Lösungen

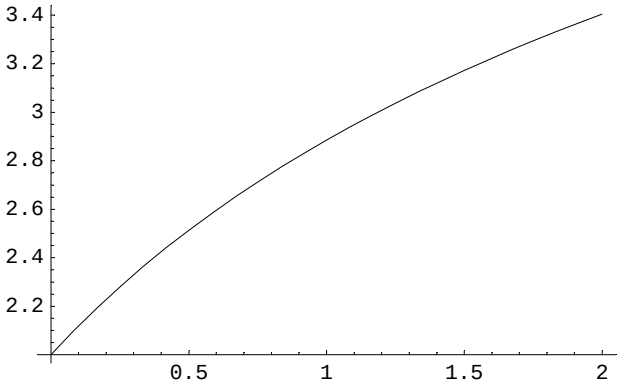
(Entwurf - Darstellung nicht überarbeitet)

1

a

```
f[x_] := Log[2 x + E] + (2 x + 2) / (x + 2)
```

```
Plot[f[x], {x, 0, 2}];
```




```
ArcTan[(f[2] - f[0]) / (2 - 0)]
```

$$\text{ArcTan}\left[\frac{1}{2}\left(-\frac{1}{2}+\text{Log}[4+e]\right)\right]$$

```
b1 = (f[2] - f[0]) / (2 - 0) // N
```

```
0.702416
```

```
a1 = ArcTan[b1] // N
```

```
0.612346
```

```
a1 / Degree
```

```
35.0848
```

```
ArcTan[(f'[x] /. x → 1)]
```

$$\text{ArcTan}\left[\frac{2}{9}+\frac{2}{2+e}\right]$$

```
b2 = (f'[x] /. x → 1) // N
```

```
0.646105
```

```
a2 = ArcTan[b2] // N
```

```
0.573632
```

```
a2 / Degree
```

```
32.8667
```

```
(b1 - b2) / b1 100
```

```
8.01674
```

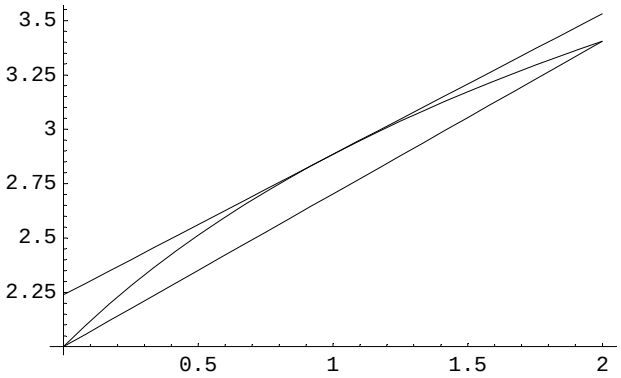
```
(a1 - a2) / a1 100
```

```
6.32213
```

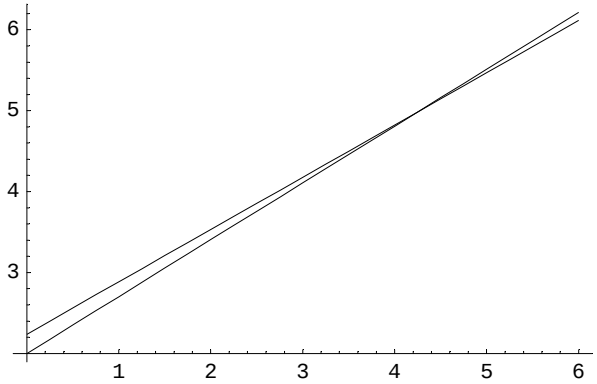
```
g[x_] := f[1] + Evaluate[(f'[v] /. v → 1)] (x - 1)
```

```
s[x_] := f[0] + (f[2] - f[0]) / (2 - 0) x
```

```
Plot[{f[x], g[x], s[x]}, {x, 0, 2}];
```




```
Plot[{g[x], s[x]}, {x, 0, 6}];
```




```
Solve[g[x] == s[x], {x}]
```

$$\left\{\left\{x \to \frac{4\ (-34 - 8\ e + 18\ \text{Log}[2 + e] + 9\ e\ \text{Log}[2 + e])}{-106 - 17\ e + 36\ \text{Log}[4 + e] + 18\ e\ \text{Log}[4 + e]}\right\}\right\}$$

```
Solve[g[x] == s[x], {x}] // N
```

```
{{x → 4.23848}}
```

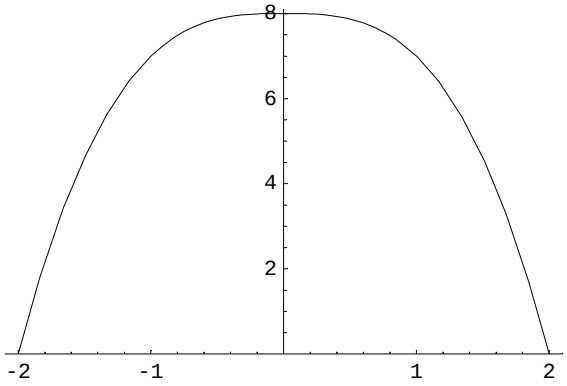
```
g[4.238482789359839`]
```

```
4.97718
```

b

```
h[x_] := -Sqrt[(x^3)^2] + 8
```

```
Plot[h[x], {x, -2, 2}, PlotRange → {0, 8.1}];
```




```
h[2]
```

```
0
```

```
Integrate[h[x], {x, -2, 2}]
```

```
24
```

```
rechteck[x_] := 2 h[x] x;
rechteck[x]
```

$$2\,x\left(8-\sqrt{x^6}\right)$$

```
rechteck'[x]
```

$$-6\sqrt{x^6}+2\left(8-\sqrt{x^6}\right)$$

```
Solve[rechteck'[x] == 0, {x}]
```

$$\{\{x\to-(-2)^{1/3}\},\{x\to(-2)^{1/3}\},\{x\to-2^{1/3}\},\{x\to 2^{1/3}\},\{x\to-(-1)^{2/3}\,2^{1/3}\},\{x\to(-1)^{2/3}\,2^{1/3}\}\}$$

```
Solve[rechteck'[x] == 0, {x}] // N
```

$$\{\{x\to-0.629961-1.09112\,i\},\{x\to 0.629961+1.09112\,i\},\{x\to-1.25992\},\{x\to 1.25992\},\{x\to 0.629961-1.09112\,i\},\{x\to-0.629961+1.09112\,i\}\}$$

```
rechteck[1.2599210498948732`]
```

```
15.1191
```

```
rechteck[1.2599210498948732`] / Integrate[h[x], {x, -2, 2}]
```

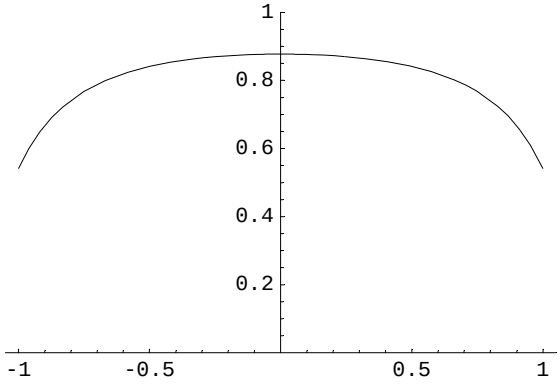
```
0.629961
```

2

```
f[x_] := Cos[1 / (2 - x^2)]; f[x]
```

$$\cos\left[\frac{1}{2-x^2}\right]$$

```
Plot[f[x], {x, -1, 1}, PlotRange → {0, 1}];
```




```
f'[x]
```

$$-\frac{2\,x\,\sin\left[\frac{1}{2-x^2}\right]}{(2-x^2)^2}$$

```
NIntegrate[Sqrt[1 + (Evaluate[f'[x]])^2], {x, -1, 1}]
```

2.21328

```
NIntegrate[2 f[x] Pi Sqrt[1 + (Evaluate[f'[x]])^2], {x, -1, 1}]
```

11.0016

```
NIntegrate[f[x]^2 Pi , {x, -1, 1}]
```

4.11864

```
Solve[Evaluate[NIntegrate[f[x]^2 Pi , {x, -1, 1}]] == r^2 Pi 2, {r}]
```

{{r → -0.809631}, {r → 0.809631}}

```
(0.8096308466898374`) / f[0] 100
```

92.2569

3

```
Remove["Global`*"]
```

```
x0Vec = {44.36, 36.85}
```

{44.36, 36.85}

```
x0 = x0Vec[[1]]
```

```
44.36
```

```
y0 = x0Vec[[2]]
```

```
36.85
```

```
A[λ_, φ_] := λ {{Cos[φ], -Sin[φ]}, {Sin[φ], Cos[φ]}}; A[λ, φ] // MatrixForm
```

$$\begin{pmatrix} \lambda \text{Cos}[\varphi] & -\lambda \text{Sin}[\varphi] \\ \lambda \text{Sin}[\varphi] & \lambda \text{Cos}[\varphi] \end{pmatrix}$$

```
uv[x_, y_, x0_, y0_, λ_, φ_] := A[λ, φ].{x - x0, y - y0}
```

```
uv[x, y, x0, y0, λ, φ]
```

```
{(-44.36 + x) λ Cos[φ] - (-36.85 + y) λ Sin[φ], (-36.85 + y) λ Cos[φ] + (-44.36 + x) λ Sin[φ]}
```

```
uv[x0, y0, x0, y0, λ, φ]
```

```
{0. λ Cos[φ] + 0. λ Sin[φ], 0. λ Cos[φ] + 0. λ Sin[φ]}
```

```
uv[x0, y0, x0, y0, λ, φ] // Chop
```

```
{0, 0}
```

```
{x1, y1} = {28.96, 43.92}
```

```
{28.96, 43.92}
```

```
{u1, v1} = {7.32, 12.88}
```

```
{7.32, 12.88}
```

```
Solve[uv[x1, y1, x0, y0, λ, φ] == {u1, v1}, {λ, φ}]
```

```
Solve::ifun : Inverse functions are being used by Solve, so some
   solutions may not be found; use Reduce for complete solution information. Mehr…
```

```
{{λ → -0.874266, φ → 1.48438}, {λ → 0.874266, φ → -1.65721}}
```

```
uv[x_, y_] := A[0.8742663363204899, -1.6572100129016138].{x - x0, y - y0}
```

```
uv[x, y] // Simplify
```

```
{-28.7493 - 0.0754546 x + 0.871004 y, 41.4182 - 0.871004 x - 0.0754546 y}
```

```
uv[44.36, 36.85]
```

```
{0., 0.}
```

```
uv[x1, y1]
```

```
{7.32, 12.88}
```

```
uv[10, 20]
```

```
{-12.0838, 31.1991}
```

```
uv[18.57, 24.24]
```

```
{-9.03739, 23.4147}
```

```
Solve[uv[x, y] == {28, 15}, {x, y}]
```

```
{{x → 24.5027, y → 67.2766}}
```

4

```
Remove["Global`*"]
```

```
a1 = {1, 0, 2};  a2 = {2, 1, 1}; a3 = {1, 2, 3};
mD = {{7, 0, 0}, {0, 14, 0}, {0, 0, -7}};
B = Transpose[{a1, a2, a3}].mD.Inverse[Transpose[{a1, a2, a3}]];
B // MatrixForm
```

$$\begin{pmatrix} 19 & -4 & -6 \\ 12 & -4 & -6 \\ 16 & -17 & -1 \end{pmatrix}$$

```
B.a1
```

```
{7, 0, 14}
```

```
B.a2
```

```
{28, 14, 14}
```

```
B.a3
```

```
{-7, -14, -21}
```

```
Det[{a1, a2, a3}] == 0
```

```
False
```

```
Eigensystem[B]
```

```
{{14, -7, 7}, {{2, 1, 1}, {1, 2, 3}, {1, 0, 2}}}
```

```
Eigensystem[Inverse[B]]
```

$$\left\{\left\{-\frac{1}{7}, \frac{1}{7}, \frac{1}{14}\right\}, \left\{\left\{\frac{1}{3}, \frac{2}{3}, 1\right\}, \left\{\frac{1}{2}, 0, 1\right\}, \{2, 1, 1\}\right\}\right\}$$

```
N[%]
```

```
{{-0.142857, 0.142857, 0.0714286},
 {{0.333333, 0.666667, 1.}, {0.5, 0., 1.}, {2., 1., 1.}}}
```

```
B.(v1 a1 + v2 a2 + v3 a3) // Simplify
```

```
{7 (v1 + 4 v2 - v3), 14 (v2 - v3), 7 (2 v1 + 2 v2 - 3 v3)}
```

```
a1 = {1, 0, 2};  a2 = {2, 1, 1}; a3 = {1, 2, 3};
pD = {{1, 0, 0}, {0, 1, 0}, {0, 0, 0}};
mC = Transpose[{a1, a2, a3}].pD.Inverse[Transpose[{a1, a2, a3}]];
mC // MatrixForm
```

$$\begin{pmatrix} \frac{9}{7} & -\frac{3}{7} & -\frac{1}{7} \\ \frac{4}{7} & \frac{1}{7} & -\frac{2}{7} \\ \frac{6}{7} & -\frac{9}{7} & \frac{4}{7} \end{pmatrix}$$

```
N[%] // MatrixForm
```

$$\begin{pmatrix} 1.28571 & -0.428571 & -0.142857 \\ 0.571429 & 0.142857 & -0.285714 \\ 0.857143 & -1.28571 & 0.571429 \end{pmatrix}$$

```
p1 = {1, 1, 1}; p2 = {4, 4, 4}; p3 = {5, 12, 20};

mC.p1
```

$$\left\{\frac{5}{7}, \frac{3}{7}, \frac{1}{7}\right\}$$

```
N[%]
```

```
{0.714286, 0.428571, 0.142857}
```

```
mC.p2
```

$$\left\{\frac{20}{7}, \frac{12}{7}, \frac{4}{7}\right\}$$

```
N[%]
```

```
{2.85714, 1.71429, 0.571429}
```

```
mC.p3
```

$$\left\{-\frac{11}{7}, -\frac{8}{7}, \frac{2}{7}\right\}$$

```
N[%]
```

```
{-1.57143, -1.14286, 0.285714}
```

```
Norm[Cross[p1 - p2, p3 - p2]] / Norm[p3 - p2]
```

$$13\sqrt{\frac{6}{107}}$$

```
u1 = N[%]
```

```
3.07841
```

```
Norm[Cross[mC.p1 - mC.p2, mC.p3 - mC.p2]] / Norm[mC.p3 - mC.p2]
```

$$\sqrt{\frac{6}{65}}$$

```
u2 = N[%]
```

```
0.303822
```

```
u2 / u1
```

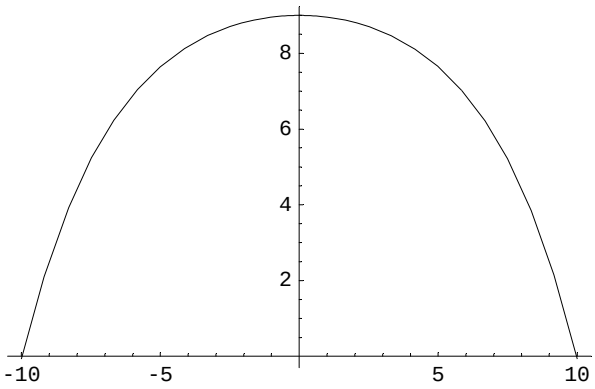
```
0.0986943
```

5

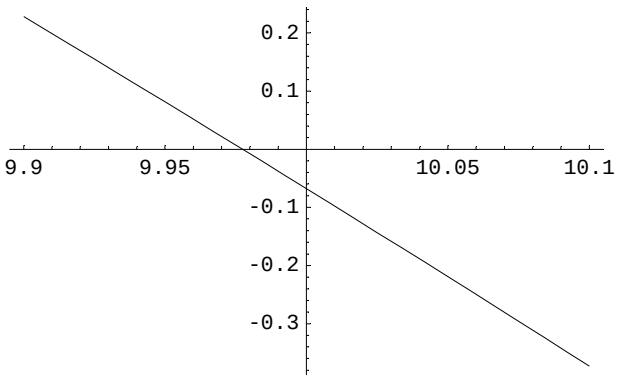
```
Remove["Global`*"]
```

```
q[x_] := 10 - (E^(3 x / 10) + E^(-3 x / 10)) / 2
```

```
Plot[q[x], {x, -10, 10}];
```




```
Plot[q[x], {x, 9.9, 10.1}];
```




```
fr = FindRoot[q[x] == 0, {x, 10}]
```

```
{x → 9.97741}
```

```
ns = x /. fr[[1]]
```

```
9.97741
```

```
FT = Integrate[q[x], {x, -ns, ns}]
```

```
133.216
```

```
VT = 2 * FT * 40000
```

$$1.06573 \times 10^{7}$$

```
VP = 230.3^2 146.6 / 3
```

$$2.59179 \times 10^6$$

```
Solve[VT == s VP, {s}]
```

$$\{\{s \to 4.11192\}\}$$

```
solv = Solve[VT == (500^3 - r^3) Pi 4 / 3, {r}] // Flatten
```

$$\{r \to -248.292 - 430.055\, i,\ r \to -248.292 + 430.055\, i,\ r \to 496.584\}$$

```
500 - r /. solv[[3]]
```

$$3.41559$$

```
f[x_] := Normal[Series[q[x], {x, 0, 6}]]
```

```
f[x]
```

$$9 - \frac{9\, x^2}{200} - \frac{27\, x^4}{80000} - \frac{81\, x^6}{80000000}$$

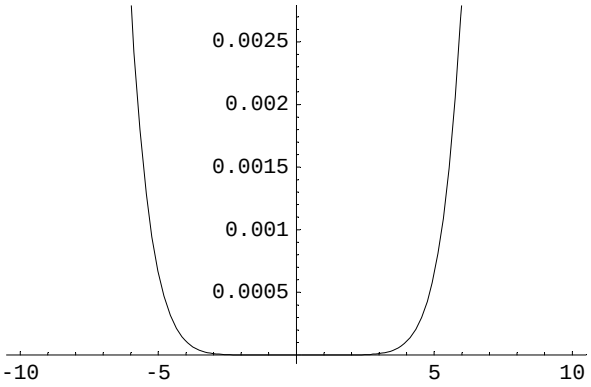
```
N[%]
```

$$9. - 0.045\, x^2 - 0.0003375\, x^4 - 1.0125 \times 10^{-6}\, x^6$$

```
f[x] - q[x] /. x → 10.
```

$$0.180162$$

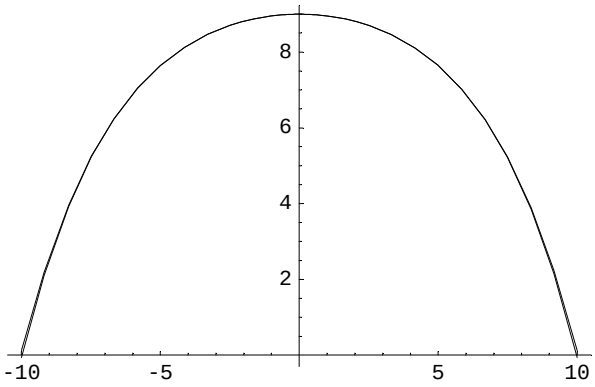
```
Plot[Evaluate[{f[x] - q[x]}], {x, -10, 10}];
```



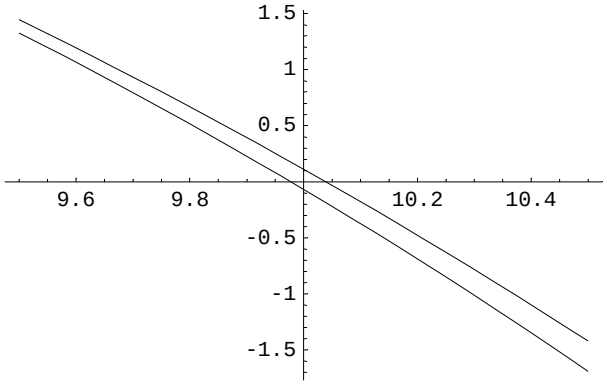

```
f[x] - q[x] /. x → ns
```

$$0.176849$$

```
Plot[Evaluate[{f[x], q[x]}], {x, -10, 10}];
```




```
Plot[Evaluate[{f[x], q[x]}], {x, 9.5, 10.5}];
```



6

```
Remove["Global`*"]
```

a

```
DSolve[{y'[x] == x^2 / 2 y[x], y[0] == 1}, {y}, x]
```

$$\left\{\left\{y \to \text{Function}\left[\{x\}, e^{\frac{x^3}{6}}\right]\right\}\right\}$$

```
e^(x^3/6) /. x → 1
```

$$e^{1/6}$$

```
N[%]
```

1.18136

b

```
p[x_, a_, b_, c_, d_] := a x^3 + b x^2 + c x + d
```

```
D[p[x, a, b, c, d], {x}] /. x → 0
```

c

```
D[p[x, a, b, c, d], {x}] /. x → -1
```

3 a − 2 b + c

```
{p[-2, a, b, c, d] == 0, p[0, a, b, c, d] == 1, (D[p[x, a, b, c, d], {x}] /. x → 0) == 0,
 (D[p[x, a, b, c, d], {x}] /. x → -1) == 0}
```

{−8 a + 4 b − 2 c + d == 0, d == 1, c == 0, 3 a − 2 b + c == 0}

```
Solve[{-8 a + 4 b == 0, 3 a - 2 b == 0
 }, {a, b}]
```

{{a → 0, b → 0}}

```
solv = Solve[
   {p[-2, a, b, c, d] == 0, p[0, a, b, c, d] == 1, (D[p[x, a, b, c, d], {x}] /. x → 0) == 0,
    (D[p[x, a, b, c, d], {x}] /. x → -1) == 0}, {a, b, c, d}] // Flatten
```

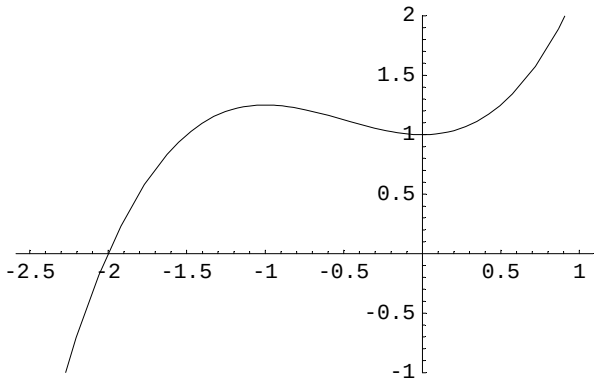
$\left\{a \to \frac{1}{2}, b \to \frac{3}{4}, c \to 0, d \to 1\right\}$

```
N[%]
```

{a → 0.5, b → 0.75, c → 0., d → 1.}

```
p[x_] := p[x, a, b, c, d] /. solv
```

```
Plot[p[x], {x, -2.5, 1}, PlotRange → {-1, 2}];
```




```
p'[x]
```

$\frac{3\,x}{2} + \frac{3\,x^2}{2}$

```
N[%]
```

1.5 x + 1.5 x^2

```
p'[x] /. x → -2
```

3

```
ArcTan[(p'[x] /. x → -2)] / Degree // N
```

```
71.5651
```