Lösungen

1

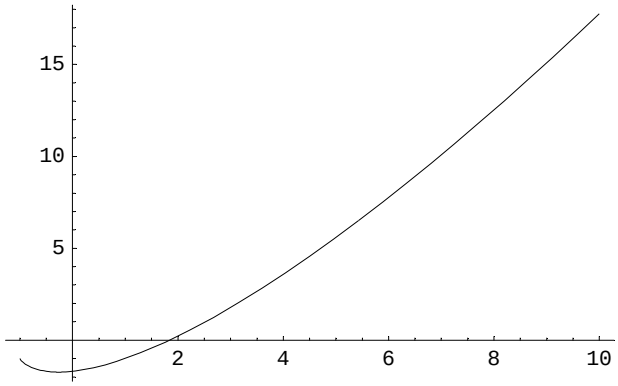
```
Remove["Global`*"]
```

```
f[x_] := (x + 1) Log[(x + 1) / 2] - 1
```

a

i

```
Plot[f[x], {x, -1, 10}];
```



ii

```
FindRoot[f[x] == 0, {x, 2}]
```

```
{x → 1.84306}
```

iii

```
f'[x]
```

$$1 + \text{Log}\left[\frac{1 + x}{2}\right]$$

```
u = f'[x] /. x → 0
```

```
1 - Log[2]
```

```
N[%]
```

```
0.306853
```

```
ArcTan[u]
```

```
ArcTan[1 - Log[2]]
```

```
N[%]
```

```
0.297732
```

```
ArcTan[1 - Log[2]] 360 / (2 Pi)
```

$$\frac{180\,\text{ArcTan}[1-\text{Log}[2]]}{\pi}$$

```
N[%]
```

```
17.0588
```

iv

```
Solve[f'[x] == 0, {x}]
```

$$\left\{\left\{x \to \frac{2-e}{e}\right\}\right\}$$

```
N[%]
```

```
{{x → -0.264241}}
```

b

```
Remove["Global`*"]
```

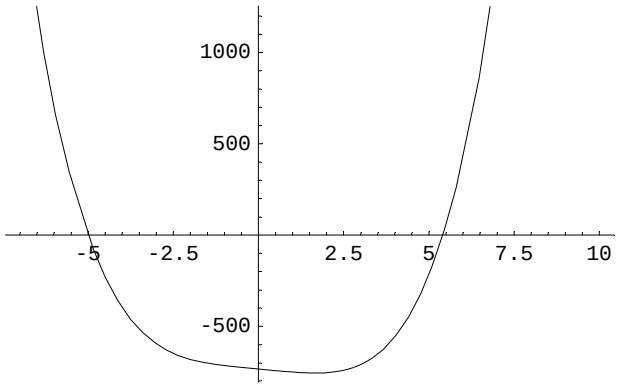
```
h[t_] := Integrate[(4 x^3 - 2 x^2 + t x - 5), {x, -5, t}] // Evaluate
```

```
h[t]
```

$$-\frac{2200}{3}-\frac{35\,t}{2}-\frac{t^3}{6}+t^4$$

i

```
Plot[h[t], {t, -7, 10}];
```



ii

```
Solve[h[t] == 0, {t}]
```

$$\left\{\{t \to -5\}, \left\{t \to \frac{1}{18}\left(31 - \frac{1829}{\left(327736 + 27\sqrt{155732965}\right)^{1/3}} + \left(327736 + 27\sqrt{155732965}\right)^{1/3}\right)\right\},\right.$$
$$\left\{t \to \frac{31}{18} + \frac{1829\left(1 + i\sqrt{3}\right)}{36\left(327736 + 27\sqrt{155732965}\right)^{1/3}} - \frac{1}{36}\left(1 - i\sqrt{3}\right)\left(327736 + 27\sqrt{155732965}\right)^{1/3}\right\},$$
$$\left.\left\{t \to \frac{31}{18} + \frac{1829\left(1 - i\sqrt{3}\right)}{36\left(327736 + 27\sqrt{155732965}\right)^{1/3}} - \frac{1}{36}\left(1 + i\sqrt{3}\right)\left(327736 + 27\sqrt{155732965}\right)^{1/3}\right\}\right\}$$

```
Solve[h[t] == 0, {t}] // N
```

```
{{t → -5.}, {t → 5.4063}, {t → -0.119816 + 5.20716 i}, {t → -0.119816 - 5.20716 i}}
```

iii

```
h'[t]
```

$$-\frac{35}{2} - \frac{t^2}{2} + 4t^3$$

```
u = h'[t] /. t → 0
```

$$-\frac{35}{2}$$

```
N[%]
```

```
-17.5
```

```
ArcTan[u]
```

$$-\text{ArcTan}\left[\frac{35}{2}\right]$$

```
N[%]
```

```
-1.51372
```

```
ArcTan[u] 360 / (2 Pi)
```

$$-\frac{180\,\text{ArcTan}\left[\frac{35}{2}\right]}{\pi}$$

```
N[%]
```

```
-86.7295
```

iv

```
Solve[h'[t] == 0, {t}]
```

$$\left\{\left\{t \to \frac{1}{24} + \frac{1}{24}\left(30241 - 24\sqrt{1587705}\right)^{1/3} + \frac{1}{24}\left(30241 + 24\sqrt{1587705}\right)^{1/3}\right\},\right.$$
$$\left\{t \to \frac{1}{24} - \frac{1}{48}\left(1 + i\sqrt{3}\right)\left(30241 - 24\sqrt{1587705}\right)^{1/3} -\right.$$
$$\left.\frac{1}{48}\left(1 - i\sqrt{3}\right)\left(30241 + 24\sqrt{1587705}\right)^{1/3}\right\}, \left\{t \to\right.$$
$$\left.\left.\frac{1}{24} - \frac{1}{48}\left(1 - i\sqrt{3}\right)\left(30241 - 24\sqrt{1587705}\right)^{1/3} - \frac{1}{48}\left(1 + i\sqrt{3}\right)\left(30241 + 24\sqrt{1587705}\right)^{1/3}\right\}\right\}$$

```
N[%]
```

$$\{\{t \to 1.67828\}, \{t \to -0.77664 + 1.41551\,i\}, \{t \to -0.77664 - 1.41551\,i\}\}$$

2

```
Remove["Global`*"]
```

```
f[x_, y_, k_] := E^k Cos[x / Pi] + Cos[y / Pi];
f[x, y, k]
```

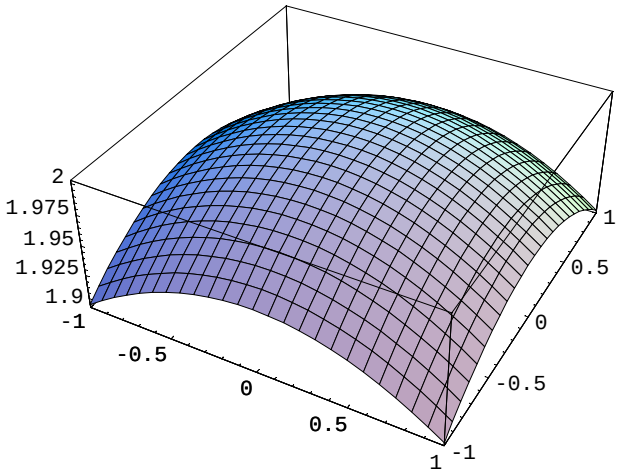
$$e^k \cos\left[\frac{x}{\pi}\right] + \cos\left[\frac{y}{\pi}\right]$$

```
f[x, y, 0]
```

$$\cos\left[\frac{x}{\pi}\right] + \cos\left[\frac{y}{\pi}\right]$$

a

```
Plot3D[f[x, y, 0], {x, -1, 1}, {y, -1, 1}];
```



b

```
V[k_] := Integrate[f[x, y, k], {x, -1, 1}, {y, -1, 1}];
V[k]
```

$$4\,(1+e^{k})\,\pi\,\text{Sin}\left[\frac{1}{\pi}\right]$$

```
N[%]
```

$$3.93279\,(1.+2.71828^{k})$$

```
%% // Expand
```

$$4\,\pi\,\text{Sin}\left[\frac{1}{\pi}\right]+4\,e^{k}\,\pi\,\text{Sin}\left[\frac{1}{\pi}\right]$$

```
N[%]
```

$$3.93279+3.93279\,2.71828^{k}$$

c

```
Solve[V[k] == 1, {k}]
```

$$\left\{\left\{k\to i\,\pi+\text{Log}\left[\frac{\text{Csc}\left[\frac{1}{\pi}\right]\,\left(-1+4\,\pi\,\text{Sin}\left[\frac{1}{\pi}\right]\right)}{4\,\pi}\right]\right\}\right\}$$

```
N[%]
```

$$\{\{k\to -0.293395+3.14159\,i\}\}$$

Keine reelle Lösung

d

```
g[x_] := 2 x - 1 / 2
```

```
f[x, g[x], k]
```

$$e^{k}\,\text{Cos}\left[\frac{x}{\pi}\right]+\text{Cos}\left[\frac{-\frac{1}{2}+2\,x}{\pi}\right]$$

```
D[f[x, 2 x - 1 / 2, k], x]
```

$$-\frac{e^{k}\,\text{Sin}\left[\frac{x}{\pi}\right]}{\pi}-\frac{2\,\text{Sin}\left[\frac{-\frac{1}{2}+2\,x}{\pi}\right]}{\pi}$$

```
D[f[x, 2 x - 1 / 2, k], x] / E^k
```

$$e^{-k}\left(-\frac{e^{k}\,\text{Sin}\left[\frac{x}{\pi}\right]}{\pi}-\frac{2\,\text{Sin}\left[\frac{-\frac{1}{2}+2\,x}{\pi}\right]}{\pi}\right)$$

```
N[%]
```

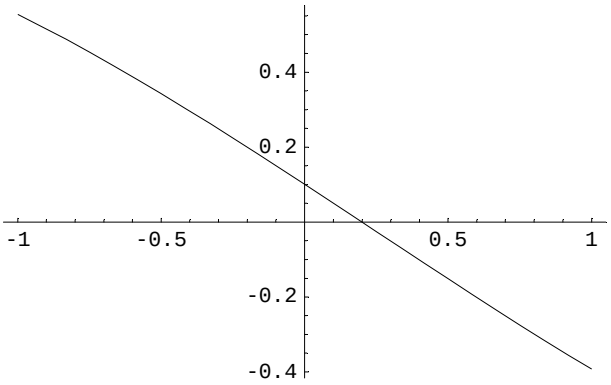
$$2.71828^{-1.\,k}\,(-0.31831\,2.71828^{k}\,\text{Sin}[0.31831\,x]-0.63662\,\text{Sin}[0.31831\,(-0.5+2.\,x)])$$

```
Solve[Evaluate[D[f[x, 2 x - 1 / 2, k], x] / E^k == 0], {x}] // N // Chop;
```

Lösung allgemein sehr lange, nicht übersichtlich

Lösung numerisch möglich für fixes k:

```
Plot[Evaluate[D[f[x, 2 x - 1 / 2, 0], x]], {x, -1, 1}];
```




```
solv = FindRoot[Evaluate[D[f[x, 2 x - 1 / 2, 0], x]], {x, 0.1}]
```

$$\{x \to 0.20002\}$$

```
x1 = x /. solv
```

```
0.20002
```

```
g[x1]
```

```
-0.0999594
```

```
f[x1, g[x1], 1]
```

```
3.71227
```

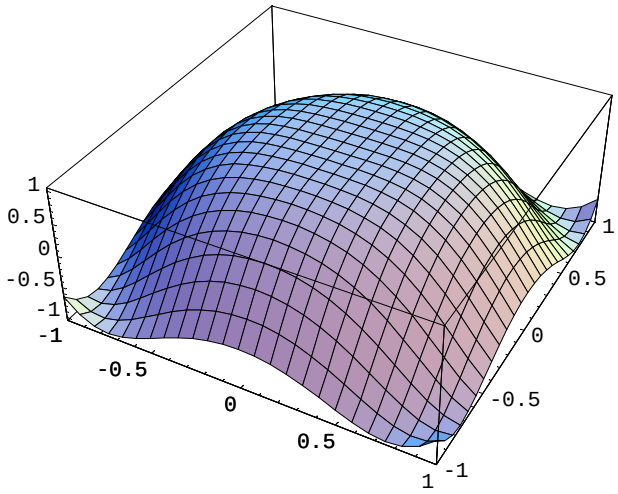
e

```
Remove["Global`*"]
```

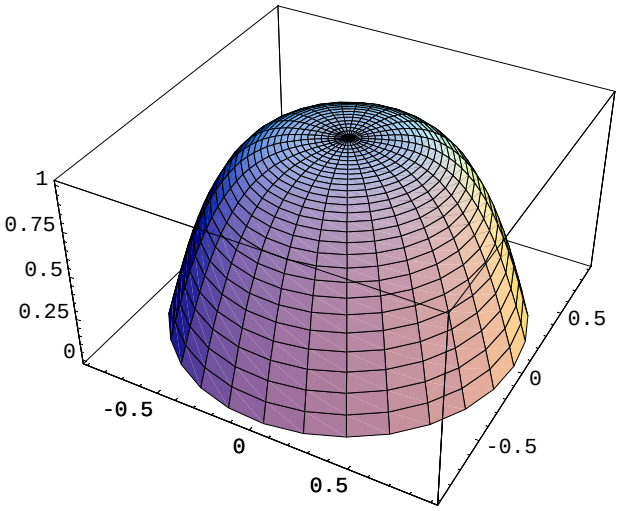
```
h[x_, y_] := Cos[(x + y) ^2 + (x - y) ^2];
h[x, y] // ExpandAll
```

$$\text{Cos}[2\,x^2+2\,y^2]$$

```
Plot3D[h[x, y], {x, -1, 1}, {y, -1, 1}];
```




```
ParametricPlot3D[{r Cos[φ], r Sin[φ], Cos[2 r^2]}, {r, 0, Sqrt[Pi / 4]}, {φ, 0, 2 Pi}];
```




```
rm = Sqrt[Pi / 4]
```

$$\frac{\sqrt{\pi}}{2}$$

```
Cos[2 r^2] /. r → rm
```

```
0
```

```
rm // N
```

```
0.886227
```

```
V1 =
 Integrate[Integrate[h[x, y], {y, -Sqrt[rm^2 - x^2], Sqrt[rm^2 - x^2]}], {x, -rm, rm}]
```

$$\int_{-\frac{\sqrt{\pi}}{2}}^{\frac{\sqrt{\pi}}{2}} \sqrt{\pi}\left(\text{Cos}[2\,x^2]\,\text{FresnelC}\left[\sqrt{1-\frac{4\,x^2}{\pi}}\,\right]-\text{FresnelS}\left[\sqrt{1-\frac{4\,x^2}{\pi}}\,\right]\text{Sin}[2\,x^2]\right)\,\mathrm{d}x$$

```
NIntegrate[NIntegrate[h[x, y], {y, -Sqrt[rm^2 - x^2], Sqrt[rm^2 - x^2]}], {x, -rm, rm}]
```

```
1.5708
```

```
NIntegrate[Integrate[h[x, y], {y, -Sqrt[rm^2 - x^2], Sqrt[rm^2 - x^2]}], {x, -rm, rm}]
```

```
1.5708
```

3

```
Remove["Global`*"]
```

```
A1 = {0, 0, 0}; A2 = {0, 0, 1 / 2}; OO = A1;
B1 = {2, 0, 0}; B2 = {2, 0, 2 / 3};
C1 = {3, 4, 0}; C2 = {3, 4, 1};
D1 = {0, 3, 0}; D2[z_] := {0, 3, z};
D2[z]
```

```
{0, 3, z}
```

b

```
Φ[λ_, μ_] := (A2 - OO) + λ (B2 - A2) + μ (C2 - A2);
Φ[λ, μ]
```

$$\left\{2\,\lambda+3\,\mu,\ 4\,\mu,\ \frac{1}{2}+\frac{\lambda}{6}+\frac{\mu}{2}\right\}$$

```
gD[t_] := (D1 - OO) + t (D2[1] - D1);
gD[t]
```

```
{0, 3, t}
```

```
solv = Solve[Φ[λ, μ] == gD[t], {t}] // Flatten
```

$$\left\{t\to\frac{11}{16}\right\}$$

```
N[%]
```

```
{t → 0.6875}
```

```
D2 = gD[t] /. solv
```

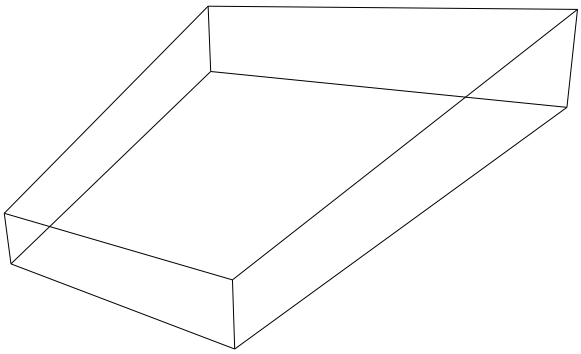
$$\left\{0,\ 3,\ \frac{11}{16}\right\}$$

```
N[%]
```

```
{0., 3., 0.6875}
```

a

```
Show[Graphics3D[Line[{A1, B1, C1, D1, A1, A2, B2, B1, C1, C2, B2, C2, D2, D1, D2, A2}]],
  Boxed -> False];
```



c

```
Inhalt = Norm[Cross[B2 - A2, C2 - A2]] / 2 + Norm[Cross[D2 - A2, C2 - A2]] / 2
```

$$\frac{17\sqrt{2329}}{96}$$

```
N[%]
```

```
8.54599
```

d1

```
A3 = A2;
B3 = {2, 0, 1 / 2}; C3 = {3, 4, 1 / 2}; D3 = {0, 3, 1 / 2};

Abs[Det[{B1 - A1, C1 - A1, A2 - A1}]] / 2 +
 Abs[Det[{B3 - A2, C3 - A2, C2 - A2}]] / 6 + Abs[Det[{B3 - A2, B2 - A2, C2 - A2}]] / 6 +

 Abs[Det[{D1 - A1, C1 - A1, A2 - A1}]] / 2 +
 Abs[Det[{D3 - A2, C3 - A2, C2 - A2}]] / 6 + Abs[Det[{D3 - A2, D2 - A2, C2 - A2}]] / 6
```

$$\frac{1777}{288}$$

```
N[%]
```

```
6.17014
```

d2

```
(Abs[Det[{B1 - A1, C1 - A1, A2 - A1}]] +
   Abs[Det[{C1 - B1, B2 - B1, A2 - B1}]] +
   Abs[Det[{A2 - C1, B2 - C1, C2 - C1}]] +

   Abs[Det[{D1 - A1, C1 - A1, A2 - A1}]] +
   Abs[Det[{C1 - D1, D2 - D1, A2 - D1}]] +
   Abs[Det[{A2 - C1, D2 - C1, C2 - C1}]]
 ) / 6
```

$$\frac{1777}{288}$$

```
N[%]
```

```
6.17014
```

4

```
Remove["Global`*"]
```

a1

```
X = Transpose[{{11, 3}, {-3, 11}}];
X // MatrixForm
```

$$\begin{pmatrix} 11 & -3 \\ 3 & 11 \end{pmatrix}$$

```
mD = {{1, 0}, {0, -1}};
mD // MatrixForm
```

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

```
Sp = X.mD.Inverse[X];
Sp // MatrixForm
```

$$\begin{pmatrix} \frac{56}{65} & \frac{33}{65} \\ \frac{33}{65} & -\frac{56}{65} \end{pmatrix}$$

```
% // N // MatrixForm
```

$$\begin{pmatrix} 0.861538 & 0.507692 \\ 0.507692 & -0.861538 \end{pmatrix}$$

a2

```
α = ArcTan[3 / 11];
α // N
```

```
0.266252
```

```
α / Degree // N
```

```
15.2551
```

```
Sp2 = {{Cos[2 α], Sin[2 α]}, {Sin[2 α], -Cos[2 α]}};
Sp2 // N // MatrixForm
```

$$\begin{pmatrix} 0.861538 & 0.507692 \\ 0.507692 & -0.861538 \end{pmatrix}$$

b

```
P1 = {6, -3}; P2 = {7, 2};
```

```
Q1 = Sp.P1
```

$$\left\{\frac{237}{65}, \frac{366}{65}\right\}$$

```
% // N
```

```
{3.64615, 5.63077}
```

```
Q2 = Sp.P2
```

$$\left\{\frac{458}{65}, \frac{119}{65}\right\}$$

```
N[%]
```

```
{7.04615, 1.83077}
```

c

```
Norm[Cross[{6, -3, 0}, {237/65, 366/65, 0}]] / 2
```

$$\frac{2907}{130}$$

```
N[%]
```

```
22.3615
```

d

```
Eigensystem[Sp]
```

$$\left\{\{-1, 1\}, \left\{\left\{-\frac{3}{11}, 1\right\}, \left\{\frac{11}{3}, 1\right\}\right\}\right\}$$

```
% // N
```

```
{{-1., 1.}, {{-0.272727, 1.}, {3.66667, 1.}}}
```

```
Eigensystem[Sp2] // N
```

```
{{-1., 1.}, {{-0.272727, 1.}, {3.66667, 1.}}}
```

Richtungsvektoren der Gerade und senkrechte Gerade geben Eingenvektoren. Eigenwerte 1 und -1.

5

```
Remove["Global`*"]
```

```
A3 = {{1, 0, 0}, {1, -1, 0}, {1, -1, 1}}; A3 // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 1 & -1 & 1 \end{pmatrix}$$

```
B3 = {{-1, 1, -1}, {0, 1, -1}, {0, 0, -1}}; B3 // MatrixForm
```

$$\begin{pmatrix} -1 & 1 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{pmatrix}$$

```
A4={{1,0,0,0},{1,-1,0,0},{1,-1,1,0},{1,-1,1,-1}}; A4//MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 1 & -1 & 1 & 0 \\ 1 & -1 & 1 & -1 \end{pmatrix}$$

```
B4={{-1,1,-1,1},{0,1,-1,1},{0,0,-1,1},{0,0,0,1}}; B4//MatrixForm
```

$$\begin{pmatrix} -1 & 1 & -1 & 1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

a

```
u1=A3.A3;u1//MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

```
v1=B3.B3; v1//MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
u1 == Transpose[v1]
```

```
True
```

```
o1 = A4.A4; o1 // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

```
q1 = B4.B4; q1 // MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
o1 == Transpose[q1]
```

```
True
```

Vermutung: Ak.Ak = Transpose[Bk.Bk].

b

```
u2=u1.A3;u2//MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 2 & -1 & 1 \end{pmatrix}$$

```
v2=v1.B3; v2//MatrixForm
```

$$\begin{pmatrix} -1 & 1 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{pmatrix}$$

```
c2=A3.B3;c2//MatrixForm
```

$$\begin{pmatrix} -1 & 1 & -1 \\ -1 & 0 & 0 \\ -1 & 0 & -1 \end{pmatrix}$$

```
u2 == Transpose[v2]
```

```
False
```

Gesetz nicht einfach offensichtlich.

c

```
A3.B3//MatrixForm
```

$$\begin{pmatrix} -1 & 1 & -1 \\ -1 & 0 & 0 \\ -1 & 0 & -1 \end{pmatrix}$$

```
B3.A3//MatrixForm
```

$$\begin{pmatrix} -1 & 0 & -1 \\ 0 & 0 & -1 \\ -1 & 1 & -1 \end{pmatrix}$$

Scheint z.B. gespiegelt am Mittelpunkt.

```
A3.A3.B3.B3//MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{pmatrix}$$

```
B3.B3.A3.A3 // MatrixForm
```

$$\begin{pmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

Scheint z.B.gespiegelt an Senkrechte auf Hauptdiagonale oder an Mittelpunkt.

```
A3.A3.A3.B3.B3.B3//MatrixForm
```

$$\begin{pmatrix} -1 & 1 & -2 \\ -1 & 0 & -1 \\ -2 & 1 & -4 \end{pmatrix}$$

```
B3.B3.B3.A3.A3.A3 // MatrixForm
```

$$\begin{pmatrix} -4 & 1 & -2 \\ -1 & 0 & -1 \\ -2 & 1 & -1 \end{pmatrix}$$

Scheint z.B.gespiegelt an Mittelpunkt.

d

```
AI3=Inverse[A3]; AI3//MatrixForm
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$$

```
BI3=Inverse[B3]; BI3//MatrixForm
```

$$\begin{pmatrix} -1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{pmatrix}$$

```
AI3.BI3//MatrixForm
```

$$\begin{pmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}$$

```
BI3.AI3//MatrixForm
```

$$\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{pmatrix}$$

```
AI3.BI3 == BI3.AI3
```

```
False
```

```
AI3.BI3 == Transpose[BI3.AI3]
```

```
False
```

e

```
(X=BI3.A3.B3.A3.BI3)//MatrixForm
```

$$\begin{pmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ -2 & 1 & 0 \end{pmatrix}$$

f

```
A3+B3//MatrixForm
```

$$\begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$$

```
Inverse[BI3 - AI3] // MatrixForm
```

$$\begin{pmatrix} -\frac{3}{4} & \frac{1}{2} & -\frac{1}{4} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{4} & \frac{1}{2} & -\frac{3}{4} \end{pmatrix}$$

```
Inverse[BI3-AI3].(A3+B3)//MatrixForm
```

$$\begin{pmatrix} \frac{1}{4} & -\frac{1}{2} & \frac{1}{4} \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{1}{4} & \frac{1}{2} & -\frac{1}{4} \end{pmatrix}$$

```
N[%] // MatrixForm
```

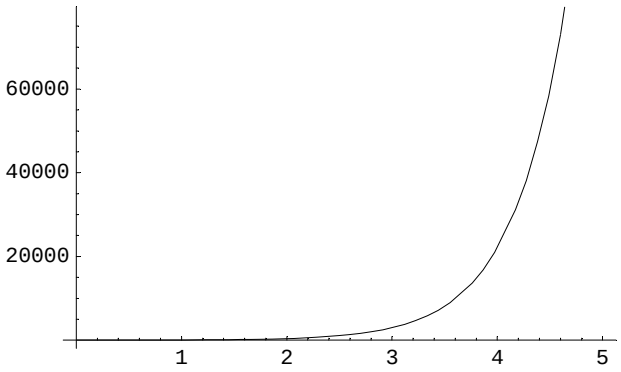
$$\begin{pmatrix} 0.25 & -0.5 & 0.25 \\ 0.5 & 0. & -0.5 \\ -0.25 & 0.5 & -0.25 \end{pmatrix}$$

6

```
Remove["Global`*"]
```

a

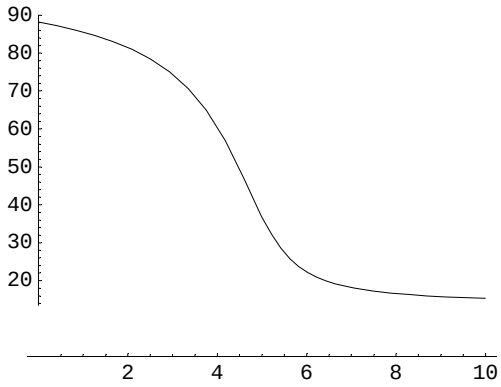
```
Plot[E^(2 x + 2), {x, 0, 5}];
```



Steigt immer, kein logistisches Wachstum.

b

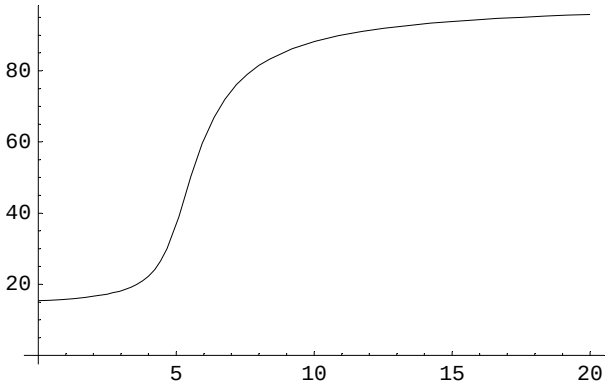
```
Plot[100 E^((-2 ArcTan[x - 5] / Pi) - 1), {x, 0, 10}, AxesOrigin → {0, 0}];
```



Fällt: So nicht geeignet für logistisches Wachstum. Nach Spiegelung und Achsenanpassung geeignet für logistisches Wachstum.

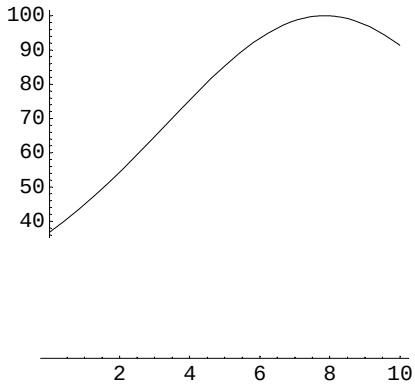
Spiegelung:

```
Plot[100 E^((2 ArcTan[x - 5] / Pi) - 1), {x, 0, 20}, AxesOrigin → {0, 0}];
```

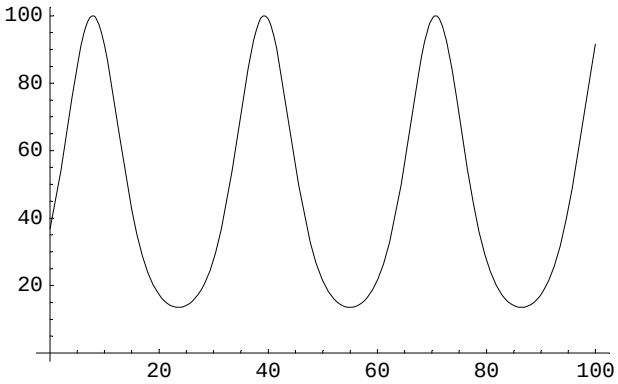


c

```
Plot[100 E^(Sin[x / 5] - 1), {x, 0, 10}, AxesOrigin → {0, 0}];
```




```
Plot[100 E^(Sin[x / 5] - 1), {x, 0, 100}, AxesOrigin → {0, 0}];
```



Oszilliert. Nicht geeignet für logistisches Wachstum.