# Lösungen

## 1

```
Remove["Global`*"]
```

### a

```
m1={{3,2,1},{1,2,3},{4,0,4}};
Det[m1]
```

```
32
```

### b

```
m2={{1,2,3},{1,2,4},{0,1,4}};
Det[m2]
```

```
-1
```

### c

```
Det[m1.m2]
```

```
-32
```

### d

```
m1.m2//MatrixForm
```

$$\begin{pmatrix} 5 & 11 & 21 \\ 3 & 9 & 23 \\ 4 & 12 & 28 \end{pmatrix}$$

### e

```
m2.m1//MatrixForm
```

$$\begin{pmatrix} 17 & 6 & 19 \\ 21 & 6 & 23 \\ 17 & 2 & 19 \end{pmatrix}$$

## f

```
b1={1,0,-1}
```

$$\{1, 0, -1\}$$

```
Solve[m1.{x1,x2,x3}==b1,{x1,x2,x3}]
```

$$\left\{\left\{x1 \to \frac{1}{8},\ x2 \to \frac{1}{2},\ x3 \to -\frac{3}{8}\right\}\right\}$$

## g

```
b2={8,0,-8}
```

$$\{8, 0, -8\}$$

```
m1.{x1,x2,x3}==b1
```

$$\{3\,x1 + 2\,x2 + x3,\ x1 + 2\,x2 + 3\,x3,\ 4\,x1 + 4\,x3\} == \{1, 0, -1\}$$

```
Solve[m1.{x1,x2,x3}==b2,{x1,x2,x3}]
```

$$\{\{x1 \to 1,\ x2 \to 4,\ x3 \to -3\}\}$$

## h

```
bT=Transpose[{b2}];
m2.bT==Transpose[Transpose[bT].Transpose[m2]]
```

```
True
```

```
Solve[(m2.m1).{x1,x2,x3}==m2.b2,{x1,x2,x3}]
```

$$\{\{x1 \to 1,\ x2 \to 4,\ x3 \to -3\}\}$$

## i

```
m3={{3,2},{1,2}};
Inverse[m3]//MatrixForm
```

$$\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{4} & \frac{3}{4} \end{pmatrix}$$

```
N[%]
```

$$\{\{0.5, -0.5\}, \{-0.25, 0.75\}\}$$

## j

```
z1=3+2I;
z2=1-I;
z1 z2
```

$5 - i$

```
1/(z1 z2)
```

$\frac{5}{26} + \frac{i}{26}$

```
N[%]
```

$0.192308 + 0.0384615\, i$

## k

```
1/(Conjugate[z1] Conjugate[z2])
```

$\frac{5}{26} - \frac{i}{26}$

```
N[%]
```

$0.192308 - 0.0384615\, i$

## l

```
(* 4*(1:5)+10  umgesetzt in Mathematica *)
```

```
4*Range[5]+10
```

$\{14, 18, 22, 26, 30\}$

```
>> 4*(1:5)+10
    14    18    22    26    30
```

## m

```
Mod[70, 12]
```

$10$

```
>> rem(70,12)
ans = 10
```

## n

```
g = I + 3; Im[g] Conjugate[g]
```

```
3 - i
```

```
>> g=j+3; imag(g)*conj(g)
ans = 3 - 1i
```

## o

```
g = I + 3; Im[g] Conjugate[g]
```

```
3 - i
```

```
a = {1, 2, 3, 4}; b = Transpose[{a, 2 a}]; b // MatrixForm
```

$$\begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \\ 4 & 8 \end{pmatrix}$$

```
>> a=[1 2 3 4];b=[a',2*a']
b =

     1     2
     2     4
     3     6
     4     8
```

## p

```
b.Transpose[b] // MatrixForm
```

$$\begin{pmatrix} 5 & 10 & 15 & 20 \\ 10 & 20 & 30 & 40 \\ 15 & 30 & 45 & 60 \\ 20 & 40 & 60 & 80 \end{pmatrix}$$

```
>> a=[1 2 3 4];b=[a',2*a'];b*b'
ans =

     5    10    15    20
    10    20    30    40
    15    30    45    60
    20    40    60    80
```

# 2

```
Remove["Global`*"]
```

## a

```
P0={1,1,-1}; P1={2,0,2}; P2={3,2,1};

Inhalt= Norm[Cross[P1-P0,P2-P0]]/2
```

$$\frac{5}{\sqrt{2}}$$

```
N[%]
```

```
3.53553
```

## b

```
v={2,5,1};
hnf[x_,y_,z_]:=(v.{x,y,z}+d)/Norm[v];
hnf[{x_,y_,z_}]:=hnf[x,y,z];
hnf[x,y,z]
```

$$\frac{d + 2\,x + 5\,y + z}{\sqrt{30}}$$

```
N[%]//Expand
```

```
0.182574 d + 0.365148 x + 0.912871 y + 0.182574 z
```

## c, d

```
hnf[1,1,-1]==0
```

$$\frac{6 + d}{\sqrt{30}} == 0$$

```
solv1=Solve[hnf[1,1,-1]==0,{d}]//Flatten
```

$$\{d \to -6\}$$

```
hnf[x_,y_,z_]:=(v.{x,y,z}+d)/Norm[v]/.solv1;
hnf[{x_,y_,z_}]:=hnf[x,y,z];
hnf[x,y,z]
```

$$\frac{-6 + 2\,x + 5\,y + z}{\sqrt{30}}$$

```
p1={2,0,2};
hnf[p1]
```

```
0
```

```
p2={3,2,1};hnf[p2]
```

$$\frac{11}{\sqrt{30}}$$

```
N[%]
```

```
2.00832
```

```
(* Nur P1 liegt in der Ebene *)
```

## e

```
g[t_]:=t p2
```

```
hnf[g[t]]
```

$$\frac{-6+17\,t}{\sqrt{30}}$$

```
solv2=Solve[hnf[g[t]]==0,{t}]//Flatten
```

$$\left\{t \to \frac{6}{17}\right\}$$

```
PS=g[t]/.solv2
```

$$\left\{\frac{18}{17}, \frac{12}{17}, \frac{6}{17}\right\}$$

```
N[%]
```

```
{1.05882, 0.705882, 0.352941}
```

```
tanA =  PS[[3]]/Sqrt[PS[[1]]^2+PS[[2]]^3]
```

$$\sqrt{\frac{17}{201}}$$

```
N[%]
```

```
0.290821
```

## f Der Abstand ist minimal, wenn die erste Spur der Ebene senkrecht zu g ist.

```
solv3=Solve[hnf[x,y,0]==0,{y}]//Flatten
```

$$\left\{y \to -\frac{1}{5}\sqrt{\frac{2}{15}}\ \left(-3\sqrt{30}+\sqrt{30}\ x\right)\right\}$$

```
N[%]
```

```
{y → -0.0730297 (-16.4317 + 5.47723 x)}
```

```
y[t_]:=(y/.solv3)/.x->t; y[a]
```

$$-\frac{1}{5}\sqrt{\frac{2}{15}}\ \left(-3\sqrt{30}+\sqrt{30}\ a\right)$$

**N[%]**

$-0.0730297\,(-16.4317+5.47723\,a)$

**y[1]-y[0]**

$-\frac{2}{5}$

**yVec= 5 {1,y[1]-y[0]}**

$\{5, -2\}$

**yVecSenkr= {-yVec[[2]],yVec[[1]]}**

$\{2, 5\}$

**lyVecSenkr=Norm[yVecSenkr]**

$\sqrt{29}$

**N[%]**

5.38516

**gyVecSenkr[t_]:= t**
**{yVecSenkr[[1]]/Norm[yVecSenkr],yVecSenkr[[2]]/Norm[yVecSenkr],tanA}; gyVecSenkr[t]**

$\left\{\frac{2\,t}{\sqrt{29}}, \frac{5\,t}{\sqrt{29}}, \sqrt{\frac{17}{201}}\,t\right\}$

**N[%]**

$\{0.371391\,t, 0.928477\,t, 0.290821\,t\}$

**(hnf[gyVecSenkr[t]]//Simplify)==0**

$\frac{-6+\left(\sqrt{\frac{17}{201}}+\sqrt{29}\right)t}{\sqrt{30}}==0$

**N[%]**

$0.182574\,(-6.+5.67599\,t)==0.$

**solv1 = Solve[(hnf[gyVecSenkr[t]]//Simplify)==0,{t}] // Simplify // Flatten**

$\left\{t\to\frac{1206}{201\sqrt{29}+\sqrt{3417}}\right\}$

**N[%]**

$\{t\to 1.05709\}$

**gyVecSenkr1[t_]:=gyVecSenkr[t]/.solv1;  gyVecSenkr1[t]//Simplify**

$\left\{\frac{2412}{5829+\sqrt{99093}}, \frac{6030}{5829+\sqrt{99093}}, \frac{6\sqrt{3417}}{201\sqrt{29}+\sqrt{3417}}\right\}$

**N[%]**

$\{0.392592, 0.981479, 0.307423\}$

```
Solve[hnf[gyVecSenkr1[t]]==0,{d}]
```

$$\left\{\left\{d \to -\frac{6\sqrt{3}\ \left(67\sqrt{87}+\sqrt{1139}\right)}{201\sqrt{29}+\sqrt{3417}}\right\}\right\}$$

```
N[%]
```

$$\{\{d \to -6.\}\}$$

---

# 3

```
Remove["Global`*"]
```

```
A={{1,0,0,0},{1,1,0,0},{1,1,1,0},{1,1,1,1}}; A//MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

```
B={{1,1,1,1},{0,1,1,1},{0,0,1,1},{0,0,0,1}}; B//MatrixForm
```

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

## a

```
u1=A.A;u1//MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 4 & 3 & 2 & 1 \end{pmatrix}$$

```
v1=B.B; v1//MatrixForm
```

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

## b

```
u2=u1.A;u2//MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 6 & 3 & 1 & 0 \\ 10 & 6 & 3 & 1 \end{pmatrix}$$

**v2=v1.B; v2//MatrixForm**

$$\begin{pmatrix} 1 & 3 & 6 & 10 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

**c2=A.B;c1//MatrixForm**

```
c1
```

## c

**A.B//MatrixForm**

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

**B.A//MatrixForm**

$$\begin{pmatrix} 4 & 3 & 2 & 1 \\ 3 & 3 & 2 & 1 \\ 2 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

**A.A.B.B//MatrixForm**

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 5 & 8 & 11 \\ 3 & 8 & 14 & 20 \\ 4 & 11 & 20 & 30 \end{pmatrix}$$

**A.A.A.B.B.B//MatrixForm**

$$\begin{pmatrix} 1 & 3 & 6 & 10 \\ 3 & 10 & 21 & 36 \\ 6 & 21 & 46 & 81 \\ 10 & 36 & 81 & 146 \end{pmatrix}$$

## d

**A1=Inverse[A]; A1//MatrixForm**

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

**B1=Inverse[B]; B1//MatrixForm**

$$\begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

**A1.B1//MatrixForm**

$$\begin{pmatrix} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{pmatrix}$$

**B1.A1//MatrixForm**

$$\begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

### e

**(B1.A.B.A.B1//MatrixForm)**

$$\begin{pmatrix} -3 & 0 & 1 & 1 \\ -2 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 10 & -1 & -2 & -3 \end{pmatrix}$$

### f

**A+B//MatrixForm**

$$\begin{pmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{pmatrix}$$

**Inverse[B1-A1].(A+B)//MatrixForm**

$$\begin{pmatrix} 2 & 3 & 2 & 3 \\ -2 & -1 & -1 & -1 \\ 1 & 1 & 1 & 2 \\ -3 & -2 & -3 & -2 \end{pmatrix}$$

---

## 4

**Remove["Global`*"]**

### a

### i

**solv=Solve[z^3==1,{z}]//Flatten**

$$\{z \to 1,\ z \to -(-1)^{1/3},\ z \to (-1)^{2/3}\}$$

```
z1=z/.solv[[1]]
```

```
1
```

```
zz[1]=z1//N
```

```
1.
```

```
z2=z/.solv[[2]]
```

```
-(-1)^(1/3)
```

```
zz[2]=z2//N
```

```
-0.5-0.866025 i
```

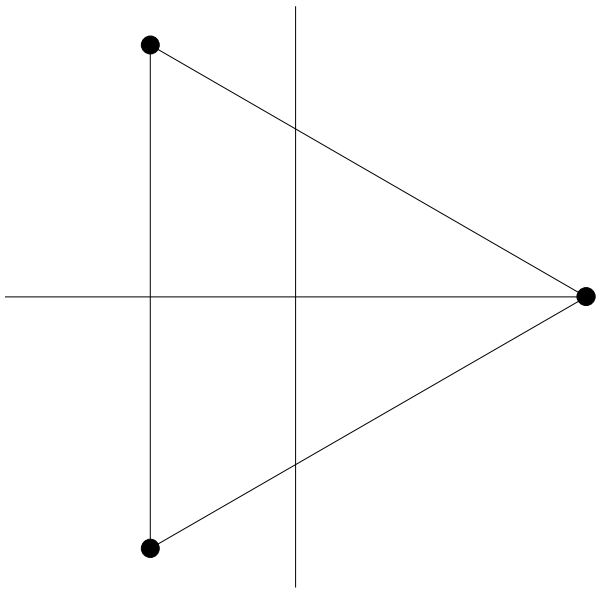
```
z3=z/.solv[[3]]
```

```
(-1)^(2/3)
```

```
zz[3]=z3//N
```

```
-0.5+0.866025 i
```

```
p[k_]:={Re[zz[k]],Im[zz[k]]}
s1=Show[Graphics[{PointSize[0.03],Point[p[1]],Point[p[2]],Point[p[3]],Line[{p[1],p[
2],p[3],p[1]}],Line[{{-1,0},{1,0}}],Line[{{0,-1},{0,1}}]}],AspectRatio->Automatic];
```



**ii**

```
w1=z1+k
```

```
1+k
```

```
w2=z2+k
```

```
-(-1)^(1/3)+k
```

**w3=z3+k**

$(-1)^{2/3} + k$

**(z-w1)(z-w2)(z-w3)//Expand//Simplify//Expand**

$-1 - k^3 + 3\,k^2\,z - 3\,k\,z^2 + z^3$

### iii

**w1=z1+2**

3

**w2=z2+2**

$2 - (-1)^{1/3}$

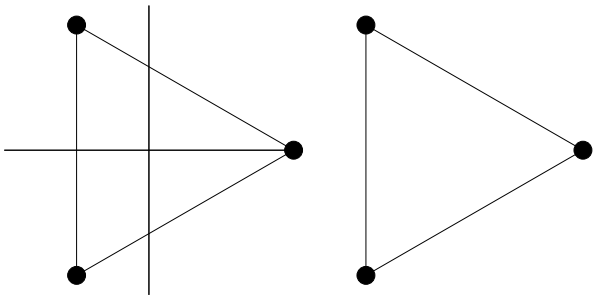
**w3=z3+2**

$2 + (-1)^{2/3}$

**(z-w1)(z-w2)(z-w3)//Expand//Simplify//Expand**

$-9 + 12\,z - 6\,z^2 + z^3$

```
ww[1]=w1//N; ww[2]=w2//N; ww[3]=w3//N;
p[k_]:={Re[ww[k]],Im[ww[k]]}
s2=Show[s1,Graphics[{PointSize[0.03],Point[p[1]],Point[p[2]],Point[p[3]],Line[{p[1]
,p[2],p[3],p[1]}],Line[{{-1,0},{1,0}}],Line[{{0,-1},{0,1}}]}],AspectRatio->Automati
c];
```



### iv

**w1=z1+2+4I**

$3 + 4\,i$

**w2=z2+2+4I**

$(2 + 4\,i) - (-1)^{1/3}$

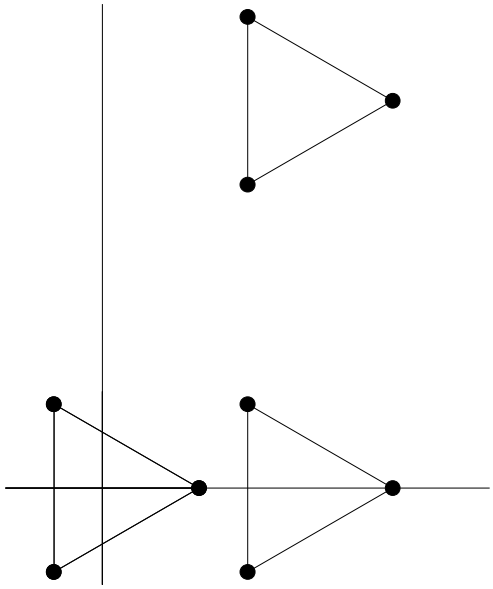
**w3=z3+2+4I**

$(2 + 4\,i) + (-1)^{2/3}$

**(z-w1)(z-w2)(z-w3)//Expand//Simplify//Expand**

$(87 + 16\,i) - (36 - 48\,i)\,z - (6 + 12\,i)\,z^2 + z^3$

```
www[1]=w1//N; www[2]=w2//N; www[3]=w3//N;
p[k_]:={Re[www[k]],Im[www[k]]}
s3=Show[Graphics[{PointSize[0.03],Point[p[1]],Point[p[2]],Point[p[3]],Line[{p[1],p[
2],p[3],p[1]}],Line[{{-1,0},{4,0}}],Line[{{0,-1},{0,5}}]}],s1,s2,AspectRatio->Autom
atic];
```



**v**

Komplex -- reell

## b

```
Remove["Global`*"]
```

**i**

```
q[x_]:=(4x^4+3x^3+2x^2+x)/((x-1)(x^2+1));
q[x]
```

$$\frac{x + 2\,x^2 + 3\,x^3 + 4\,x^4}{(-1 + x)\,(1 + x^2)}$$

```
Apart[q[x]]
```

$$7 + \frac{5}{-1 + x} + 4\,x - \frac{2}{1 + x^2}$$

**ii**

```
(x-1)(x^2+1)//Expand
```

$$-1 + x - x^2 + x^3$$

```
solv=Solve[(x-1)(x^2+1)==0,{x}]//Flatten
```

$\{x \to -i,\ x \to i,\ x \to 1\}$

```
w[1]=x/.solv[[1]]
```

$-i$

```
w[2]=x/.solv[[2]]
```

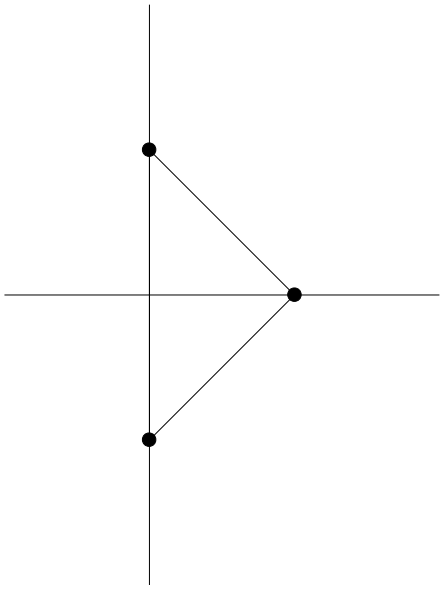
$i$

```
w[3]=x/.solv[[3]]
```

1

```
p[k_]:={Re[w[k]],Im[w[k]]}
```

```
s4=Show[Graphics[{PointSize[0.03],Point[p[1]],Point[p[2]],Point[p[3]],Line[{p[1],p[
2],p[3],p[1]}],Line[{{-1,0},{2,0}}],Line[{{0,-2},{0,2}}]}],AspectRatio->Automatic];
```



**iii**

```
Flaeche[v1_,v2_]:=v1[[1]] v2[[2]] - v1[[2]] v2[[1]]
```

```
A1 = Flaeche[p[3]-p[1],p[2]-p[1]]/2
```

1

```
Umfang1 = Norm[p[3]-p[2]]+Norm[p[2]-p[1]]+Norm[p[1]-p[3]]
```

$2 + 2\sqrt{2}$

```
N[%]
```

4.82843

**A1/Umfang1**

$$\frac{1}{2+2\sqrt{2}}$$

**N[%]**

0.207107

**iv**

```
pp[k_]:={Re[1/w[k]],Im[1/w[k]]}
```

**A2 = Flaeche[pp[3]-pp[1],pp[2]-pp[1]]/2**

−1

**N[%]**

−1.

**Umfang2 = Norm[pp[3]-pp[2]]+Norm[pp[2]-pp[1]]+Norm[pp[1]-pp[3]//Simplify]**

$2+2\sqrt{2}$

**N[%]**

4.82843

**A2/Umfang2**

$$-\frac{1}{2+2\sqrt{2}}$$

**N[%]**

−0.207107

**v**

**ww[1]=w[2]+2**

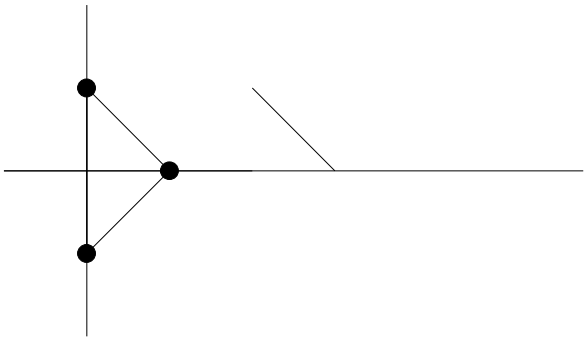
$2+i$

**ww[2]=w[3]+2**

3

```
pw[1]={Re[ww[1]],Im[ww[1]]};
pw[2]={Re[ww[2]],Im[ww[2]]};
```

```
s5=Show[s4,Graphics[{PointSize[0.03],Line[{pw[1],pw[2]}],Line[{{-1,0},{6,0}}],Line[
{{0,-1},{0,1}}]}],AspectRatio->Automatic];
```



```
winkel=-(pw[1]-pw[2]).{1,0}/Norm[pw[1]-pw[2]]
```

$$\frac{1}{\sqrt{2}}$$

```
ArcCos[winkel]
```

$$\frac{\pi}{4}$$

```
ArcCos[winkel]/Degree//N
```

```
45.
```

---

## 5

Das Oktaeder ist belanglos. Es dient nur der Einkleidung der Aufgabe, kann also überlesen werden.

Die weitere Lösung ist dem Leser überlassen.