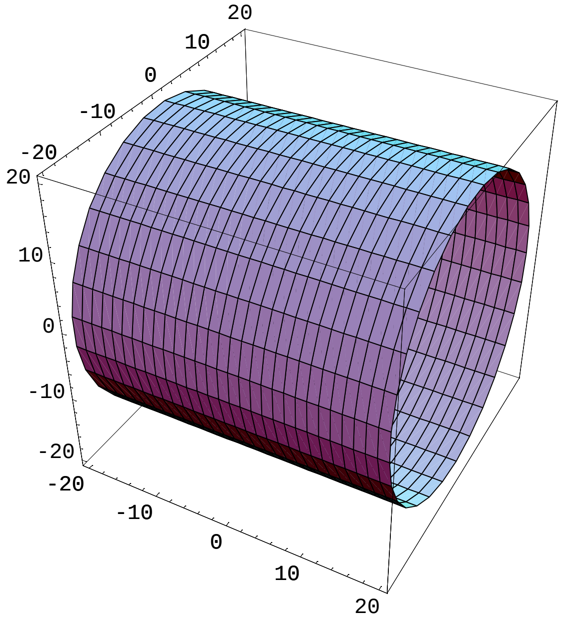
Lösungen Teil 1

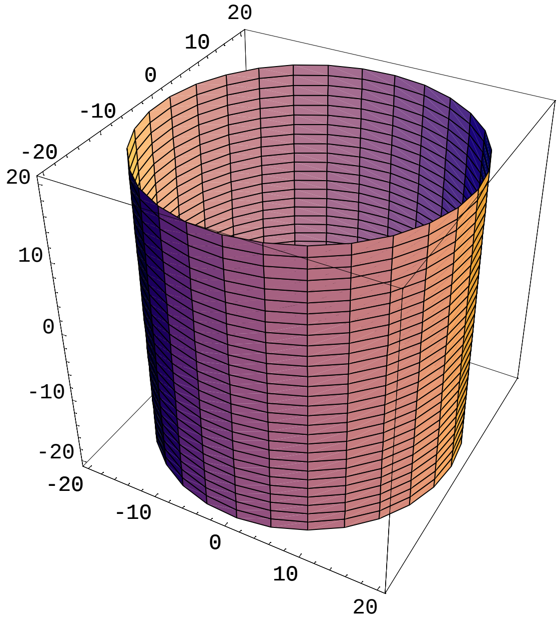
1

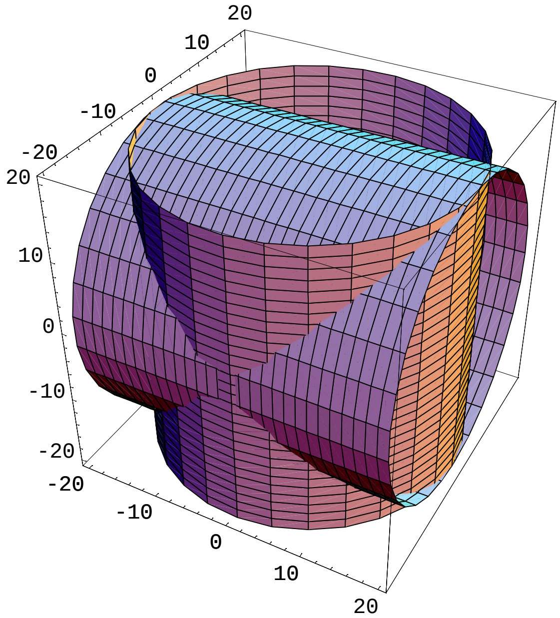
```
Remove["Global`*"]
```

a

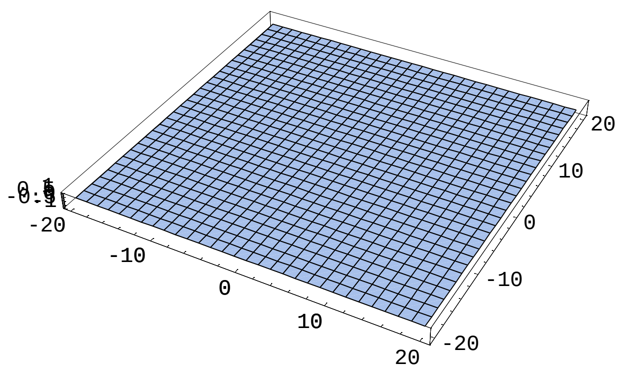
```
f1[r_, φ_, x_] := {x, r Cos[φ], r Sin[φ]};
r = 20;
p1 = ParametricPlot3D[f1[r, φ, x], {φ, 0, 2 Pi}, { x, -r, r}];
f2[r_, φ_, z_] := {r Cos[φ], r Sin[φ], z};
p2 = ParametricPlot3D[f2[r, φ, z], {φ, 0, 2 Pi}, { z, -r, r}];
Show[p1, p2];
```

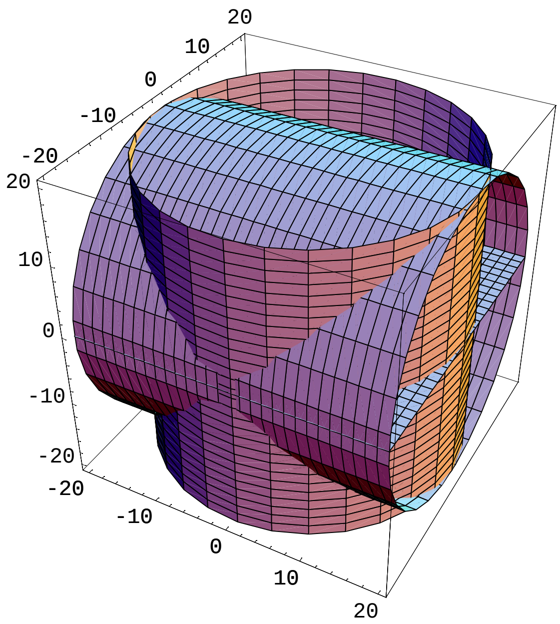




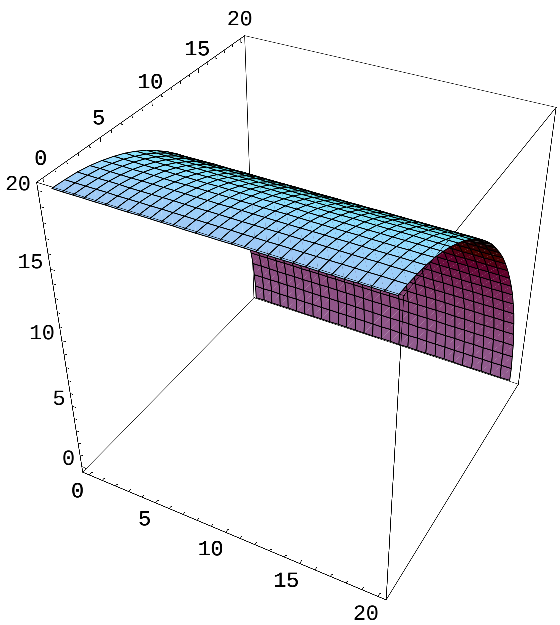


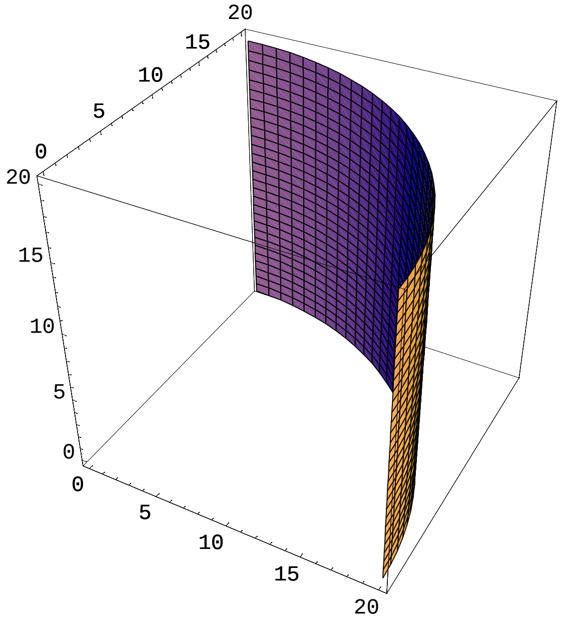

```
f3[x_, y_, z_] := {x, y, 0};
p3 = ParametricPlot3D[f3[x, y, z], {x, -r, r}, { y, -r, r}];
Show[p1, p2, p3];
```

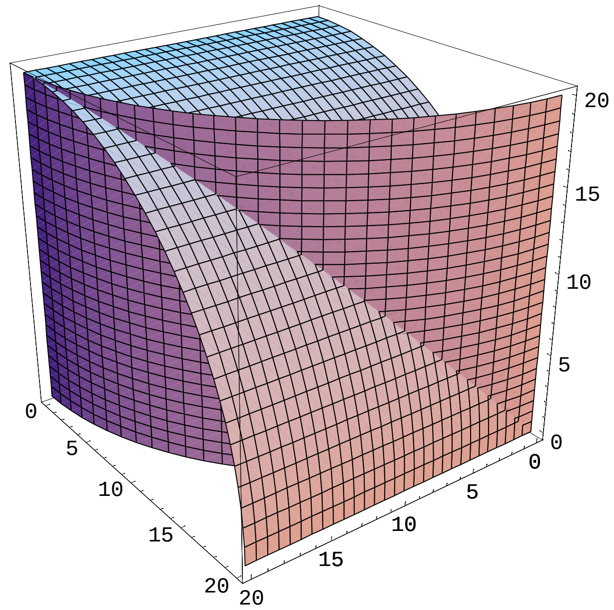





```
f1[r_, φ_, x_] := {x, r Cos[φ], r Sin[φ]};
r = 20;
p1 = ParametricPlot3D[f1[r, φ, x], {φ, 0, Pi / 2}, { x, 0, r}];
f2[r_, φ_, z_] := {r Cos[φ], r Sin[φ], z};
p2 = ParametricPlot3D[f2[r, φ, z], {φ, 0, Pi / 2}, { z, 0, r}];
Show[p1, p2, ViewPoint -> {1.848, 2.455, 1.418}];
```

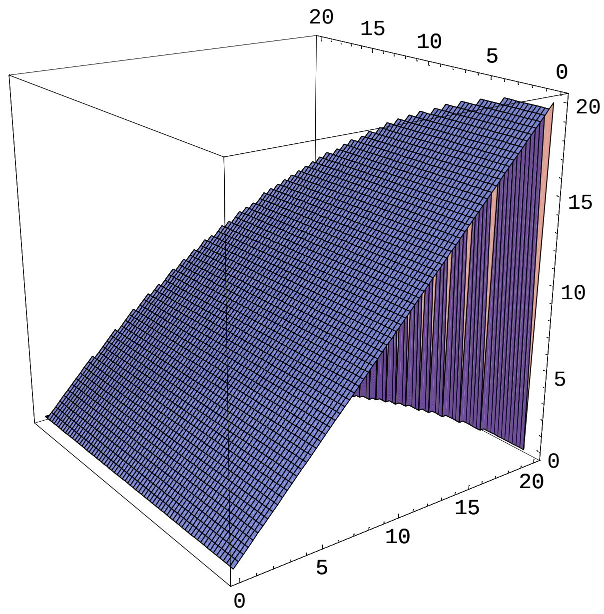


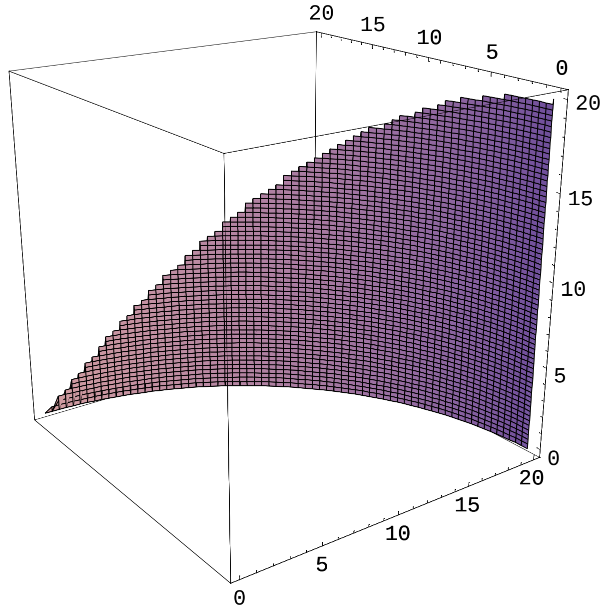




```
Remove["Global`*"]

r = 20;
f1New[r_, x_, y_] := {x, y, 0} /; x^2 + y^2 > r^2;
f1New[r_, x_, y_] := {x, y, x} /; x^2 + y^2 ≤ r^2 ;
f2New[r_, φ_, z_] := {r Cos[φ], r Sin[φ], 0} /; z > r Cos[φ];
f2New[r_, φ_, z_] := {r Cos[φ], r Sin[φ], z} /; z ≤ r Cos[φ];
p1 = ParametricPlot3D[f1New[r, x, y], {x, 0, r},
   { y, 0, r}, ViewPoint -> {-1.867, -2.569, 1.168}, PlotPoints → 70];
p2 = ParametricPlot3D[f2New[r, φ, z], {φ, 0, Pi / 2}, { z, 0, r},
   ViewPoint -> {-1.867, -2.569, 1.168}, PlotPoints → 70];
```





b

Der gesuchte Körper besteht aus 8 mal 2 mal sochen kongruenten Teilen.

```
Integrate[1, {z, 0, x}]
```

x

```
r = 20;
int =
 Integrate[Integrate[Integrate[1, {z, 0, x}], {x, 0, Sqrt[20^2 - y^2]}], {y, 0, r}]
```

$\frac{8000}{3}$

```
N[%]
```

2666.67

```
V = 2 * 8 * int
```

$\frac{128000}{3}$

```
N[%]
```

42666.7

```
intN = 2 * 8 * NIntegrate[1, {y, 0, r}, {x, 0, Sqrt[20^2 - y^2]}, {z, 0, x}]
```

42666.7

2

```
Remove["Global`*"]
```

a

```
LaplaceTransform[Sin [3 t] + Sinh [5 t], t, s]
```

$\frac{5}{-25 + s^2} + \frac{3}{9 + s^2}$

```
% // Together
```

$\frac{2 (-15 + 4 s^2)}{(-25 + s^2) (9 + s^2)}$

```
% // ExpandAll // Together
```

$\frac{2 (-15 + 4 s^2)}{-225 - 16 s^2 + s^4}$

```
% // Apart
```

$\frac{1}{2 (-5 + s)} - \frac{1}{2 (5 + s)} + \frac{3}{9 + s^2}$

b

```
LaplaceTransform[E^(t - 3) * E^(t + 3), t, s]
```

$\frac{1}{-2 + s}$

c

```
LaplaceTransform[ⅇ^t (1 + Sin[t]), t, s]
```

$$\frac{1}{-1+s}+\frac{1}{2-2s+s^2}$$

```
% // Together
```

$$\frac{1-s+s^2}{(-1+s)(2-2s+s^2)}$$

```
% // ExpandAll // Together
```

$$\frac{1-s+s^2}{-2+4s-3s^2+s^3}$$

d

```
LaplaceTransform[t/(1 + t^2), t, s]
```

$$-\mathrm{Cos}[s]\,\mathrm{CosIntegral}[s]+\frac{1}{2}\mathrm{Sin}[s]\,(\pi-2\,\mathrm{SinIntegral}[s])$$

e

```
LaplaceTransform[DiracDelta[t] + (1 + t)^2, t, s]
```

$$1+\frac{2}{s^3}+\frac{2}{s^2}+\frac{1}{s}$$

```
% // Together
```

$$\frac{2+2s+s^2+s^3}{s^3}$$

```
% // Factor
```

$$\frac{(1+s)(2+s^2)}{s^3}$$

f

```
InverseLaplaceTransform[2 s / (4 s^2 + 1), s, t]
```

$$\frac{1}{2}\mathrm{Cos}\left[\frac{t}{2}\right]$$

g

```
InverseLaplaceTransform[(s^2) / (s^2 + 1), s, t]
```

```
DiracDelta[t] - Sin[t]
```

h

```
InverseLaplaceTransform[1 / s + 1 / s^2 + 1 / s^3, s, t]
```

$1 + t + \frac{t^2}{2}$

i

```
InverseLaplaceTransform[E^(-3) / (s - 1) + E^3 / (s - 1), s, t]
```

$e^{-3+t}\,(1 + e^6)$

```
% // ExpandAll
```

$e^{-3+t} + e^{3+t}$

k

$$\text{InverseLaplaceTransform}\left[\frac{2}{s^3} + \frac{1}{1+s^2} + \frac{4}{s+s^2}, s, t\right]$$

$4 - 4\,e^{-t} + t^2 + \text{Sin}[t]$

3

```
Remove["Global`*"]
```

a

```
DSolve[y'''[t] - 3 y''[t] + 3 y'[t] - y[t] == E^(-t), y, t]
```

$$\left\{\left\{y \to \text{Function}\left[\{t\}, -\frac{e^{-t}}{8} + e^t\,C[1] + e^t\,t\,C[2] + e^t\,t^2\,C[3]\right]\right\}\right\}$$

$$f[t\_, C1\_, C2\_, C3\_] := -\frac{e^{-t}}{8} + e^t\,C1 + e^t\,t\,C2 + e^t\,t^2\,C3;$$

```
f[t, C[1], C[2], C[3]]
```

$$-\frac{e^{-t}}{8} + e^t\,C[1] + e^t\,t\,C[2] + e^t\,t^2\,C[3]$$

```
{f[t, C[1], C[2], C[3]] == 0, Evaluate[D[f[t, C[1], C[2], C[3]], t]] == 0,
  Evaluate[D[f[t, C[1], C[2], C[3]], {t, 2}]] == 0} /. t → 0
```

$$\left\{-\frac{1}{8} + C[1] == 0,\ \frac{1}{8} + C[1] + C[2] == 0,\ -\frac{1}{8} + C[1] + 2\,C[2] + 2\,C[3] == 0\right\}$$

```
solv = Solve[{-1/8 + C[1] == 0, 1/8 + C[1] + C[2] == 0, -1/8 + C[1] + 2 C[2] + 2 C[3] == 0},
   {C[1], C[2], C[3]}] // Flatten
```

$\{C[1] \to \frac{1}{8},\ C[2] \to -\frac{1}{4},\ C[3] \to \frac{1}{4}\}$

Eine einzige Lösung bei diesen Bedingungen

b

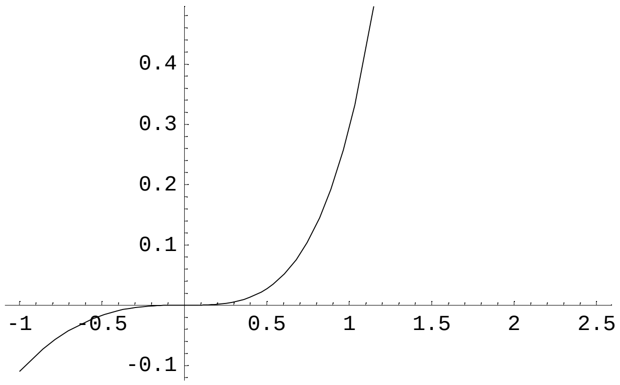
```
f[t_] := f[t, C[1], C[2], C[3]] /. solv; f[t]
```

$-\frac{e^{-t}}{8} + \frac{e^{t}}{8} - \frac{e^{t}\,t}{4} + \frac{e^{t}\,t^2}{4}$

```
% // Simplify
```

$\frac{1}{8} e^{-t} \left(-1 + e^{2t} \left(1 - 2t + 2t^2\right)\right)$

```
Plot[f[t], {t, -1, 2.5}];
```



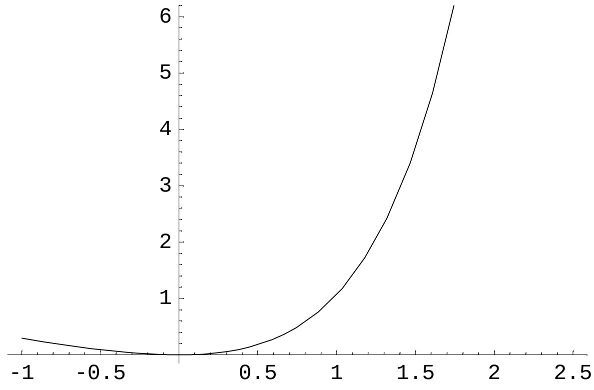
c

```
Evaluate[D[f[t], t]]
```

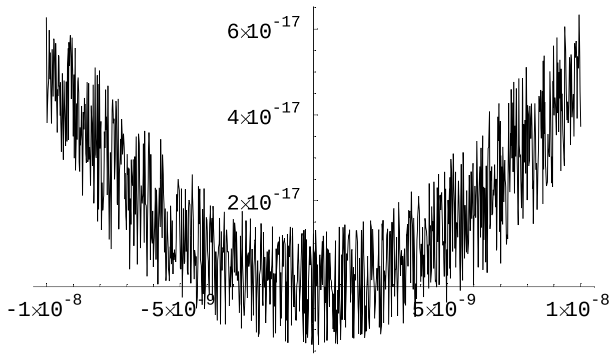
$\frac{e^{-t}}{8} - \frac{e^{t}}{8} + \frac{e^{t}\,t}{4} + \frac{e^{t}\,t^2}{4}$

Steigt mindestens exponentiell

```
Plot[(Evaluate[D[f[t], t]] /. t → t1), {t1, -1, 2.5}];
```




```
Plot[(Evaluate[D[f[t], t]] /. t → t1), {t1, -10^-8, 10^-8}];
```



Präzision muss höher gestellt werden - wird hier des Aufwandes wegen unterlassen

```
FindRoot[Evaluate[D[f[t], t]] == 0, {t, 0}]
```

{t → 0.}

```
FindRoot[Evaluate[D[f[t], t]] == 0, {t, -1}]
```

$\{t \to -9.72491 \times 10^{-9}\}$

Problem der schwach eingestellten Präzision ==> Abbruch der Iteration im Bereich von 10^{-8}

```
FindRoot[Evaluate[D[f[t], t]] == 0, {t, 1}]
```

$\{t \to 7.16076 \times 10^{-9}\}$

Problem der schwach eingestellten Präzision ==> Abbruch der Iteration im Bereich von 10^{-8}

Resultat: Nur ein Wendepunkt - wegen der Monotonie

d

```
d = NIntegrate[Sqrt[1 + ((Evaluate[D[f[t], t]] /. t → t1))^2], {t1, -1, 2.5}]
```

14.9074

```
d = NIntegrate[Sqrt[1 + ((Evaluate[D[f[t], t]] /. t → t1)) ^2], {t1, -1, 6}]
```

3078.13

e

```
loes = DSolve[{y'''[t] - 3 y''[t] + 3 y'[t] - y[t] == t^2,
    y[0] == -1, y'[0] == 0, y''[0] == 0}, y, t] // Flatten
```

$\left\{y \to \text{Function}\left[\{t\},\ \frac{1}{2}\left(-24 + 22\,e^t - 12\,t - 10\,e^t\,t - 2\,t^2 + e^t\,t^2\right)\right]\right\}$

```
yLoes[t_] := y[t] /. loes;  yLoes[t]
```

$\frac{1}{2}\left(-24 + 22\,e^t - 12\,t - 10\,e^t\,t - 2\,t^2 + e^t\,t^2\right)$

```
yLoes[t] // Expand // Simplify
```

$-12 - 6\,t - t^2 + \frac{1}{2}\,e^t\left(22 - 10\,t + t^2\right)$

4

```
Remove["Global`*"]
```

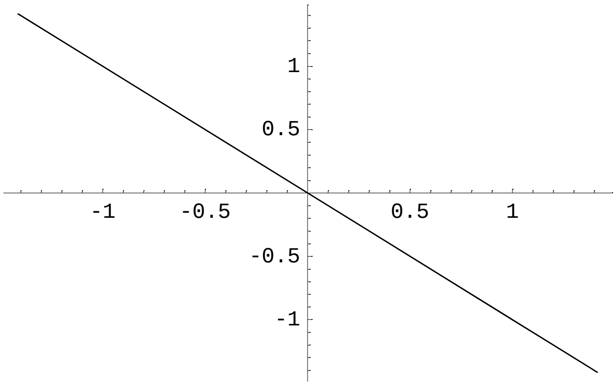
a

```
DSolve[
 {x''[t] - y[t] == 0, y[t] - x'[t] == 2 Sin[t], x[0] == 1, y[0] == -1, x'[0] == -1}, {y, x}, t]
```

{{x → Function[{t}, Cos[t] - Sin[t]], y → Function[{t}, -Cos[t] + Sin[t]]}}

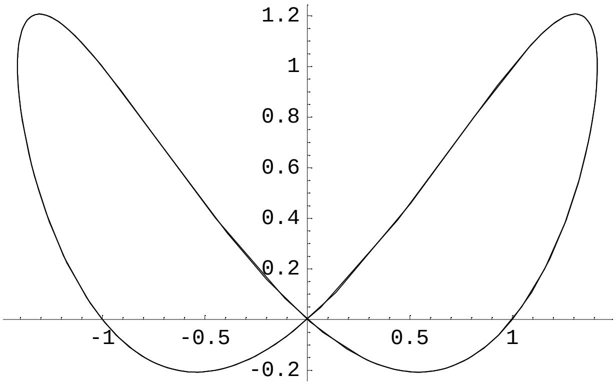
```
x[t_] := Cos[t] - Sin[t];  y[t_] := -Cos[t] + Sin[t];
```

```
v[t_] := {x[t], y[t]};
ParametricPlot[v[t], {t, 0, 4 Pi}];
```



b

```
v[t_] := {x[t], Sin[t] y[t]};
ParametricPlot[v[t], {t, 0, 4 Pi}];
```



5

```
Remove["Global`*"]
```

a

```
Integrate[ (2 x - y) , {x, 0, 2 Pi}, {y, 0, 4 Pi}]
```

0

```
Integrate[ (2 x - y) + Sin[x y], {x, 0, 2 Pi}, {y, 0, 4 Pi}]
```

EulerGamma - CosIntegral[8 π^2] + Log[8] + 2 Log[π]

```
NIntegrate[ (2 x - y) + Sin[x y], {x, 0, 2 Pi}, {y, 0, 4 Pi}]
```

4.9511

```
Integrate[(2 x - y) + Sin[x + y], {x, 0, 2 Pi}, {y, 0, 4 Pi}]
```

0

```
NIntegrate[(2 x - y) + Sin[x + y], {x, 0, 2 Pi}, {y, 0, 4 Pi}]
```

2.84217×10^{-14}

```
% // Chop
```

0

6

```
Remove["Global`*"]
```

```
v'[t] == g - c / m v[t]^2
```

$$v'[t] == g - \frac{c\, v[t]^2}{m}$$

```
c == cw ϱ A / 2
```

$$c == \frac{A\, cw\, \varrho}{2}$$

```
v'[Inf] == 0 == g - c / m v[Inf]^2
```

$$v'[Inf] == 0 == g - \frac{c\, v[Inf]^2}{m}$$

```
Solve[0 == g - c / m v[Inf]^2, { v[Inf]}]
```

$$\left\{\left\{v[Inf] \to -\frac{\sqrt{g}\,\sqrt{m}}{\sqrt{c}}\right\}, \left\{v[Inf] \to \frac{\sqrt{g}\,\sqrt{m}}{\sqrt{c}}\right\}\right\}$$

$$\mathbf{v[Inf] == \frac{\sqrt{g}\,\sqrt{m}}{\sqrt{c}};\ v[Inf]\hat{}2 == \left(\frac{\sqrt{g}\,\sqrt{m}}{\sqrt{c}}\right)\hat{}2}$$

$$v[Inf]^2 == \frac{g\, m}{c}$$

$$\mathbf{Evaluate\left[v[Inf]^2 == \left(\frac{g\, m}{c}\ /.\ (m / c) \to 1 / k\right)\right]}$$

$$v[Inf]^2 == \frac{g}{k}$$

$$\mathbf{Solve\left[Evaluate\left[v[Inf]^2 == \left(\frac{g\, m}{c}\ /.\ (m / c) \to 1 / k\right)\right],\ \{k\}\right]}$$

$$\left\{\left\{k \to \frac{g}{v[Inf]^2}\right\}\right\}$$

$$\mathbf{v'[t] == g - \frac{g}{vInf^2}\ v[t]\hat{}2}$$

$$v'[t] == g - \frac{g\, v[t]^2}{vInf^2}$$

```
Solve[0 == g - c / m vInf^2, {vInf}]
```

$$\left\{\left\{vInf \to -\frac{\sqrt{g}\,\sqrt{m}}{\sqrt{c}}\right\}, \left\{vInf \to \frac{\sqrt{g}\,\sqrt{m}}{\sqrt{c}}\right\}\right\}$$

$$\mathbf{DSolve\left[v'[t] == g - \frac{g}{vInf^2}\ v[t]\hat{}2,\ v,\ t\right]}$$

$$\left\{\left\{v \to Function\left[\{t\},\ vInf\, Tanh\left[\frac{g\, t}{vInf} + vInf\, C[1]\right]\right]\right\}\right\}$$

```
solv = DSolve[v'[t] == g - g/vInf^2 v[t]^2, v, t] /. vInf -> (Sqrt[g] Sqrt[m])/Sqrt[c] // Simplify // Flatten
```

$$\left\{v\to \text{Function}\left[\{t\},\ \frac{\left(\sqrt{g}\,\sqrt{m}\right)\text{Tanh}\left[\frac{g\,t}{\frac{\sqrt{g}\,\sqrt{m}}{\sqrt{c}}}+\frac{\left(\sqrt{g}\,\sqrt{m}\right)C[1]}{\sqrt{c}}\right]}{\sqrt{c}}\right]\right\}$$

```
vNeu[t_] := v[t] /. solv; vNeu[t]
```

$$\frac{\sqrt{g}\,\sqrt{m}\,\text{Tanh}\left[\frac{\sqrt{c}\,\sqrt{g}\,t}{\sqrt{m}}+\frac{\sqrt{g}\,\sqrt{m}\,C[1]}{\sqrt{c}}\right]}{\sqrt{c}}$$

```
vNeu[t_] := Simplify[%] /. C[1] -> 0; vNeu[t]
```

$$\frac{\sqrt{g}\,\sqrt{m}\,\text{Tanh}\left[\frac{\sqrt{c}\,\sqrt{g}\,t}{\sqrt{m}}\right]}{\sqrt{c}}$$

```
h = 1000; g = 9.81; m = 100; cw = 1.33; ϱ = 1.2; A = 25; c = cw ϱ A / 2;
```

```
DSolve[v'[t] == g - g/vInf^2 v[t]^2, v, t] /. vInf -> (Sqrt[g] Sqrt[m])/Sqrt[c] // Simplify
```

$$\left\{\left\{v\to \text{Function}\left[\{t\},\ 7.01234\,\text{Tanh}\left[0.01\left(\frac{981.\,t}{7.01234}+100.\ 7.01234\,C[1]\right)\right]\right]\right\}\right\}$$

```
Solve[(7.012341429995196` Tanh[
    0.01` (981.` t/7.012341429995196` + 100.` 7.012341429995196` C[1])] /. t -> 0) == 0, {C[1]}]
```

$$\{\{C[1]\to 0.\}\}$$

```
(DSolve[v'[t] == g - g/vInf^2 v[t]^2, v, t] /. {vInf -> (Sqrt[g] Sqrt[m])/Sqrt[c], C[1] -> 0}) // Simplify
```

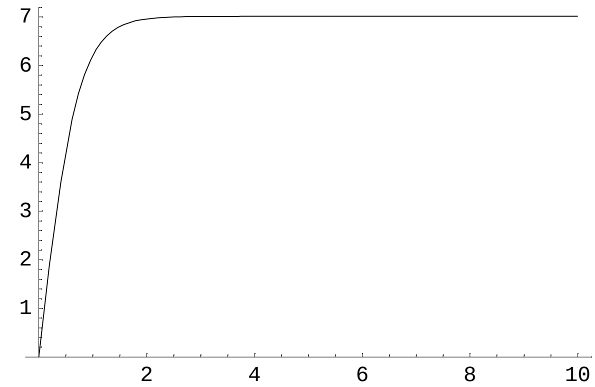
$$\left\{\left\{v\to \text{Function}\left[\{t\},\ 7.01234\,\text{Tanh}\left[0.01\left(\frac{981.\,t}{7.01234}+100.\ 7.01234\ 0\right)\right]\right]\right\}\right\}$$

```
v[t_] := 7.012341429995196`
   Tanh[0.01` (981.` t/7.012341429995196` + 100.` 7.012341429995196` 0)] // Chop
```

```
v[t]
```

$$7.01234\,\text{Tanh}[1.39896\,t]$$

```
Plot[v[t], {t, 0, 10}, PlotRange → {0, 7.2}];
```



b

```
sNeu[t_] := h - Integrate[vNeu[t1], {t1, 0, t},
   Assumptions → {Element[t1, Reals], Element[t, Reals] && t > 0}]; sNeu[t]
```

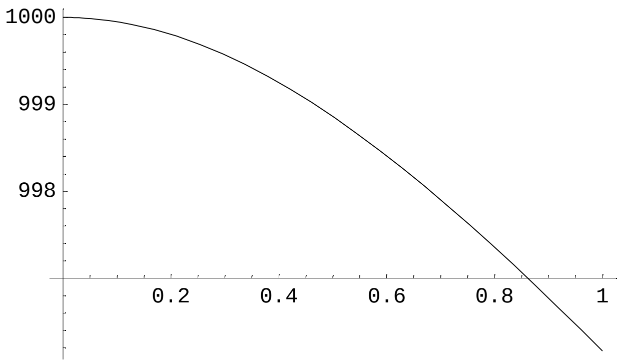
```
1000 - t (- Graphics -)
```

```
s[t_] := h - Integrate[v[t1], {t1, 0, t},
   Assumptions → {Element[t1, Reals], Element[t, Reals] && t > 0}]; s[t]
```

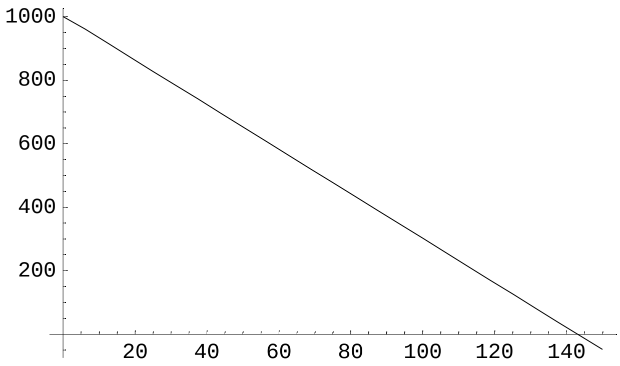
```
1000. - 5.01253 Log[Cosh[1.39896 t]]
```

```
s[t_] := 1000.` - 5.012531328320802` Log[Cosh[1.3989621152840417` t]]
```

```
Plot[s[t], {t, 0, 1}];
```



```
Plot[s[t], {t, 0, 150}];
```



c

```
FindRoot[s[t] == 0, {t, 150}]
```

```
{t → 143.101}
```

7

```
T[x_, t_] := 40 - 4 x;
T[0, t]
```

```
40
```

```
T[10, t]
```

```
0
```

```
T[x, t]
```

```
40 - 4 x
```

Unabhängig von t

```
D[T[x, t], t]
```

```
0
```

```
D[T[x, t], {x, 2}]
```

```
0
```

```
D[T[x, t], t] == D[T[x, t], {x, 2}]
```

```
True
```

Lösung gefunden: T[x,t]=40-4 x

Lösungen Teil 2

1

```
Remove["Global`*"]
```

a

```
120.8 == 1 / 500 (sumFix + 76 110)
```

$$120.8 == \frac{8360 + \text{sumFix}}{500}$$

```
solv1 = Solve[120.8 == 1 / 500 (sumFix + 76 110), {sumFix}] // Flatten;
sumFix = sumFix /. solv1
```

```
52040.
```

```
mittelwert = 1 / 500 (sumFix + 76 130)
```

```
123.84
```

b

```
(500 - 1) / 500 6.24^2 == 1 / 500 (QS +  76 110^2) - 120.8^2
```

$$38.8597 == -14592.6 + \frac{919600 + \text{QS}}{500}$$

```
solv2 = Solve[(500 - 1) / 500 6.24^2 == 1 / 500 (QS +  76 110^2) - 120.8^2, {QS}] // Flatten;
QS = QS /. solv2
```

6.39615×10^{6}

```
s = Sqrt[1 / (500 - 1) (QS  + 76 130^2) - 123.84`^2]
```

```
7.45237
```

2

Der Fall 4 mit 0 wird gestrichen. 9 mal kommt "+" vor und 2 mal "-" von den 11 Fällen.
P(X   9) = 1 - P(X   8) := P,   p=q=0.5

```
p = 0.5; q = p;
P = 1 - Sum[Binomial[11, k] p^k q^(11 - k), {k, 0, 8}]
```

```
0.0327148
```

P = 0.0327148... ist hier nicht kleiner als = 0.01. Daher kann H1 nicht auf signifgikante Weise angenommen werden. H0 muss daher belassen werden.
Zum Beispiel zu = 0.035. Könne man H1 auf signifgikante Weise annehmen. H0 müsste man dann ablehnen.

3

```
Remove["Global`*"]
```

```
dat1 = {45, 7, 4, 49, 11, 7, 7, 5, 7, 41, 53, 1, 4, 41, 41, 22, 21, 2, 8, 2};
dat2 = {27, 44, 4, 1, 16, 4, 12, 49, 2, 23, 68, 3, 13, 13, 50, 4, 42, 7, 27, 1};
```

a

```
Mean[dat1] // N
```

```
18.9
```

```
Mean[dat2] // N
```

```
20.5
```

Beanstandung nicht nachvollziehbar

b

```
StandardDeviation[dat1] // N
```

```
18.5043
```

```
StandardDeviation[dat2] // N
```

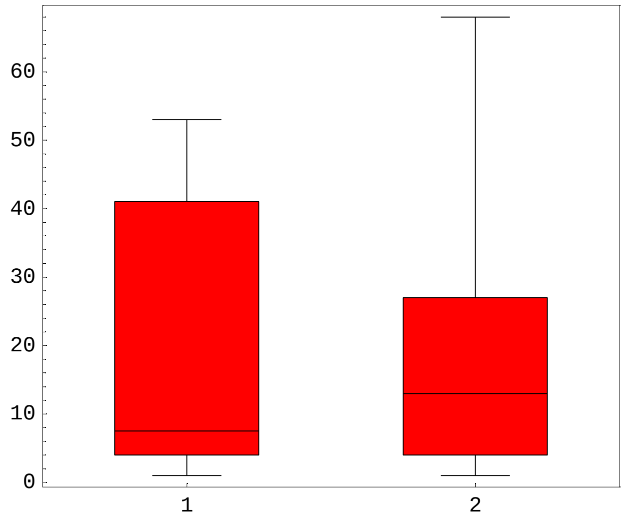
```
20.0749
```

Beanstandung nicht nachvollziehbar

c

```
<< Statistics`StatisticsPlots`
```

```
BoxWhiskerPlot[Transpose[{dat1, dat2}]];
```




```
{Median[dat1], Median[dat2]} // N
```

```
{7.5, 13.}
```

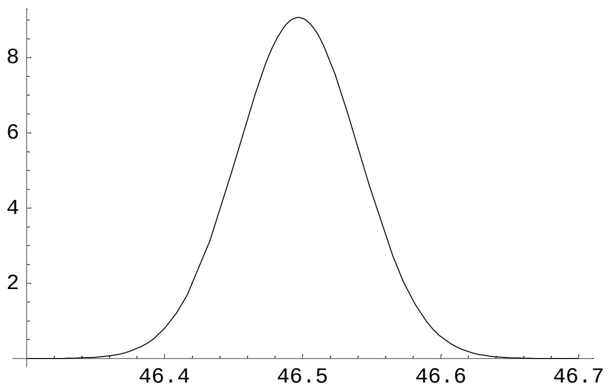
Im BoxWhiskerPlotsieht man, dass die Datensätze sehr ungleich streuen. Dazu ist der Median des einen Datensatzes fast doppelt so gross wie der andere!

4

```
Remove["Global`*"]
```

a

```
a0 = 46.50; Δa0 = 0.01;  a1 = 46.497; s = 0.044;
φ[x_, μ_, s_] := 1 / (s Sqrt[2 Pi]) E^(-(x - μ)^2 / (2 s^2));
schranke = a0 + 2 Δa0;
Plot[φ[x, a1, s], {x, 46.3, 46.7}];
```




```
test = Integrate[φ[x, a1, s], {x, -100, 200}]
```

```
1.
```

```
WahrscheinlichkeitAusschuss = Integrate[φ[x, a1, s], {x, a0 + 2 Δa0, Infinity}]
```

```
0.300582
```

b

```
FindRoot[Integrate[φ[x, a1, sNeu], {x, a0 + 2 Δa0, Infinity}] == 0.01, {sNeu, 0.01}]
```

```
{sNeu → 0.00988674}
```

0.00988...ist von einer anderen Grössenordnung als 0.044. Eine Genauigkeit derart zu erhähen wird wohl mit grossen Kosten verbunden sein.

c

```
FindRoot[Integrate[φ[x, a1Neu, s], {x, a0 + 2 Δa0, Infinity}] == 0.01, {a1Neu, 46.0}]
```

```
{a1Neu → 46.4176}
```

46.4176...ist von einer ähnlicher Grössenordnung wie 46.497. Hier liesse sich vielleicht was machen.

5

```
Remove["Global`*"]
```

a

```
N[1 - (364 / 365) ^365] (* Bei 365 *)
```

```
0.632625
```

```
N[1 - (364 / 365) ^20] (* Bei 20 *)
```

```
0.0533915
```

b

```
P = 1 - (1 - 0.03) ^20
```

```
0.456206
```

c

```
p = 1 / 20; q = 1 - p;
PXk[r_, k_] = Binomial[r + k - 1, r - 1] p^r q^k;
PXk[1, 0] + PXk[1, 1] + PXk[1, 2] + PXk[1, 3] // N
```

0.185494

```
{ PXk[1, 0], PXk[1, 1], PXk[1, 2], PXk[1, 3]} // N
```

{0.05, 0.0475, 0.045125, 0.0428688}

```
Sum[PXk[1, u], {u, 0, 20}] // N
```

0.659438

```
Sum[PXk[1, u], {u, 0, 40}] // N
```

0.877913

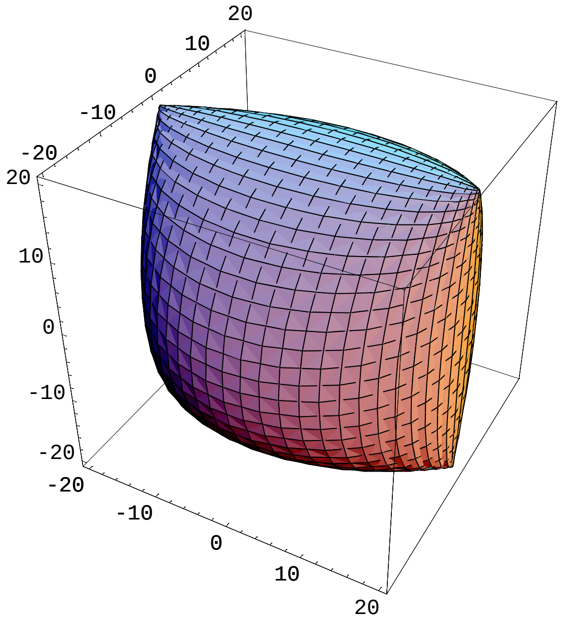
```
Sum[PXk[1, u], {u, 0, 100}] // N
```

0.994375

Work

Work Kissen

```
f[r_, φ_, ϑ_] := {r Sin[φ], r Cos[φ] Cos[ϑ], r Sin[ϑ] };
ParametricPlot3D[f[20, φ, ϑ], {φ, 0, 2 Pi}, {ϑ, 0, 2 Pi}];
```




```
ParametricPlot3D[f[20, φ, ϑ], {φ, 0, 2 Pi},
  {ϑ, 0, 2 Pi}, ViewPoint -> {2.537, -1.875, -0.304}];
```