pC-pA]+mV[pA] // N  // MatrixForm
```

$$\begin{pmatrix} 6.30769 \\ -2.53846 \end{pmatrix}$$

```
flat[v_]:= Flatten[Transpose[v]];
pS2 = flat[mSp[α].mV[pS-pA]+mV[pA]] // N
```

```
{5.10256, -0.512821}
```

```
pC2 = flat[mSp[α].mV[pC-pA]+mV[pA]] // N
```

```
{6.30769, -2.53846}
```

## e

```
str[v_]:={2v[[1]],v[[2]]};
pS3 = str[pS2]
```

```
{10.2051, -0.512821}
```

```
pS4 = mSp[α].mV[pS3-pA]+ mV[pA]  // MatrixForm
```

$$\begin{pmatrix} 9.37673 \\ 3.62919 \end{pmatrix}$$

```
pS4 = flat[mSp[α].mV[pS3-pA]+mV[pA]] // N
```

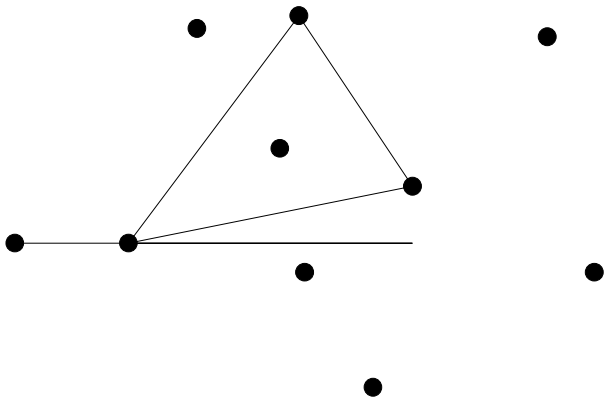
```
{9.37673, 3.62919}
```

## f

```
points =
Prepend[Map[Point,{{0,0},pA,pB,pC,pS,pS1,pS2,pC2,pS3,pS4}//N],PointSize[0.03]];
lin = {Line[{pA,pB,pC,pA,{7,0},{0,0}}]};
allP = Join[points,lin]
```

```
{PointSize[0.03], Point[{0., 0.}], Point[{2., 0.}], Point[{7., 1.}], Point[{5., 4.}],
 Point[{4.66667, 1.66667}], Point[{3.20812, 3.77671}], Point[{5.10256, -0.512821}],
 Point[{6.30769, -2.53846}], Point[{10.2051, -0.512821}],
 Point[{9.37673, 3.62919}], Line[{{2, 0}, {7, 1}, {5, 4}, {2, 0}, {7, 0}, {0, 0}}]}
```

```
Show[Graphics[allP],AspectRatio->Automatic];
```



# 3B

## a

```
r = 4;
h[a_,b_]:= Sqrt[r^2-(a/2)^2-(b/2)^2];
h[a,b]
```

$$\sqrt{16 - \frac{a^2}{4} - \frac{b^2}{4}}$$

## b

```
V[a_,b_]:= a b h[a,b] + a b (r-h[a,b])/3;
V[a,b] // Simplify
```

$$\frac{1}{3} a b \left(4 + \sqrt{64 - a^2 - b^2}\right)$$

## c

```
Solve[Evaluate[{D[V[a,b],a]==0, D[V[a,b],b]==0}],{a,b}]
```

$$\left\{\{a \to 0,\ b \to 0\},\ \left\{a \to -\sqrt{\frac{176}{9} + \frac{16\sqrt{13}}{9}},\ b \to -\sqrt{\frac{176}{9} + \frac{16\sqrt{13}}{9}}\right\},\right.$$
$$\left\{a \to -\sqrt{\frac{176}{9} + \frac{16\sqrt{13}}{9}},\ b \to \sqrt{\frac{176}{9} + \frac{16\sqrt{13}}{9}}\right\},$$
$$\left\{a \to \sqrt{\frac{176}{9} + \frac{16\sqrt{13}}{9}},\ b \to -\sqrt{\frac{176}{9} + \frac{16\sqrt{13}}{9}}\right\},$$
$$\left.\left\{a \to \sqrt{\frac{176}{9} + \frac{16\sqrt{13}}{9}},\ b \to \sqrt{\frac{176}{9} + \frac{16\sqrt{13}}{9}}\right\}\right\}$$

```
N[%]
```

$$\{\{a \to 0.,\ b \to 0.\},\ \{a \to -5.09563,\ b \to -5.09563\},\ \{a \to -5.09563,\ b \to 5.09563\},$$
$$\{a \to 5.09563,\ b \to -5.09563\},\ \{a \to 5.09563,\ b \to 5.09563\}\}$$

## d

```
Remove["Global`*"]

r = 4; V = 20;

Solve[V==a b h+ a b (r-h)/3,{h}]
```

$$\left\{\left\{h \to -\frac{2\,(-15 + a\,b)}{a\,b}\right\}\right\}$$

```
h[a_, b_] := -2 (-15 + a b)/(a b)
```

```
h[a,b]// Apart
```

$$-2+\frac{30}{a\,b}$$

## e

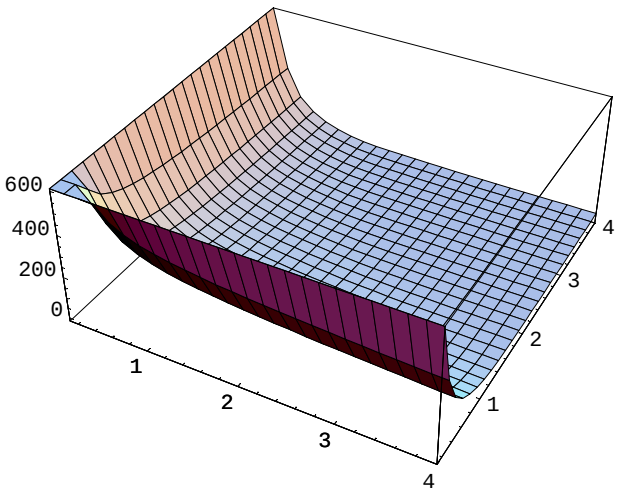
```
Ob[a_,b_]:= 2 a h[a,b] +2 b h[a,b]+ 2 a Sqrt[(r-h[a,b])^2+(b/2)^2]/2+ 2 b
Sqrt[(r-h[a,b])^2+(a/2)^2]/2;
Ob[a,b] //Simplify
```

$$\frac{60}{a}-4\,a+\frac{60}{b}-4\,b+b\sqrt{\frac{a^2}{4}+\frac{36\,(-5+a\,b)^2}{a^2\,b^2}}+a\sqrt{\frac{b^2}{4}+\frac{36\,(-5+a\,b)^2}{a^2\,b^2}}$$

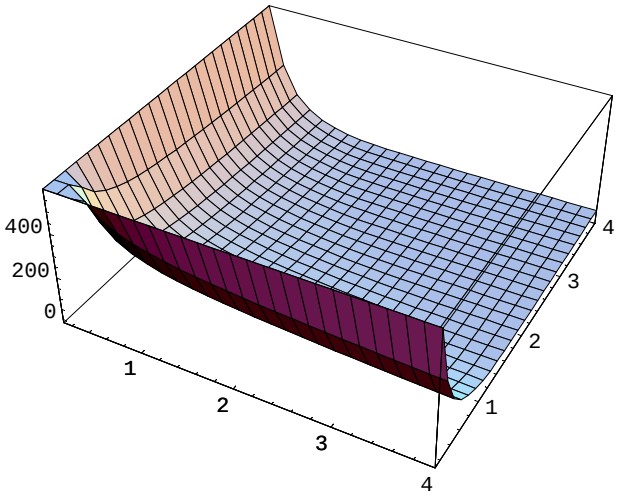
```
Ob1[a_,b_]:= Ob[a,b]+ a b;
Ob1[a,b] //Simplify
```

$$\frac{60}{a}-4\,a+\frac{60}{b}-4\,b+a\,b+b\sqrt{\frac{a^2}{4}+\frac{36\,(-5+a\,b)^2}{a^2\,b^2}}+a\sqrt{\frac{b^2}{4}+\frac{36\,(-5+a\,b)^2}{a^2\,b^2}}$$

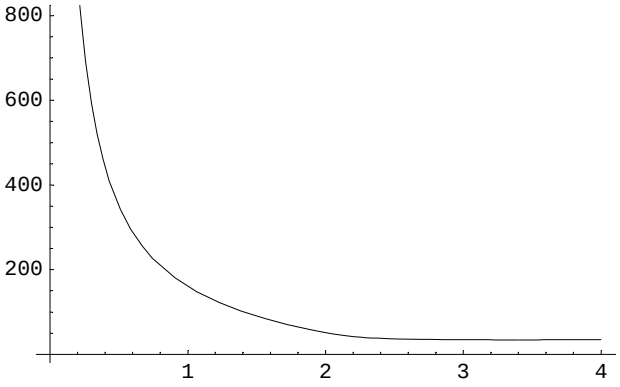
```
Plot3D[Ob[a,b],{a,0.1,4},{b,0.1,4}];
```



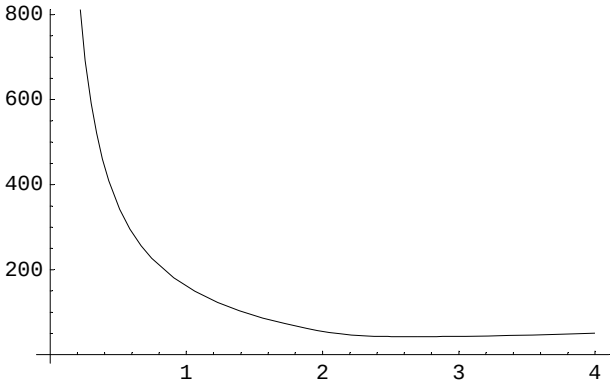
```
Plot3D[Ob1[a,b],{a,0.1,4},{b,0.1,4}];
```



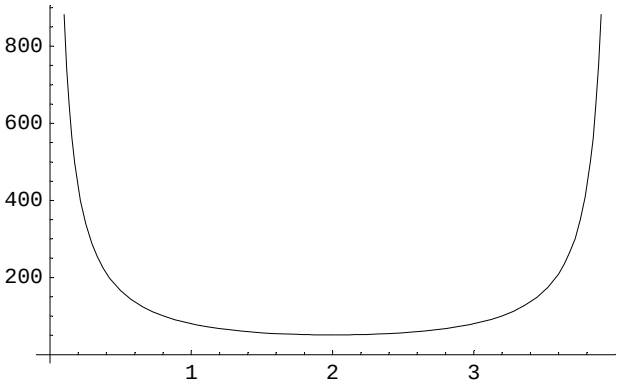

```
Plot[Ob[a,a],{a,0.1,4}];
```



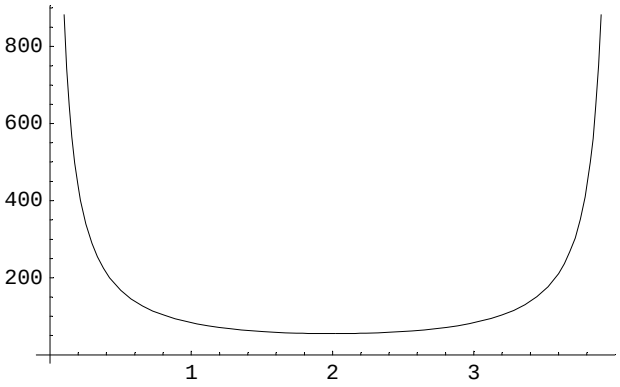

```
Plot[Ob1[a,a],{a,0.1,4}];
```



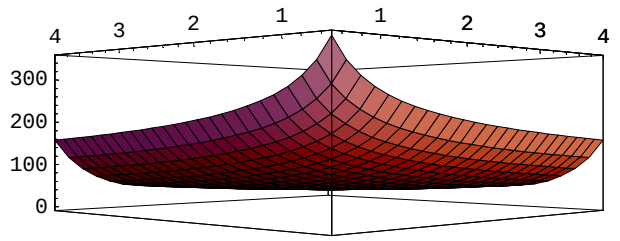
```
Plot[Ob[a,4-a],{a,0.1,3.9}];
```




```
Plot[Ob1[a,4-a],{a,0.1,3.9}];
```




```
Plot3D[Ob[a,b],{a,0.5,4},{b,0.5,4},ViewPoint->{-2.056, -2.031, -0.0}];
```




```
(* Ableitungen zu kompliziert: Graphisch Schnitte studieren *)
```

```
D[Ob[a,b],a] // Simplify
```

$$-4-\frac{4\,b}{a}+\frac{4\,(-15+a\,b)}{a^2}+\frac{360\,(-5+a\,b)}{a^2\,b^2\sqrt{144+\frac{3600}{a^2\,b^2}-\frac{1440}{a\,b}+b^2}}+$$

$$\frac{-3600+720\,a\,b+a^4\,b^2}{2\,a^3\sqrt{144+a^2+\frac{3600}{a^2\,b^2}-\frac{1440}{a\,b}}\,b}+\sqrt{\frac{b^2}{4}+\frac{36\,(-5+a\,b)^2}{a^2\,b^2}}$$

```
D[Ob[a,b],b] // Simplify
```

$$-4-\frac{4\,a}{b}+\frac{4\,(-15+a\,b)}{b^2}+\frac{360\,(-5+a\,b)}{a^2\sqrt{144+a^2+\frac{3600}{a^2\,b^2}-\frac{1440}{a\,b}}\,b^2}+$$

$$\frac{-3600+720\,a\,b+a^2\,b^4}{2\,a\,b^3\sqrt{144+\frac{3600}{a^2\,b^2}-\frac{1440}{a\,b}+b^2}}+\sqrt{\frac{a^2}{4}+\frac{36\,(-5+a\,b)^2}{a^2\,b^2}}$$

```
NSolve[Evaluate[{D[Ob[a,b],a]==0,D[Ob[a,b],b]==0}],{a,b}]
```

```
{{a → 12.3229, b → -12.3229}, {a → -12.3229, b → 12.3229}, {a → -4.15846, b → 4.15846},
 {a → 4.15846, b → -4.15846}, {a → 1.57562 + 6.28115 i, b → -1.57562 + 6.28115 i},
 {a → 1.57562 - 6.28115 i, b → -1.57562 - 6.28115 i},
 {a → -1.57562 + 6.28115 i, b → 1.57562 + 6.28115 i},
 {a → -1.57562 - 6.28115 i, b → 1.57562 - 6.28115 i},
 {a → 0. + 6.39557 i, b → 0. + 6.39557 i}, {a → 0. - 6.39557 i, b → 0. - 6.39557 i},
 {a → 3.3916, b → 3.3916}, {a → -3.3916, b → -3.3916}}
```

```
(*   {a→3.3916,b→3.3916}   *)
```

```
NSolve[Evaluate[{D[Ob1[a,b],a]==0,D[Ob1[a,b],b]==0}],{a,b}]
```

```
{{a → -2.14339 - 14.3597 i, b → -2.14339 + 14.3597 i},
 {a → -2.14339 + 14.3597 i, b → -2.14339 - 14.3597 i},
 {a → -1.36982, b → 12.1225}, {a → -0.901512 + 1.14831 i, b → 1.20755 + 13.1863 i},
 {a → -0.901512 - 1.14831 i, b → 1.20755 - 13.1863 i},
 {a → 1.06262 - 0.962986 i, b → -3.44155 + 10.6807 i},
 {a → 1.06262 + 0.962986 i, b → -3.44155 - 10.6807 i},
 {a → 12.1225, b → -1.36982}, {a → -3.44155 - 10.6807 i, b → 1.06262 + 0.962986 i},
 {a → -3.44155 + 10.6807 i, b → 1.06262 - 0.962986 i},
 {a → 1.20755 - 13.1863 i, b → -0.901512 - 1.14831 i},
 {a → 1.20755 + 13.1863 i, b → -0.901512 + 1.14831 i},
 {a → -2.39753 + 2.93073 i, b → -2.39753 + 2.93073 i},
 {a → -2.39753 - 2.93073 i, b → -2.39753 - 2.93073 i}, {a → 2.66569, b → 2.66569},
 {a → -2.17742 + 0.264196 i, b → -2.17742 + 0.264196 i},
 {a → -2.17742 - 0.264196 i, b → -2.17742 - 0.264196 i}}
```

```
(*   {a→2.66569,b→2.66569}   *)
```

```
(*  Achtung Zeitaufwand!!!
Solve[Evaluate[{D[Ob[a,b],a]==0,D[Ob[a,b],b]==0}],{a,b}] *)
```

---

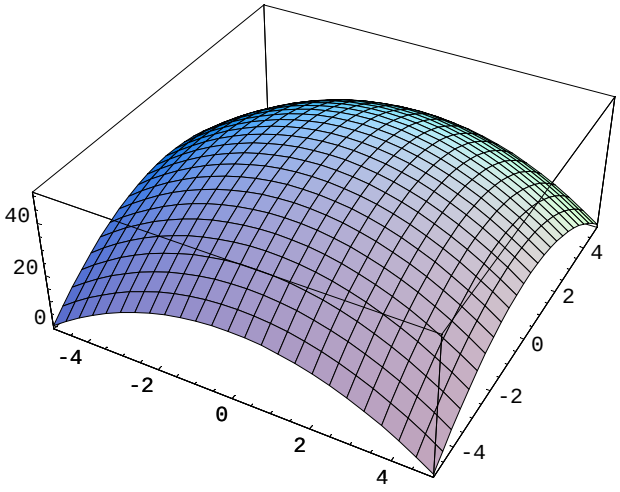
# 4B

## a

```
Remove["Global`*"]
```

```
f[x_,y_]:= 50-x^2-y^2
```

```
Plot3D[f[x,y],{x,-5,5},{y,-5,5},AspectRatio->Automatic];
```




## b

```
Integrate[f[x,y],{x,-5,5},{y,-5,5}]
```

$$\frac{10000}{3}$$

```
N[%]
```

```
3333.33
```

## c

```
f[x,5]
```

$$25 - x^2$$

```
len = Integrate[Evaluate[Sqrt[1+D[f[x,5],x]^2]],{x,-5,5}]
```

$$5\sqrt{101} + \frac{\text{ArcSinh}[10]}{2}$$

```
N[%]
```

```
51.7485
```

## d

```
Integrate[f[x,5],{x,-5,5}]
```

$$\frac{500}{3}$$

```
N[%]
```

```
166.667
```

### e

```
v[x_,y_]:={x,y,f[x,y]};
Norm[Cross[D[v[x,y],x],D[v[x,y],y]]]
```

$$\sqrt{1 + 4\,\text{Abs}[x]^2 + 4\,\text{Abs}[y]^2}$$

```
Sqrt[Cross[D[v[x,y],x],D[v[x,y],y]].Cross[D[v[x,y],x],D[v[x,y],y]]]
```

$$\sqrt{1 + 4\,x^2 + 4\,y^2}$$

```
Integrate[Evaluate[Sqrt[Cross[D[v[x,y],x],D[v[x,y],y]].Cross[D[v[x,y],x],D[v[x,y],y
]]]], {x,-5,5}]
```

$$\frac{1}{4}\left(20\sqrt{101 + 4\,y^2} - (1 + 4\,y^2)\,\text{Log}\left[-10 + \sqrt{101 + 4\,y^2}\right] + (1 + 4\,y^2)\,\text{Log}\left[10 + \sqrt{101 + 4\,y^2}\right]\right)$$

```
Integrate[Integrate[Evaluate[Sqrt[Cross[D[v[x,y],x],D[v[x,y],y]].Cross[D[v[x,y],x],
D[v[x,y],y]]]], {x,-5,5}], {y,-5,5}]
```

$$\frac{1}{6}\left(200\sqrt{201} + 1030\,\text{ArcSinh}\left[\frac{10}{\sqrt{101}}\right] - 2\,\text{ArcTan}\left[\frac{100}{\sqrt{201}}\right] - 515\,\text{Log}\left[-10 + \sqrt{201}\right] + 515\,\text{Log}\left[10 + \sqrt{201}\right]\right)$$

```
N[%]
```

```
773.504
```

```
NIntegrate[Evaluate[Sqrt[Cross[D[v[x,y],x],D[v[x,y],y]].Cross[D[v[x,y],x],D[v[x,y],
y]]]], {x,-5,5}, {y,-5,5}]
```

```
773.504
```

---

## B5

```
Remove["Global`*"]
```

### a

#### I

```
(* b in Abh. von a *)

f[a_,x_,n_]:= a x^n;
h[b_,x_]:= 2-b x^2;
Solve[f[a,2,n]==h[b,2],{b}]
```

$$\left\{\left\{b \to \frac{1}{4}\,(2 - 2^n\,a)\right\}\right\}$$

```
h[a_, x_, n_] := 2 - 1/4 (2 - 2^n a) x^2;
```

## II

```
res[a_,n_]:=Integrate[f[a,x,n]-h[a,x,n],{x,0,2},GenerateConditions→False]
```

```
res[a,n]
```

$$-\frac{2\,(2^n\,a\,(-2+n)+4\,(1+n))}{3\,(1+n)}$$

```
res[a,n]//Apart
```

$$-\frac{8}{3}-\frac{2^{1+n}\,a\,(-2+n)}{3\,(1+n)}$$

```
(* Ex. Max oder Min , in Abh. von a? - Unabh. vom Rand *)
```

```
D[res[a,n],a]
```

$$-\frac{2^{1+n}\,(-2+n)}{3\,(1+n)}$$

## III

```
(* ist immer 0 für n = 2, sonst nie, d.h. monoton *)
```

## IV

```
(* Tangenten senkrecht *)
```

```
D[f[a,x,n],{x}]
```

$$a\,n\,x^{-1+n}$$

```
-1/D[h[a,x,n],{x}]
```

$$\frac{2}{(2-2^n\,a)\,x}$$

```
Solve[(Evaluate[D[f[a,x,n],{x}]==-1/D[h[a,x,n],{x}]]/.x->2),{a}]
```

$$\left\{\left\{a\to\frac{2^{-1-2n}\left(2^{1+n}\,n-2\sqrt{-2^{1+2n}\,n+2^{2n}\,n^2}\right)}{n}\right\},\ \left\{a\to\frac{2^{-1-2n}\left(2^{1+n}\,n+2\sqrt{-2^{1+2n}\,n+2^{2n}\,n^2}\right)}{n}\right\}\right\}$$

```
Solve[(Evaluate[D[f[a,x,n],{x}]==-1/D[h[a,x,n],{x}]]/.x->2),{a}]/.n->2
```

$$\left\{\left\{a\to\frac{1}{4}\right\},\ \left\{a\to\frac{1}{4}\right\}\right\}$$

```
Solve[(Evaluate[D[f[a,x,n],{x}]==-1/D[h[a,x,n],{x}]]/.x->2),{a}]/.n->3
```

$$\left\{\left\{a\to\frac{1}{384}\left(48-16\sqrt{3}\right)\right\},\ \left\{a\to\frac{1}{384}\left(48+16\sqrt{3}\right)\right\}\right\}$$

```
Solve[(Evaluate[D[f[a,x,n],{x}]==-1/D[h[a,x,n],{x}]]/.x->2),{a}]/.n->4
```

$$\left\{\left\{a\to\frac{128-64\sqrt{2}}{2048}\right\},\ \left\{a\to\frac{128+64\sqrt{2}}{2048}\right\}\right\}$$

## b

```
Remove["Global`*"]
```

### I

```
A[a_]:={{a,-1},{1,2}};
A[a]//N//Eigensystem//Simplify
```

$$\left\{\left\{1.+0.5\,a-0.5\,\sqrt{a}\,\sqrt{-4.+1.\,a},\ 1.+0.5\,a+0.5\,\sqrt{a}\,\sqrt{-4.+1.\,a}\right\},\right.$$
$$\left.\left\{\left\{-1.+0.5\,a-0.5\,\sqrt{a}\,\sqrt{-4.+1.\,a},\ 1.\right\},\ \left\{-1.+0.5\,a+0.5\,\sqrt{a}\,\sqrt{-4.+1.\,a},\ 1.\right\}\right\}\right\}$$

```
A[a] // MatrixForm
```

$$\begin{pmatrix} a & -1 \\ 1 & 2 \end{pmatrix}$$

```
Eigenvalues[A[a]]
```

$$\left\{\frac{1}{2}\left(2-\sqrt{-4+a}\,\sqrt{a}+a\right),\ \frac{1}{2}\left(2+\sqrt{-4+a}\,\sqrt{a}+a\right)\right\}$$

```
(* Bedingung für 2 EW: a gröser 4 *)
```

### II

```
A[5]// Eigensystem
```

$$\left\{\left\{\frac{1}{2}\left(7+\sqrt{5}\right),\ \frac{1}{2}\left(7-\sqrt{5}\right)\right\},\ \left\{\left\{-2+\frac{1}{2}\left(7+\sqrt{5}\right),\ 1\right\},\ \left\{-2+\frac{1}{2}\left(7-\sqrt{5}\right),\ 1\right\}\right\}\right\}$$

```
(A[5]//N//Eigensystem)
```

$$\{\{4.61803,\ 2.38197\},\ \{\{0.934172,\ 0.356822\},\ \{0.356822,\ 0.934172\}\}\}$$

```
Eigenvalues[A[5]]
```

$$\left\{\frac{1}{2}\left(7+\sqrt{5}\right),\ \frac{1}{2}\left(7-\sqrt{5}\right)\right\}$$

```
 Eigenvectors[A[5]]
```

$$\left\{\left\{-2+\frac{1}{2}\left(7+\sqrt{5}\right),\ 1\right\},\ \left\{-2+\frac{1}{2}\left(7-\sqrt{5}\right),\ 1\right\}\right\}$$

```
A[5].(t {{1/2 (3-Sqrt[5])},{1}})//MatrixForm
```

$$\begin{pmatrix} -t+\frac{5}{2}\left(3-\sqrt{5}\right)t \\ 2\,t+\frac{1}{2}\left(3-\sqrt{5}\right)t \end{pmatrix}$$

```
%//Simplify//MatrixForm
```

$$\begin{pmatrix} \frac{1}{2}\left(13-5\sqrt{5}\right)t \\ -\frac{1}{2}\left(-7+\sqrt{5}\right)t \end{pmatrix}$$

```
N[%]
```

```
{{0.90983 t}, {2.38197 t}}
```

```
(* Geraden Richtung Eigenvektoren sind Fixgeraden *)
```

## III

```
(Eigenvectors[A[5]])[[1]]
```

$$\left\{-2+\frac{1}{2}\left(7+\sqrt{5}\right),\ 1\right\}$$

```
(Eigenvalues[A[5]])[[2]]
```

$$\frac{1}{2}\left(7-\sqrt{5}\right)$$

```
(A[5].(Eigenvectors[A[5]])//Flatten)//Simplify
```

$$\left\{3\left(2+\sqrt{5}\right),\ 4,\ \frac{1}{2}\left(9-\sqrt{5}\right),\ 3\right\}$$

```
(A[5].(Eigenvectors[A[5]])//Flatten)[[1]]//Simplify
```

$$3\left(2+\sqrt{5}\right)$$

```
(* 2.Koordinate = EW *)
```

```
(A[5].({{1/2 (3-Sqrt[5])},{1}})//Flatten)[[2]] //Simplify
```

$$\frac{1}{2}\left(7-\sqrt{5}\right)$$